Mathematical and philosophical manifesto, declaring numerous theorems, problems, postulates, corollaries, axioms, poropsitions, rules and facts, hitherto unknown in science, and naturally growing out of the extraordinary and most significant discovery of a lacking link in the demonstration of the world-renowned Pythagorean problem, utterly disproving its absolute truth, although demonstrated as such for twenty-three centuries; and by this discovery establishing the fact of the existence of perfect harmony between arithmetic and geometry as a law of nature; and calculated to settle forever the famous dispute between two great philosophical schools, By Theodore Faber.

Faber, Theodore.
New York, E. S. Dodge & co., 1872.

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Mathematical and Philosophical MANIFESTO,

DECLARING

NUMEROUS THEOREMS, PROBLEMS, POSTULATES, COROLLARIES, AXIOMS, PROPOSITIONS, RULES, AND FACTS, HITHERTO UNKNOWN IN SCIENCE, AND NATURALLY GROWING OUT OF THE

EXTRAORDINARY AND MOST SIGNIFICANT DISCOVERY

OF A

Lacking Link

IN THE DEMONSTRATION OF THE WORLD-RENOWNED

PYTHAGOREAN PROBLEM,

UTTERLY DISPROVING ITS ABSOLUTE TRUTH, ALTHOUGH DEMONSTRATED AS SUCH FOR TWENTY-THREE CENTURIES; AND BY THIS DISCOVERY ESTABLISHING THE FACT OF THE EXISTENCE OF

Perfect Harmony between Arithmetic and Geometry as a Law of Nature;

AND CALCULATED TO SETTLE FOREVER THE FAMOUS DISPUTE BETWEEN THE

TWO GREAT PHILOSOPHICAL SCHOOLS.

BY

THEODORE FABER,

A citizen of the United States of America, New York.

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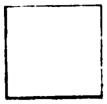
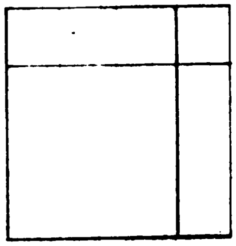
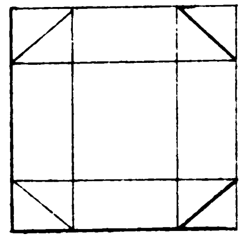
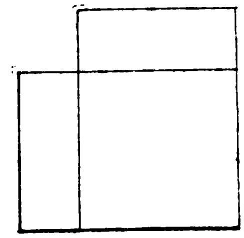
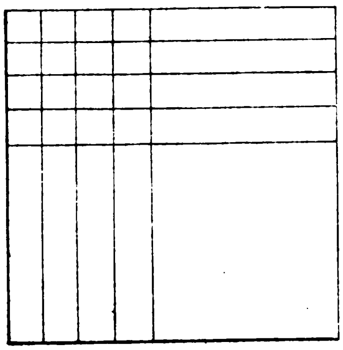
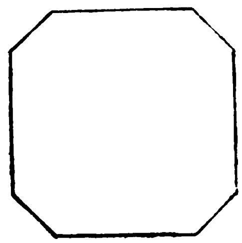
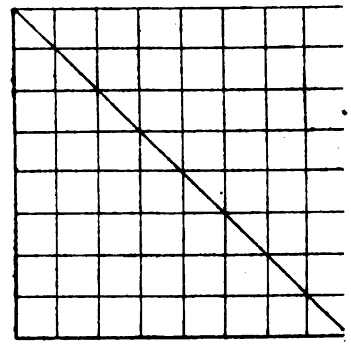
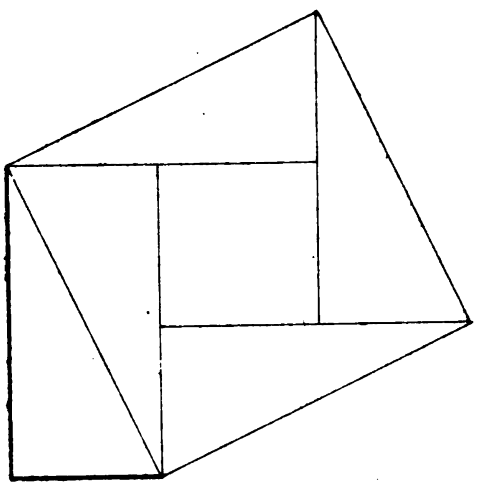
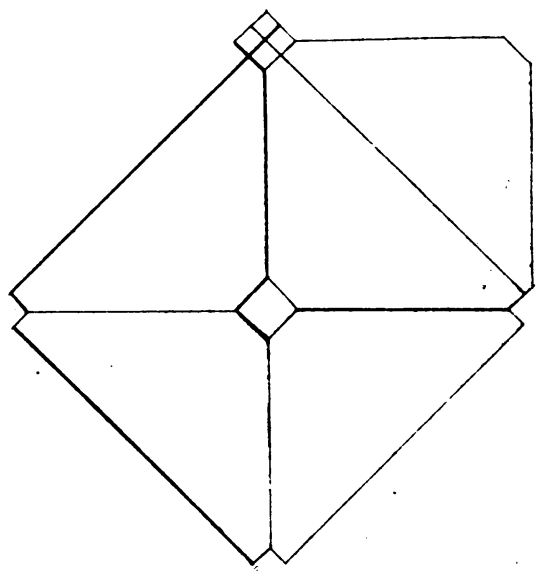
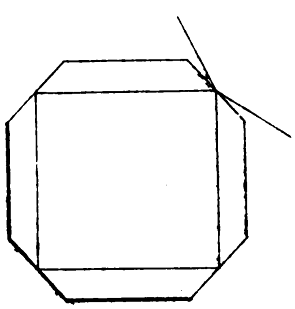
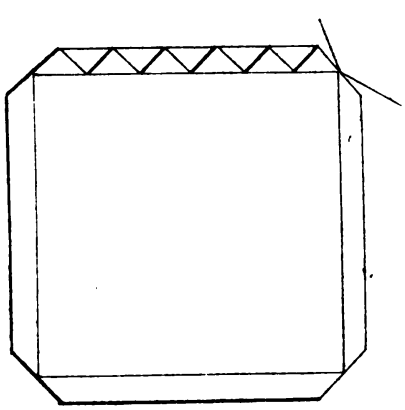
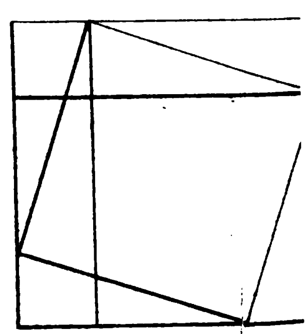

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A Citizen of the United States of America, New York.

