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NUMEROLOGY

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NUMEROLOGY

BY

E. T. BELL, Ph.D.

Professor of Mathematics in the California Institute of Technology

BALTIMORE

THE WILLIAMS & WILKINS COMPANY

1933

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INTRODUCTION: HOW IT HAPPENED

A little over a year ago, the lady who used to sweep out the living room occasionally, asked if she might turn on the radio. She said she was anxious not to miss Swami Something-or-another's Half Hour with the Numbers and the Stars. She should have been home hours ago anyway, and would I mind? I told her she might stand on her head if she liked, as I was going to take a bath behind two locked doors. She turned the infernal thing on full blast.

At first I tried not to listen to the devastating barrage of "Numerology, the Science of Number, Life and Truth" crashing through both doors and drowning the roar of two faucets turned on full. Then, as the delightful heat of the water began to get in its work, I found myself slipping into the mood of the snooty musical critic who had been asked to report a recital of modern music for his newspaper. At first hostile and amazed, he had gradually relaxed to the danger point. Realizing that he was about to be deboshed forever, he leapt to his feet, shouting "Let me out of here before I begin to like the damned stuff!" For obvious reasons I could not dash out and turn off the radio. Hence my passion for numerology. The sweeping lady mothered my passion, but I certainly was not its father. It just came.

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Since then I have followed numerology about everywhere like a lover. Making my living as I do in a scientific institute, and living within a stonesthrow of Hollywood, I have had unique opportunities for trailing the loved one. Some of my colleagues (I hope they will be un-numerological enough to continue letting me work in their give-and-take fellowship) are numerologically hard-boiled; others are as tender as raw eggs. The like holds for the distinguished scientists from abroad who honor us from time to time with their lectures. I have often wondered just how much some of these famous men believe Americans can swallow and digest. In fact that wonder was a contributing cause to my present delinquency. Many Americans will gladly admit their lack of "culture," but how many will confess to being suckers? Few are, as a matter of fact; most are just incurably polite.

My little jaunt into numerology was undertaken to answer the following question. Is there more science in numerology than there is numerology in science? I have given enough on both sides of the question to enable any reader to settle it according to his own liking.

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Chapter I

THROUGH HOLLYWOOD TO SCIENCE

COMEDY OR TRAGEDY?

Although numbers cannot lie, they have a positive genius for telling the truth with intention to deceive. Not only have eminent philosophers—including Auguste Comte and the incomparable Plato—been numerologists of the first rank, but many a man whose name is remembered in astronomy, mathematics, physics, or chemistry has also at some stage of his career believed implicitly in the miraculous properties of numbers.

Leaders of thought in the past then have been numerologists. The same is true today. If you disagree with the last, you should at least be unnumerological enough to examine the evidence. Some of it is presented in the following chapters, and there is a lot more where this came from.

Need we be surprised therefore when we read news items like the following in our morning paper?

"Despite the 1,500,000 letters she received the past year, bringing to her the problems that beset mortals in all walks of life, ... popular radio numerologist, takes the optimistic view that the human race is improving right along."

The lady in question is a radio broadcaster in the

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Eastern United States. The item goes on to say that "Women wanting to know about their husbands, husbands wanting to know what to do about their wives, young men and women wanting to know about business careers, parents wanting to know what to do with their children, business men wanting to know what to do about business—all these laid their troubles at the door of the attractive numerologist." It was "a veritable avalanche of appeals for help."

Most significant is the further statement that "If anyone thinks that numerology may be a woman's fad, witness the fact that forty-eight per cent of Miss ———'s mail last year was from men."

So it was about fifty-fifty. Personally, from the lady's picture, I should have guessed about ninety-eight instead of a trifling forty-eight per cent. The lady is undoubtedly attractive; and I swear I would gladly believe anything she might be gracious enough to tell me. The news item is dated April 7, 1932.