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E. S. DODGE & CO., STEAM BOOK AND JOB PRINTERS.
No. 84 JOHN STREET.

1872.

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FULL AND EXPLICIT EXPLANATION

OF THE

Singularly Significant Discovery

OF

A LACKING LINK IN THE DEMONSTRATION OF THE WORLD-RENOWNED PYTHAGOREAN PROBLEM.

ANY man advancing a pretension like the above, is either a sage or a fool! If the former, he will reap a sage's reward, if the latter, a fool's reward. No man of sense and of constitutional diffidence, will expose himself to a fool's reward, and would rather cut off his little finger, than publicly advance mathematical doctrines, the truth of which he could not strictly mathematically demonstrate.

An error in the "positive and exact science!"—and that error remaining undiscovered for a period of 23 centuries—must indeed have been veiled by *a sophism of a most subtle nature!* Any claim to the solution of the mystery of "squares with infinite roots," equal to a paradox, will of course have to be so clear in its demonstration, as to brand any doubting Thomas of the science with blindness and stupidity. Truth is preferable to error; but, notwithstanding this trite saying, the mathematical world, at least, may for a long time resist a public acknowledgment of our discovery, from the consciousness of a certain measure of disgrace, that might be involved in the exclamation of the outside world, "Behold the positive and exact science convicted of error!" But

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disciples of Pythagoras and Euclid! Outside of the scarlet lady, that sits on the eternal seven hills, the infallibility doctrine has no advocates. Truth is an irresistible force, and will prevail! Accept the inevitable with good grace! We cordially, however, bear this testimony to the democratic spirit of the scientific republic, that we have met with but comparatively few of its members so "hide-bound" with prejudice in favor of ancient authority, as to refuse as much as even to look at our discovery, with the exlamation, "Preposterous, in the face of 23 centuries of demonstration!" We do not blame them; they are sturdy upholders of infallibility! Mathematicians may be divided into two classes—the philosophical and the simply practical; or spiritual and material. The first have a knowledge, or some conception of first mathematical principles, but the second often have none whatever, and learn the science only as the parrot learns language, by simple imitation. To the second, discoveries in the science would be barely possible, and mathematics has no other significance to them, than as it is applicable to mechanical use; while to the former class, mathematics is the "science of all created things," as the great philosopher Leibniz called it. Members of the first class have recognized the truth of our discovery almost at first sight. We rejoice already in numerous converts, and among them men of scientific eminence. We have yet to meet with the first attempt at the refutation of our doctrine, although we have repeatedly tried to provoke attack by public call. Hence we can invite readers with boldness of confidence to an examination of our claim of discovery.

"What is truth?" We have often expressed the opinion among friends, that none but a philospher could

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have put that question; and Pilate was no doubt greatly disappointed at receiving no answer. We have often attempted to answer this question to ourselves, but without fair success, until we discovered the truth, when we found that harmony is truth, and truth is harmony, and harmony is order, and order is the law of the Universe! When Plato poetically spoke of the music of the Spheres, he meant nothing but the harmony in the order and motions of the heavenly bodies. Any son of Adam and daughter of Eve, can have the privilege of hearing that celestial music. Harmony is bliss, and discord is misery, as every lover of music will allow. And man is happy only in so far as there is harmony with himself and all outside things. Truth then, which is harmony, is the pearl of great price, and the finders and teachers of it—and among them the philosphers and inventors—are the great benefactors of mankind. Truth too, is promotive of probity and order, and merchants make a great mistake in supposing that religion and philosophy has no connection with or relation to their avocation. The popularity of Young Men's Christian Associations, with their scientific and philosophical lectures, is a promising token for the correction of this mistake—calculated greatly to advance the interests, as well as to purify the moral atmosphere of commercial communities. Any member of such community, who is so singularly happy as to discover a philosophical truth of importance, confers honor on that community, and is deserving of its sympathy and support.

Truth is a unity, and it has even been hyperbolically said, that "a partial truth is a positive falsehood." Many philosophers have doubted man's ability to recognize truth in its unity; these are the skeptics. Others

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thought, if they could find but a single truth in its unity, it would afford a foundation for other truths. Twice 2 is 4 is a truth, that must of necessity be valid throughout the universe; and it is indeed a happy thing for us to know, that there is such a truth, insignificant as it may appear to ordinary minds. Numerical figures, in themselves, as mere symbolical representatives of quantity, do not possess the attributes of quantity, which are length, breadth and depth: Outside of their application to concrete objects, they have no significance, unless we call a number of thoughts, a quantity. "2 substracted from 2, is equal to nothing," is an ideal truth, but not a real truth, when applied to objects, since in the realm of objects, there is no such thing as nothing. Subtraction means separation, or removal of one object from another; it is another form of division. In division, the quotient does not represent a number of parts of the dividend, but only the *number of times* the divisor is contained in the dividend. We perceive then, that the number expressing the quotient, has a very different significance from the number expressing a remainder; the one expresses *times*, and the other *parts*. Annihilation of matter, as such, is not recognized in philosophy. When matter ceases to be matter, it is spirit, for spirit and matter are the only primary elements recognized in philosophy. What is not one, must be the other, and *vice versa*. We cannot divide 0 or zero, however much mathematicians may attempt such absurdity. Zero is the negation of quantity. To mathematicians, who try to puzzle us with the question, "What is the square root of — *a*?" we would respectfully suggest that, since a positive must exist in conception before a negative can be predicated of it, they call the *root* positive, but

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the *square* of the same, negative—thus, $\sqrt{-a} = \sqrt{a-}$; $\sqrt{a-}^2 = -a$. Quantity being always a finite conception, and the finite being ever divisible, that is, subject to measure and number, we cannot but see, that harmony between measure and number, that is, between geometry and arithmetic, must be *a law of nature*, since there is nothing indivisible in nature, excluding "God," or "space," from our meaning of this word, and the divisible is ever subject to measure and number. We have met with numerous mathematicians, who still stoutly believe in an indivisible line, and no wonder, when the Pythagorean problem itself, is based on that doctrine, and is the sole support of the same ; yet the great Newton already expressed his *doubts* in the existence of the indivisible in nature, as not in harmony with his reason ; but his simple *doubt* would to-day, under the light of our discovery, be quickly converted into *conviction!* The philosopher Berkeley, conscious of many inconsistencies in the science of mathematics, proposed numerous reforms. We mention these facts to the outside public, that they may understand that the term "positive and exact" is not applicable to the science as a whole, in its present form, and that there is room in it for discoveries! Mathematics is emphatically a science of measures and of numbers, and how the *indivisible line* found a place in it, transcends our comprehension. It is a purely ideal conception, not derived from any thing *visible*, except as its necessary opposite, hence we scarcely wonder at the idealistic philosophers calling it an innate, inborn idea, not derived through experience! But as the indivisible cannot possibly serve for a basis of a science of measures and of numbers, it is not one of the *first principles* of the science ; and space, being absolute, one and indivisible,

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cannot serve as a first principle of the science, nor can we have any innate, inborn idea of space, since the infant is apt to grasp after the moon for a toy, and therefore has not the slightest notion of *space*. "Good heavens!" We hear mathematicians exclaim, "What are we coming to? Has not the science all along taught, that pure mathematics deals only with abstract space?" Impossible! We cannot measure space *per se*, but only the things in space and their separation; and the use of the word "space," as a substitute for *surface*, to which alone measure and number are applicable, is highly improper. An object in space is not one quantity, and the space it occupies another quantity! Every measure, as such, is a quantity, and every quantity, as such, may serve as a measure; hence, there is harmony between quantity and measure, and the expression "incommensurable quantity," in itself, is a paradox! Surface, as such, having length and breadth only, is not quantity *per se*; but the cube measure unit is a quantity, and we use one of its surfaces for a measure of surfaces, since like can only be measured with like, and through the medium of the surfaces we measure quantities; hence we call the surface symbolically a quantity. Length, by itself, is not a quantity; a foot long, a yard long, a mile long, expresses only the *side* of a *square* standard measure unit. We image surfaces on paper by *lines*, to represent the boundaries of the surfaces; but a boundary away from its object has only an ideal, or abstract significance, and mathematics, dealing with these ideal surfaces and lines, is therefore called an abstract science. A "line," as representative of a boundary, is not quantitative, but it is quantitative as representative of *a square root*, consisting of a straight line of square units, as one line out of a

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number of equal divisions of a square surface, so that such *line* or *square root*, multiplied by itself, gives the number of square measure units contained in the square, and hence the "square root" may also be called the "side" of the square. We cannot multiply one side of a geometrical figure with another, without identifying the word "side" with "square root," since multiplication is only possible with *units*. $a+b$, as representative of a "line," is the inquantitative side of *one* unit, be that a triangle or a square, and as such, cannot be multiplied with a similar side, for 1×1, is not *multiplication!* We admire the French division of unit measures into "lines;" but *these* lines *are quantitative*, and have nothing about them calculated to mislead the mind into a paradox, as the "indivisible line" does. We perceive at once, that under this doctrine the side of a square cannot be coincident with the side of a right-angled triangle, while the Pythagorean problem *assumes* such coincidence, *without previous proof*, and bases its whole demonstration upon such coincidence, since it speaks of the *squares* of the *sides* of a right-angled *triangle*, in its very statement. How can the side of a triangle serve as a square root? The law of mathematical demonstration admits of no assumptions without proof, and we hold Pythagoras himself amenable to this law! Prove the coincidence, and we will believe in the truth of the famous problem, although it be equal to saying that twice 2 is 3! Will mathematicians exclaim, "What becomes, under your doctrine, of the center of the universe—our 'mathematical point?' How can we do without an indivisible point into which all the indivisible radii of the circle center?" What is the primeordeal

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form of matter, a cube or a sphere? We would not get far in the construction of a globe by beginning with a cube atom of matter for a center! Hence the center of a globe must necessarily be a spherical atom, and mathematicians will perceive at once that all radii from the periphery of the globe must strike the periphery of the central atom, and afford room for every one of the radii, however much divided; and we may conceive a central atom, within the central atom, to infinity of division, on account of the necessarily infinite divisibility of matter; we say, necessarily, because the moment matter ceases to be divisible, it would no longer be matter, but spirit of the absolute nature, one and indivisible. It is clear then, that infinite divisibility is the *sine qua non* condition of *individual* existence; and only the indivisible, the absolute, being entitled to the attribute of perfection, it follows, that imperfection is an inexorable law in individual existence, since without it, existence, as individuals, would be impossible. Can we not call God an individual? Certainly not! He is the sum of all individuality, and infinitely more! Is God a "person?" Yes, if we apply the word solely to consciousness and not to *Spirit*, nor to *Being*—two words in reality of identical meaning. We are *of* God, and in so far we are *Being*, and impersonal, as God is as Spirit; but consciousness is divisible, as involving *distinction*, and it is our consciousness, which constitutes us "persons," and it is God's consciousness, which constitutes Him a person. We *are* in God, but we *exist* outside—not of his Being, but of his consciousness; that is, we are excluded from the possession of His consciousness, while his consciousness includes our own, for to Him all thoughts are known.

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Hence, since there is really but one Being, and not several Beings, we rather improperly substitute that word for *existences.* The imperfect, or finite, is, of necessity dependant on the perfect or absolute infinite, as effect is dependent on cause, and *forms* are an *effect*, and mathematics has to do with forms, and nothing with their cause, unless it be simply to recognize the cause as necessarily existing, since there can be no effect without a cause. This existing, or rather *being*, by necessity, is *all* we "mathematically" know of the "First cause."

Before closing our remarks on the "first principles of mathematics," we wish to say something on the *origin* of a *line.* Mathematicians place the origin of a line in the motion of an indivisible point, without length, breadth and depth. How *such* an object can possibly be conceived to have *motion*, transcends altogether our comprehension. If we were to define *nothing*, although that word has in reality no predicate, we should just use the definition of a "mathematical point," and what philosopher would ascribe motion to *nothing!* What *basis* for a science of measures and of numbers! The legitimate basis of that science is a square unit, infinitely divisible, so that a line of infinitesimal square units, might even become invisible, without losing the characteristic of entity, and such a line should satisfy the idealist. But a square unit, being infinitely divisible in its angles, and, hence, not closing in an indivisible point, is *not absolute*, but is *all but absolute.* We approach the absolute by division, and recede from it by multiplication. We shall have to refer to this fact again, in our analysis of the hypothenuse, together with its square, so called.