Before taking leave of this wistful lady whose name is blanked above, I should like to give beautiful lady numerologists a valuable tip. Why not use the professional name Theano? It would be most fitting, for it is the beautiful name of the beautiful young girl who had more influence on numerology than all the rest of her sex combined. We shall meet this charmer later. If any lady numerologist avails herself of my tip I will leave it

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entirely to her conscience what rakeoff I am to get. I am sure I can depend upon the generous promptings of her conscience.

Numerology is not defined in the dictionaries printed ten or more years ago. There is a place for it—there is a place for everything, in a dictionary—between numero and numerosity.

An older equivalent of the term is number-mysticism. Under this name there is a vast literature of numerology in the histories of philosophy and mathematics. Nor should the careers of science and theology be overlooked. Numerology has played its part there also.

In the past ten years numerology has had a new flowering, more luxuriant than ever before. The practice of the art has been elevated to a profession. As such it is today a serious rival of the much older profession of astrology. Neither need be jealous of the other. The human race is diversified enough to accomodate both astrology and numerology.

Such is the plain fact. It is upheld by the testimony of history for at least 2600 years, and probably for 5000. Those who dislike either facts or history are just as powerless as are those who revel in both to change either.

Instead of quarreling with a condition as it exists, it is more amusing to see what brought it about. Why did numerology flourish in the past, and why does it flourish today? Unfortunately I cannot

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find any satisfying answer. The best I can do is to give some account of the work of a few of the greater numerologists, and let everyone draw his own conclusion.

I shall endeavor to keep my own opinions in the background as far as possible. The story will neither commend nor condemn. The verdict on numerology will be left entirely to the reader.

To give my short account of numerology a reasonably up-to-date flavor, I shall include illustrations from our own times. The current researches of Sir Arthur Stanley Eddington, for example, on 137 make a very great contribution to numerology, and one which is characteristic of our age, profoundly preoccupied as it is with the speculations of physical science.

I quote nothing that has not been printed in good faith by its author or authoress over his or her own name. Where names and references are suppressed, this has been done for humane reasons. All of the matter quoted is readily accessible to anyone who may be sufficiently curious to look for it. Some of it has appeared quite recently in technical scientific journals, not primarily as elaborations of numerology, but as professedly serious contributions to science. Nevertheless it makes excellent numerology in the true tradition of the Babylonians and Pythagoras.

Occasionally a little technical language is indulged in where it makes the meaning clearer.

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But fair warning is always given, and those who are not interested can skip.

Before we go on to the great geniuses of numerology, the modern meaning of numerology may be illustrated by three very recent applications of the pursuit. The first of these concerns Hollywood and theology; the second, the stock market, and the third, modern physics. The reader who wishes may defer the third example until he has read Chapter IX, where a little more is said regarding the terms used. An unsympathetic critic might harshly refuse to see the human appeal of these samples, and confuse tragedy with comedy.

HOLLYWOOD IN THE MIDDLE AGES

Believe it or not, this is a true story. The scene is a severely businesslike office somewhere in Hollywood. Behind the slick mahogany desk sits a slicker looking Englishman of about forty. His sporty morning suit is concealed behind a white smock-apron disguise, such as physicians often wear when receiving patients. But for the books on the desk at the consultant's left he might be a psychiatrist, or an eye, ear, nose and throat specialist. One of the books is a table of logarithms; another, the commonest English table of squares, cubes, square roots and cube roots.

Two dazzling young women are announced by the primly practical secretary: "Miss Lispeth Planette and Miss Gloria Moone."

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Miss Planette, whose name in private life is Mrs. Abraham Finkelbaum, is the patient; Miss Moone, who has no private life to speak of, is her sympathetic friend.

Miss Moone is as platinized and as hardboiled as they make them. She is sharper than a tack. Knowing well that numerology is sometimes little more than a sucker racket in Hollywood no less than in New York, Miss Moone has accompanied her easy friend Lispeth to see that she is not overcharged. In fact, just before the secretary showed them in she snapped, "Lispeth! If you pay Sir Charles a cent over a thousand dollars, I'll never take you shopping again. All that hooey about complications in your case is just boloney. I know, because my vibrations harmonize with yours, and there was nothing wrong with me and Jack."