We perceive that the divisible is a *separation* from the

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indivisible, but not so, as to destroy all relation ; in fact, the divisible could not exist, if the indivisible did not first exist ; hence the two have the relation of cause and effect. Then, might we not say, that a line is the effect of an indivisible point? But the indivisible transcends all our powers of comprehension, and a *line* is a thing that we have to comprehend *as it is*, or we know nothing of it, and can do nothing with it, in a science of measures and of numbers, which deals only with the finite and limited—the divisible. Hence we perceive that an indivisible line, becomes by pure reflection upon itself, an impracticable object, and we shall further on, mathematically prove its impossibility. If the square unit cannot possibly be absolute in its angles, then the *side* of a square unit, or of a square consisting of square units, cannot be an absolute unity, though it may *appear* so to our vision ; nor can a plane surface be absolute. In fact, under an adequate magnifier, the most exquisitely polished surface of the densest material, would appear waving in its character. How is this to be accounted for? It can only be accounted for by the fact of the primary *spherical form* of material atoms. These are so minute, that a plane surface composed of them appears to our limited power of vision as a perfectly continuous, absolute plane. And such *appearance* of surface we *measure*, and we measure it with the *apparent* surface of the measure unit, since like can only be measured with like. We might perhaps get as far in geometry, by the use of a circular unit measure, provided it be infinitesimal enough to obliterate to our vision the distinction between a curve and a straight line! What is apparently the longest and straightest line in nature? Is it not the sun-

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beam? Yet, Sir Isaac Newton calls it a wave line! And probably with perfect justice, since, if light be material, it must be composed of spherical atoms, and a sunbeam may thus be looked upon as an immensely long string of bead-like atoms. We cannot in this essay dwell on all the philosophical deductions to be drawn from our reflections; we will, however, mention that spherical atoms serve as an adequate explanation of the *porosity* of the *densest* masses of matter. The philosopher Fichte placed the curious question before mathematicians, "Are there any other than straight lines?" Our above remarks are a full answer to that question! We might, yankee-like, retort with the question, "Are there any other but curve lines?" We use the water-level for a standard of level, while we *know* that every water-line must be a *curve line.* What is the *origin* of a *line?!* Man's knowledge is almost exclusively limited to things as they appear, and not as they are in themselves. This is never to be ignored in science. We perceive that our perception of straightness is totally dependent on our sense of vision, and hence we can have no innate, inborn idea of a straight line! The idealistic philosophers will have to "cave in!" And we rather regret the necessity, since we would feign have "fought on the side of our fathers." But truth knows no fatherland but Heaven. The sun of truth shines for the whole universe!

Now, what is it, we propose mathematically to prove? By the universal veneration for the ancient sage Pythagoras, it is no small thing! For we propose to refute the absolute truth of his world-renowned problem—*that* problem, which serves as the sole support of the fundamental doctrines of the celebrated idealistic school of

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philosophy, of which Kant is the reputed father, but Pythagoras, is the real father, since the conception of the indivisible, hence absolute line, is the very basis of his problem. The German philosopher did not originate that conception, but accepted it as truth, but finding that such a conception could not have its source in experience, he sought for such source, in the internal springs of the mind itself, as most nearly allied to the absolute. Thus he called the conception an innate, inborn idea. With the fall of the indivisible line, falls, as a matter of course, the conception of innate, inborn ideas, and the doctrines of the Empirical school of philosophers, to which Newton and most of the English and Scotch philosophers belong, that experience is the primary source of all knowledge, thus becomes fully confirmed and established for all time. Hence it is no small thing, *this*, that we propose to prove! We proceed by taking a look at the famous problem, $A^2+B^2=C^2$, or, "the sum of the squares of the sides of a right-angled triangle, is equal to the square of the hypothenuse."

What right, we demand in the first place, had the sage Pythagoras, to say, "The *squares* of the sides of a *triangle?*" The inexorable law of demonstration required of him first to prove his ability to construct a square on the side of a triangle, that is, that he could use the side of a triangle for a *square root.* We deny the coincidence of the two respective sides of a square, and of a triangle.

What line of argument shall we pursue to arrive at truth in the shortest and best manner? A truth discovered at the bottom of a well of 23 centuries, the wheel of which well has been revolved over and over again by the wisest sages and mathematicians of all that period, to all

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of whom "a square with infinite root," cannot but have been a stumbling-block; we say, the discovery of such a truth could not have been the inspiration of a moment; it has cost more than a decade of intense reflection, in such leisure hours, as an exciting business would afford, and to bring the discovery into the present concentrated shape, was no trifling task! The truth, when found, is always so simple, that it excites the utmost wonder, how it was, that it was not discovered before!

In order to get at the bottom of the paradox of "a square with infinite root," it is absolutely necessary that we should fully understand the constitution of *a square*, not so simple a thing as even mathematicians are apt to imagine. $a^2+2\ a\ b+b^2$, is the algebraic formula of *a square*. Let us see how many changes we can ring on that formula; we must, however, set by its side another formula of a square, fully as valid, viz., $a^2+4\ a\ b+4\ b^2$. This square is, of course, of a larger compass than the other, by $2\ a\ b+3\ b^2$. $a\ b+b^2=c$, is the *root* of the first formula, or side of the square. $a+b=c$, would simply represent the *boundary* of the *side*, and as such cannot be quantitative. $a\ b+2\ b^2=c+b^2$. This is the *root* of the second formula. $a^2=c^2-2\ a\ b+b^2$. $a^2+2\ a\ b=c^2-b^2$ =a parallelogram. $a^2+2\ a\ b+b^2=c^2$=a square. $2\ a\ b=c^2-a^2+b^2$. $2\ a\ b+b^2=c^2-a^2$ (growth of square.) $a^2+b^2=c^2-2\ a\ b$ (not equal to c^2!) $a^2-b^2=2\ a\ b-(a-b)^2$. $a^2+b^2=2\ a\ b+(a-b)^2$ (not equal to c^2!) $(a+b)^2=a^2+b^2+2\ a\ b$. $(a+b)^2-b^2=a^2+2\ a\ b+b^2-b^2$=an irrational quantity. $a^2+2\ a\ b+b^2$, is not the simplest constitution of a square, since, if we call $a=1$ and $b=1$, we shall obtain $a^2+2\ a+b^2=4$. We perceive then, that the square next to the measure unit itself requires *two* units in addi-

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tion to a^2+b^2, in order to constitute *a square*; therefore, please to mark, that $a^2+b^2=2\,a\,b+(a-b)^2$, is certainly not equal to c^2, and hence can never be equal to *a square*; therefore, the Pythagorean problem lacks *two square units* from being the absolute truth. We may imagine a measure unit, so minute, as to bring the value of the two lacking units nigh unto 0; but they can never be absolutely extinguished, owing to the inexorable law of infinite divisibility, which *excludes* all *existing* things from *absolute being*—that is, from being one and indivisible. There are not several beings; being is one, and only ascribable to Him, who calls his name, "I Am." What creature can claim to say in the same sense, "I am?" "As far as we *know* anything, we are God," says Fichte, but, oh! how infinitesimal is this degree or portion of being, if we may so express ourselves. God *is* all things and infinitely more; but all things being *less* than *He*, they *exist in* Him, but have no such *being* as *He!* "Man is the image of God;" but an image, capable of infinite development towards an approach to the original, if not into final unity with the same! Who can measure the immortal dignity of man? Two infinitesimal units are of no account in human mechanics, but the infinitesimal atoms of the universe are numbered and weighed in the scales of its architect. The infinitesimal dust on the wings of a butterfly, as well as the infinitesimal atoms, composing the nebula in the "milky way," are *numbered* in the "harmony of the spheres!" No discord there of "a lacking link!"

How far our discovery will enable astronomers to take a seat nearer to the celestial orchestra, we will not pretend to say, but they do have to do with infinitesimal

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particles of *time* and *motion*, and *exactness* is the acme of their astronomical bliss! "Truth is harmony, and harmony is bliss." Nor do we profess to be omniscient, or to encompass with our poor limited powers of reason the whole chain of consequences of our discovery, but that it constitutes the initiation of a new era in philosophy may be confidently asserted, since truth is a two-edged sword, cutting error asunder! Atheism, materialism, pantheism, and other isms of like ilk, have to vanish before the light of our discovery, like mist before the sun! The unknown can only be discovered by the known: reflection upon the external impressions—and reflection upon the reflections, are the source of our conceptions. But to resume the thread of our mathematical proof. $a^2+2\,a\,b+b^2$, begins with a square, namely a^2, and *grows* into another square, by the addition of $2\,a\,b+b^2$, that is, a square grows into another by the addition of twice its *side*, plus the square of the breadth of its side. Each unit addition to the *square root* increases the *area* of the square by two units, this is called the *ratio* of increase, which is as 1 to twice the side of the square, and serves for the basis of the "infinitesimal calculus," which method transcends common sense, the moment it forsakes the doctrine of infinite divisibility. Let us here pause a moment at the enunciation of this inexorable law (all other growth formula being simply equations of this), and ask the Pythagoreans by what manner of legerdemain they jump this law into "a square with infinite root?" We can well imagine, how the ancient sage, in spite of his known enthusiasm for the harmony of numbers, fell into the delusion of that extremely paradoxical conception of an indivisible line—

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vertically divisible, but literally indivisible; we had almost christened it a bastard conception! But that the fact of the paradox, as a paradox, should have escaped the attention of all the mathematicians since his time, excites our wonder to the utmost degree, excepting the truly great Newton, whose sagacious mind rebelled against it, but failing to explain it to himself, he contented himself by simply expressing his *doubt* in the existence of the indivisible in nature. All other mathematicians accepted the paradox as absolute truth, believing that nature in some mysterious manner performs the creation of a square, containing an irrational quantity! And yet philosophers hesitate to believe in the miracles of Christ, not half as miraculous as this miracle of Pythagoras, which would be impossible for even a God? Mathematical truth is eternal truth!

The *square*, then, is considered as always divided into a number of square units, and the *whole number* of those units is called *a square number*. We perceive then that no number whatever, except such a number as to which the formula $a^2+2\,a\,b+b^2$ (excepting the number 4) is applicable, can possibly be equal to *a square*, and a square is a surface (divided or divisible into square units) bounded by four equal sides at right angles. The *number* of square units contained in the square, is *identical* with the *geometrical quantity* of the *square*. There is complete harmony here between arithmetic and geometry. We will here take occasion to make the remark, equal to an axiom, that no undivided whole can be measured without previous division, real or implied, into square units or their fractions, since it is only by the square unit that we can measure, else, how could we *number* or *count*

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the measure of a quantity? b^2, however much divided, is always the measure unit of the square. We beg particular attention to these *laws* of geometry, for it is to them we appeal. A square, then, minus one square unit, is not a square number, and cannot be *a square.* It is an irrational number$=a^2+2\ a\ b+b^2-b^2$, or $a^2+2\ a\ b=$a rectangle parallelogram; therefore, such a parallelogram, containing an irrational quantity, cannot be equal to a square. This formula of an irrational quantity is the very basis of our discovery. b^2 is the eternal difference between an irrational quantity and a square, however infinitesimal the square unit may be assumed. We have now pretty exhaustively considered the constitution of a square, the most markable feature of which is the *two units ratio of increase* between one square and the next square in order of progression or *growth*, which the Pythagoreans seem to have, *most strangely*, ignored, when comparing the area of *two* squares to that of a *third* square, since there must *needs* be a *growth* between *two* squares! Let us now examine into the constitution of a right angled triangle, and see what there is in its characteristics to have warranted Pythagoras in his assumption of coincidence between its side and that of a square. A square of square units, diagonally divided, is equal to two right angled triangles. Does each of these consist of all square units? No! Why not? Because one diagonal line of them has been cut in two, thus creating two half lines of half square units, one belonging to one triangle and one to the other, and the *side* of the triangle has been *diminished* by half a square unit. How then can such a side serve for a square root? The only warrant for Pythagoras to think that he could construct *a square*

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on such a side, lies in a *mere visual appearance* of the *boundary* of the *side* of the *triangle.* But the moment we reflect on the *infinite permanence* of the relation of a *right* angle at one extremity of that side, and an *acute* angle at the other extremity, while the side of a square has a right angle at both extremities, *even this appearance proves a delusion!* Besides, a boundary line is *not quantitative*, and hence, cannot constitute *a square root.* Why do mathematicians, in measuring the contents of a triangle, almost always first double the same into a square or a parallelogram, and then, by dividing the same by 2, thus obtain the content of the triangle, because only squares and parallelograms of rectangles can be *measured* by the *square unit*, and the triangle is equal to one-half of these figures. If they measure the triangle by triangular units, they have to double these to obtain the square unit content, because *we have no standard triangular measure unit.* Now a geometrical square is equal to a *square number* of square units, and its triangular *half* is equal *in surface* to half the number of the units of the square, or of the parallelogram. But a square number divided by 2, is *always* equal to an *irrational number*, that is, a number from which no finite square root can be extracted, because *it is not a square.* Since *number* in geometry represents *surfacial quantity*, it is plain, that an *irrational quantity* can never be equal to a geometrical *square.* How then can "a circumscribed square be equal to twice its inscribed square"? Notice, well, that the *ratio of increase* also equally applies inversely to *decrease*, and hence the *half* of a *number.* square cannot be equal to a square. What, then, is the inscribed square equal to? We have seen that the ratio of increase