The ladies are seated. Sir Charles suavely and scientifically informs the palpitant Planette that he has at last deciphered all the numerological complications of her extremely interesting case. As he emphasizes the "interesting," he keeps one glittering, wary eye cocked on the pugnacious Gloria. "Interesting" is his last bridge should he be forced to retreat from his demand of $5,000 for his professional services. After considerable diplomacy Miss Planette pops her seasick question, "How much?"

For $5,000 Sir Charles will adjust all of her marital and talkie difficulties by presenting her with

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the outcome of his researches. From her birthdate (correctly stated, in the strictest confidence), her numerological name giving her vibration number, and the like for the names of Messrs. Finklebaum and Blaustein, Sir Charles has discovered the cause of Miss Planette's mediocre success in the talkies, and the sure means for her instant rise to stardom. All this for $5,000.

"Nothing doing," snaps Gloria. She rises, smooths her shapely hips, and the panicky Planette seems to hang suspended in the fourth dimension, neither on her chair nor off it. "A thousand is all Miss Planette can pay."

"Exactly," Sir Charles agrees with his disdainful Oxford smile. "I was just about to say the extreme interest of the complications in Miss Planette's case almost induces me to waive the fee entirely. But we numerologists must live, you know."

"Oh," says Gloria, giving him a filthy look. "Then it's a thousand."

The data of the problem are these. Miss Planette's husband, Abraham Finklebaum, is a camera man. Her director is Mr. Blaustein, who recently has been casting appreciative glances on her lithe shapeliness. The severest complication is that Miss Planette is fundamentally a boob, a tender-hearted, small-town girl who rather likes her husband, and she does not particularly desire the 225 perspiring pounds of Mr. Blaustein as a

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bedfellow. Shall she give the hardworking, affectionate Abie the air after all he did for her in getting her taken on? It doesn't look quite the square thing. Still, if her number is at odds with her husband's she would be little better than a common hussy to keep on living with him.

As might have been guessed, Sir Charles disharmonizes Miss Planette and her husband, and harmonizes her with her director, all for the nominal fee of $1,000. He even shows the critically protective Gloria exactly how he did it, with the higher mathematics of logarithms and cube roots. That neither he nor the hardboiled Gloria could tell a logarithm from the cube root of sin makes not the slightest difference; he does the trick. Only when he gets toward the end of his higher mathematics and comes down to common addition does the platinized peach unwrinkle her lovely forehead. She understands it all now, and Sir Charles ain't been doing the trusting Planette no dirt.

The historical interest of this incident is in the numerology of the last part of Sir Charles' demonstration. Write down any three consecutive whole numbers, the largest of which is exactly divisible by three. For example, 2691642, 2691641, 2691640. Or, if this is too much for titled numerologists in Hollywood, take any number, multiply it by 3, and subtract 1 from it, then subtract 1 again. In the above example I proceeded thus from 897214. Now add the three

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numbers. The above example gives 8074923. Add all the digits of this number, $8 + 0 + 7 + 4 + 9 + 2 + 3 = 33$. Add the digits of this number, $3 + 3 = 6$. Now, believe it or not, you will always get a number (or vibration) 6 as the final result if you do this trick to *any* 3 consecutive numbers, the largest of which is exactly divisible by 3. But if the largest is not exactly divisible by 3, you won't get a number or vibration 6 as the final result.

Really it was a shame to take Miss Planette's thousand. By means of his logarithms and cube roots Sir Charles readily assigned three numbers to each of Messrs. Finklebaum and Blaustein and to Miss Planette. Curiously enough the Blaustein and Planette triads give 6 as the final vibration rate, while the unlucky husband's gave something else. The rest is obvious.

Now, when we come to glance at what I have ventured to call sacred numerology as opposed to profane, we shall see that the good men of the Middle Ages did quite similar tricks with numbers and names. So successful were they that one wrathful theologian (Hippolytus) in his *Refutation of All Heresies* undertook to prove that all such numerology is bunk by just this trick with 6. Poor optimist! He succeeded only in convincing everybody that the "Pythagorean calculus"—as he called numerology—is *true*. If he had not been a theologian he would have sworn.