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between two adjacent squares is equal to *two* units, while, on the other hand, we have seen that the difference between a square quantity and an irrational quantity is equal to *one* square unit$=b^2$; hence the irrational quantity lies *between* two squares, and the inscribed square is equal to the square *next in order* to the irrational *half* of the *square*. Half the circumscribed square would be equal to a square with lopped corners! There has been no formula hitherto in mathematics for an irrational quantity. We have constituted one from the formula of the square by subtracting b^2, namely, $a^2+2\,a\,b+b^2—b^2$ or $a^2+2\,a\,b=$a parallelogram. Extract the square root of any irrational number, converting the *remainder* (of the given number over and above the square of the integers of the root) into a vulgar fraction, by using this remainder for a numerator, and twice the integers, for a denominator, then squaring this artificial or "imaginary root," we obtain a square, which yields the given irrational number, *plus* the square of the fraction ($=b^2$). Hence the irrational quantity is equal to a square, *minus* a square unit, and lies between two squares in geometrical order. This is a *most significant discovery*, pregnant with numerous deductions of extraordinary importance to philosophy. In the first exuberance of our joy at this discovery, which at once affected the truth of the Pythagorean problem (the evident paradox of which we had *fixed* our mind on to fathom), we made a communication of it to the Royal Society of England, during the most critical period of our rebellion war, by a letter to Sir Roderick Murchison, to which we little expected an answer, as we were just then at dagger's points with England; but commend us to the unextinguishable

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politeness of a "fine old English gentleman!" Almost by return of mail, we received a reply from Sir Roderick, couched in not only polite, but even complimentary language, assuring us of his having handed our communication over to General Sabine, the President of the Society. Soon after, we noticed in one of our daily papers, an extract from the London Times, headed, "Extraordinary mathematical discovery!" and eulogising a certain mathematical professor of Cambridge, "*Member of the Royal Society*," for this discovery. A most singular *coincidence!* Is it not? If the American public should ever wake up to the importance of the discovery, and to the national honor reflected by it, this singular coincidence may become a question of international investigation.

But we have not got through our explanation yet, for the mere denial of the truth of the famous problem does not satisfy us: we must dive down to the very bottom of the paradox and show the reason *why* it is a paradox, and the *exact difference* between the paradox and the truth.

We have stated the constitution of a square and of a right-angle triangle; of a square quantity and irrational quantity; of a square number and an irrational number; and have shown how the sum of two squares cannot be equal to a *third* square. To this latter proposition there is, however, one apparent exception, namely, $3^2+4^2=5^2$ $=9+16=25$, the same relation existing between *all the multiples* of these roots. How is this? Call $16=a^2$ and $9=b^2$ and what would be the square, *according to the formula?* $=49$! A pretty $A^2+B^2=C^2$! $25=49$! The *apparent* truth lies in this, that $9=2\times4+1=2\,a\,b+b^2$.

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Call $4=a$ and $b=1$, we obtain $16+2\times1\times4+1=25$. Thus, we see, that $9+16=25$ is a mere *coincidence*, as applied to the problem, these three numbers being *successive growths of squares*, which succession *in growth* so that the third shall equal the two preceeding occurs in no other three numbers, excepting in the multiples of these three. Nor is it possible for any four equal irrational numbers to be equal to a *square number*. But we have seen that a square is equal to two irrational quantities, equal to two right-angled triangles; hence, *four such triangles* combined, can never be equal to *a square*. Yet, *that* is the square of the hypothenuse! The four triangles are equal to two squares; but the *ratio of increase* to constitute the two square into a *third* square, *requires two additional units;* hence, it follows, that the so-called square of the hypothenuse, lacks two square units in order to be equal to a full square! We are able to furnish a diagram of the Pythagorian problem within the formula of a square, which amounts to an *occular* self-evidence of the truth of the problem, and to which truth any mathematician would be ready to swear at first sight; but the ideal eye of *reason* pierces deeper into the essential nature of things than the sensual eye! "Ah, here we have got you!" we hear idealists exclaim, "you acknowledge then an *internal source of knowledge?*" Not too fast, if you please, respected friends! Would a man, deprived of his five senses, at his birth, be likely to acquire any knowledge, even supposing that he could live? Do we not perceive, then, that *outside impressions* are the sine qua non condition of knowledge. The unknown can only be recognized by the known. The square root of a negative quantity must be a positive!

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There must be *objects* to become conscious of before we can become conscious of any thing, and although we may become conscious of a *pure idea*, by means of *two* distinct ideas, since the sine qua non condition of consciousness is *distinction*, yet, if ideas are not images of objects, they are nothing. All our ideals and abstract ideas, are reflection upon reflections, and have their original source in knowledge obtained through the medium of the senses, or in other words, *through experience*, which involves *development*, in fact, infinite development, till *absolute truth* be evolved, such truth as must be law for the universe, and in so far only as we are in possession of absolute truth, can we correctly say "we are", in a similar sense as God says "I am!" Preciously infinitesimal is our portion of Being thus far! But there is room for comfort in the idea, that it is the final destiny of man to be "heir of all things," to became possessed of the universe, by the growing knowledge of God in all His works! We ask pardon of our mathematical readers for straying into the field of metaphysics, but it actually requires a powerful restraint upon our emotions to repress the hosts of thoughts that rush in upon us as an effect of our discoveries.

Let us take up again the thread of our mathematical reasoning. If the square of the hypothenuse be not a *square*, what *is* its geometrical constitution? The square of that Greek abstraction, sometimes finite, sometimes infinite, requires indeed a subtle analysis! We must begin by a reference to our first *limitation* of the *absolute* in mathematics.

We called the square unit "all but absolute," in consequence of the infinite divisibility of the angle. We said,

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that we approach the absolute by division, as we recede from it by multiplication. Magnifying is multiplication. Then, if we multiply an all but absolute square unit by an adequate magnifier, tenfold for instance, it will appear tenfold *less* than *all but absolute*. The finest imaginable *point* will appear comparatively *blunt* under an adequate magnifier, and this bluntness will increase in the ratio of the square of the magnifying power. A tenfold magnified square unit will not then have the appearance of a square of four equal sides, but be like a square with lopped corners! We perceive at once that the diagonal of such a square is a *finite*, and not an *infinite* line, and if such a square unit be diagonally divided, half the bluntness of the angles will belong to one of the triangular divisions and one-half to the other. Now, if we arrange four such triangles into the form of a square, with the hypothenuse for a base, we shall find that such square lacks a minute square unit at each corner, equal to the square of *half the bluntness of the angle* of one of the two squares from which the 4 triangles were taken, and then, four minus square units are equal to the square of the whole bluntness; and in the centre of the so-called square of the hypothenuse, there will be another minus square unit, equal to the square of the whole bluntness, so that this *imperfect square* of the hypothenuse, will be equal to the sum of the *two* squares, called by Pythagoras squares of the sides of the right-angled triangle. We perceive that the perfect square would *exceed* the imperfect square by exactly two units, the exact requirement of the law of geometrical growth. But the boundary line of this perfect square, would extend slightly beyond the extension of the line of the hypothenuse. Therefore, we may consider the square of the hypothenuse from two dif-

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ferent standpoints, namely, either as an *imperfect square* or as a *perfect square*, the latter being equal to a *square number*, and the former to an *irrational number* with an infinite root. If we adopt the former, we cannot call the square of the hypothenuse *a square*, and, if the latter, the square of the hypothenuse exceeds in quantity the sum of the two squares, with which it is compared by Pythagoras. A true problem, versus the Pythagorean problem, would then have to be worded as follows: "The sum of the squares of the adjacent two sides of a square, or of a parallelogram, is equal to the imperfect square of the diagonal (hypothenuse), the perfect square exceeding the imperfect square by b^2"; for the side of the imperfect square, being increased by the sides of the two minus units at its extremities, the increased root not only covers the corner minus units, but also the central minus unit, thus fulfilling the law of geometrical growth. Now we can perceive that the half of "a circumscribed square," which is necessarily an imperfect square, equal to an irrational quantity, must be adopted as a perfect square, before twice the inscribed square can be equal to it. Hence, it follows clearly, that the two quantities compared by the Pythagorean problem, may infinitely approach to coincidence, but can never dissolve into absolute unity, however infinitesimal a measure unit we may employ. This guarantees the *infinite divisibility* of a *line*, and, in fact, the infinite divisibility of all material things, and the *absence of the absolute* in *matter*, as such, that is, so long as it exists as extension. The infinitely divisible cannot be self-existing, as this attribute can only be ascribed to the absolute, the one and indivisible. *Space*, is only another name for the Absolute Infinite—

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the one and indivisible. The infinite universe of matter is infinitely divisible, so that the absolute is infinite, but the Infinite, *per se*, is not absolute! This difference or distinction between the Absolute and Infinite has never been adequately recognized in philosophy; hence, the frequent confounding of the two terms as identical. Time is infinite, but Eternity is absolute. The term "infinite," as applied to something that may have an end, seems rather inappropriate; but conception will ever fail to define the *exact line* between the finite and infinite, and between the infinite and absolute. When God shall be "all in all," time must cease, but it does not prevent Him from creating a new universe! A new Infinite within the absolute. The condition of chaos, only another name for formless Being, may come again!

An irrational quantity being equal to a square, more or less blunt in its angles, according to the number, we perceive that the various crystallic forms may be owing to irrational numbers of particles of matter! An interesting discovery for crystallographists, if not for chemists. The materialists, who consider thought as "a chemical process," might possibly by our discovery determine what *number* of *thoughts* would go to the cube inch, or what quantity of them would form an octahedron! Joke aside, we have discovered to ourselves, whether before known or not, a method to enclose, with perfect precision, any irrational number into a regular geometrical figure. Image to yourself, good reader, a square with blunt corners, such as would appear by diagonally cutting off half of each of the four corner square units of the square. Within this figure inscribe *a square*, so that its angles shall rest on the middle of each blunt corner. This figure

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will be equal to $a^2+4\ a\ \mathrm{b}+\frac{4\ b^2}{2}$. The chess board will furnish us with a square=64; the next square in order below that is $49=7^2$. The next below that is $36=(6^2=a^2)+(6\times1\times4=4\ a\ b)+(\frac{4b^2}{2})=62$. Here then we have enclosed an irrational number into a square with blunt angles; two units less than a square will always fall under the tangent line of the square, here represented by the diagonal cut off from the corner square units—an easy operation; but how to enclose 63, for instance, would puzzle many a mathematician. $63-36=27$; $\frac{27}{4}=6\frac{3}{4}$; $\frac{6\frac{3}{4}}{11}=\frac{27}{44}$; $\frac{27}{44}\div\frac{1}{2}=\frac{27}{88}$; $\frac{27}{88}\times\frac{1}{2}=\frac{27}{44}$; $\frac{27}{44}\times11=\frac{297}{44}=6\frac{33}{44}=6\frac{3}{4}$; $6\frac{3}{4}\times4=27$; $27+36=63$. But where do we get the divisor 11 from? That number represents the content of 11 isoscele triangles, each containing $\frac{27}{44}$; these triangles originate by assuming the base of each square unit of the root of 36 to be the base of an isoscele triangle, which gives us 6 such triangles on the base line=6 $(=a)$, and 5 more on the line resting on the summits of the 6, hanging downward into the vacant spaces between the lower tier of triangles. We have then 11 such isoscele triangles to divide into $6\frac{3}{4}$ ($=\frac{1}{4}$ of 27, the remainder, after subtracting the square 36 from the irrational number 63), $\frac{6\frac{3}{4}}{11}=\frac{27}{44}$, as content of each triangle, and by dividing this content by half the base$=\frac{1}{2}$, we obtain the necessary height, here$=\frac{27}{88}$. But the side of this isoscele triangle, height and base not being equal, must necessarily fall *outside* of the tangent, and thus create an obtuse angle at the corners of the figure, and the form of the angle will vary according to the irrational number to be enclosed. This method is applicable to all irrational quantities, or square quantities either, for in the latter case, the minus in the angle will simply be filled. We perceive thus that there is a regular geometrical relation

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between square quantities and irrational quantities. *Perfect* harmony between Arithmetic and Geometry, as a *law of nature!* From this fact we deduce the possibility of making all mathematical calculations without the use of decimals, with perfect precision! Let us try the experiment with two irrational quantities, multiplying their roots:

$\sqrt{12}=3$ and 3 remaining. $9\times3=27$

$\sqrt{15}=3$ " 6 " $9\times6=54$

$3^2=9$ $3\times6=18$

Divisor twice $9=18)99(4$

$18\times4=72$

27

Quotient square$=16$

Remainder of new root, 11

$+4=$quotient.

$13+11$ remainder. $13^2=169+11=180$

$13\frac{11}{26}{}^2=180+\frac{11}{26}{}^2$. $12\times15=180$

Who will deny ingeniousness to this method? Extracting the root from the product of the given numbers, gives us the same result, and this is the common method. Two methods of the same thing is always a gain! But what method have mathematicians for addition? None! In fact, Chamberlin's Encyclopædia, in its article on the Quadrature of the Circle, says, "If an equation could be discovered for $\sqrt{a}+\sqrt{b}$, a and b representing irrational quantities, it would be welcomed as the solution of the grand problem." Here it is $\sqrt{a}+\sqrt{b}=\sqrt{(a^2+b^2+\sqrt{4a^2b^2})}$.

The result is, of course, an irrational quantity, as the

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area of the Circle, and equal to a parallelogram, which according to the known rule of mathematicians, is convertible into a square; but not according to *our rule*, which is, that no parallelogram, containing an irrational quantity, can be equal to a square. It will forever lack one square unit, however infinitesimal the measure unit may be assumed. According to old mathematical rules we have squared the Circle! But it is not such a square, as *we* call a square! It is a less than all but absolute square, though it may infinitely approach the all but absolute square, the only conception it is possible for us to have of a *square.* The square of an infinite root, even if the decimals reached to the sun, would only infinitely *approach our result.* Hence, it seems to us, that logarithms may be dispensed with, as far as decimals are concerned, and this was the instant deduction made by a French mathematical gentleman, at the first recognition of our discovery. We would respectfully suggest that in mathematical tables, in future, after our method shall have become fully familiar to the public, simply the *remainders* be given with the integers of the roots, with which to restore, at pleasure, the square of the root, from which those, who like to use decimals, may extract them to their heart's content.

We will finally recapitulate the various discoveries, to which we lay claim, independent of all deductions from them, viz:

1. A lacking link in the demonstration of the Pythagorean Problem.

2. Formula for an irrational quantity.

3. Artificial or imaginary root of an irrational quantity.

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4. Difference between a square and an irrational quantity.

5. New Problem in the Pythagorean sense, $C^2 = 2\ AB + (A - B)_2$

6. Non-coincidence between the side of a square and the side of a triangle. $9+16=25$

7. The fact, that outside of $3^2+4^2=5^2$ and their multiplies, there are no other two squares, the sum of which is equal to a square number. $5^2+11^2=13^2$
$25+121=169$

8. The fact, that no four equal irrational quantities are together equal to a square quantity.

9. The fact, that the diagonal division of a square constitutes two irrational quantities, neither of which can be equal to a square.

10. Harmony between Arithmetic and Geometry, as a law of nature.

11. The fact, that the proposition "a circumscribed square, is equal to twice its inscribed square," is not true.

12. True problem versus Pythagorean problem.

13. The fact of the non-absoluteness of the square unit.

14. The fact of the infinite divisibility of a line, and that consequently, there is nothing indivisible in nature, excluding absolute space.

15. The fact, that infinite divisibility is the inexorable law of individual existence.

16. The fact, that we cannot measure an undivided whole, without previous real or implied division.

17. Equation for $\sqrt{a} + \sqrt{b}$, a and b representing irrational quantities.

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18. Method for multiplying the roots of irrational numbers, without decimals, and with precise results.

19. Method to represent irrational numbers in all possible geometrical figures with precision, interesting to crystallographists.

20. Sensual self-evident representation of the Pythagorean problem, within the diagram of the formula for a square.

21. The fact, that the adoption of the indivisible as a *basis* for a science of measures and of numbers, is an absurdity.

22. The fact, that the square unit, infinitely divisible, is the only legitimate basis for geometry.

23. The fact, that "straightness," in itself, is but an appearance, owing to the limited powers of our vision, which prevents us from seeing things as they are, and that there is, in reality, no such thing as an absolute straight line.

24. The fact, that the origin of a line must of necessity be a *curve*, owing to the known fact of the spherical primary form of matter. Hence, we might get as far in geometry with a circular, as with a square unit, if we did not measure simply the *appearing* surface of things, for which the square unit only will answer, since like has to be measured with like. We exclude the mechanical formation of a line from our consideration.

25. Difference between the absolute infinite and the infinite *per se*.

26. The fact, that we have no innate, inborn ideas of first mathematical principles.

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We might multiply numerous new philosophical deductions from our discovery!

We cannot take leave of our readers without a parting word. It is a desire towards an effort to remove a rather general and not a little pernicious prejudice, against the science of mathematics, as the driest and least interesting of all studies! If it proves to be so, it is certainly owing to an uninteresting method of teaching it. When we reflect, that all forms of matter and all motions of the same, manifested in either organic or inorganic existence, follow the laws of mathematical proportions, be it in the magnitudes, densities and motions of the stars, or the chemical changes occurring in the elements of their composition; in the formation of crystals; in the growth and forms of plants and animals; even in that which affords harmony to music and poetry; when we consider that all arts and sciences are more or less dependent on mathematical knowledge; that the universe, in fact, is constructed on mathematical principles: can we longer wonder, that philosophers have gone so far as to call it "a divine science?" How few enthusiasts there are to see it in that light! To the majority of men it is a sealed book! And to many men the book of Nature only fairly opens with the breaking of that seal; and not the book of Nature only, but also the Book of Revelation. We can fairly avow that a great many expressions in the sacred scriptures have only become clear to us in consequence of our mathematical discovery! It proved to us *to be* the opening of a sealed book of philosophy. *Error* is the *seal* which keeps men from the *recognition* of *truth!* We have broken a seal which remained unbroken for 23 centuries: may the revelation of truth do its master's

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work! We are but His humble instrument. Truth is always beneficient, and to the God of Truth be all the glory! And we desire here, before the whole world, to bear witness that God hears prayer, for we distinctly remember an ardent prayer of our early manhood, when sharply feeling the nothingness of virile life without some act or thought beneficial to the race, that God might grant us the discovery of some beneficient truth! And God heard our prayer, although taking his own time for its fulfilment. We hope the learned professor Tyndall may read this!

THEODORE FABER.

NEW YORK, *October*, 1872.

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