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The whole episode is extremely interesting as it is probably the closest that theology ever got to Hollywood, or Hollywood to theology. After this we can almost agree with the lady numerologist quoted earlier. If the human race is not exactly improving right along, it certainly is getting no dumber than it was in the Middle Ages, at least in Hollywood.

VIBRATIONS, BULLS AND BEARS

In 1931 a prominent stock broker in New York had his name legally changed for what, to him and to the scholarly jurist who sanctioned the change, appeared good and sufficient reasons.

The broker had been losing money. The panic of 1929 had caught him short and he had been short ever since. What to do about it?

Instead of dashing to the nearest fortune teller or astrologist as a less cautious loser might have done, the broker consulted one of the many leading numerologists who make New York their headquarters. This expert quickly uncovered the root of the trouble.

As is well known to all professional numerologists, the fundamental cause of harmony or discord is not vibration but number. The vibrations which determine—in numerology—a man's character, his personality, and his financial success are merely the outward and sensible manifestations of an

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inner and insensible number. If the number be ill-adjusted to the numerological sum of the letters in a man's name, only irreparable disaster can result from any of that unfortunate man's undertakings until the name be changed.

The rest is obvious. A change of name was prescribed, sanctioned by the court, and adopted by the broker.

Unfortunately the sequel to this bit of history is not available. Whether the new number with which the court endowed the harassed broker enabled him to climb out of the depression, or whether the new name harmonized no better than the old with the vibrations of the ticker, is not known. But all good numerologists will unite in hoping that the unhappy broker was not slashed to ribbons by the bears or trampled to mush by the bulls. If either of these fates overtook him the fault was not with numerology but with arithmetic. Even the most expert numerologist occasionally puts 3 for 7, or makes some trivial slip in the decimal point. The latter indeed is the secret of the following mishap. For its suggestiveness in the higher stages of modern Pythagoreanism this example merits the closest attention. The explanation of the miracle suggested here is by no means the simplest possible in this particular instance. But as the explanation has very wide applications, I have alluded to it, rather than to a simpler one.

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A MODERN MIRACLE

To most scientists one of the gratifying things of recent years is the avid popular interest in science which has swept the world since the Great War. There has been nothing approaching it since those heroic squabbles of the infancy of evolution, when even dignified English bishops so far forgot their holy office as to lay out—or try to lay out—the saints of Darwinism.

All this adds to the joy of life, whether one be pro-science or anti-science. Only an occasional scientist now and then gets fed up with too much appreciative publicity of himself or his work, and wishes the world would leave him some time to dream. But Einstein is in the minority; possibly he is the minority itself.

For our present purposes this eager appreciation of science is all to the good. It relieves any mere mathematician who is trying to describe scientific numerology from the necessity of explaining what electrons, protons, neutrons, Planck's constant, and quanta are, to say nothing of photons and Boltzmann's constant. These physical terms have passed into common language and are well understood by everyone—except possibly a confirmed mathematician. But mathematicians are notoriously thick when it comes to common things, and their failure to comprehend what others understand can be ignored.

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The scientific miracle which follows concerns the velocity of light, the mass of the electron, the mass of the proton, Planck's constant, the gravitational constant, and Boltzmann's constant. Any physicist will admit that each and every one of these is fundamental in physics. The author of the numerological miracle connecting all these constants is therefore within his rights in calling them fundamental physical constants. To these must be added another constant, a purely mathematical one, precisely as the author of the miracle does.

This mathematical constant is the villain of the play. Anyone who recalls anything of his school arithmetic will remember pi (π), the number by which the diameter of a circle must be multiplied to give the circumference. Roughly, pi is three and one-seventh; to seven decimals it is 3.1415926.

There is no exact, terminated decimal expression for pi. Nor can pi be got by dividing one whole number by another. As the mathematicians express it, pi is *irrational*. Worse, it is transcendental—in the mathematical, not the numerological, sense of the word.

To proceed with the miracle. This was reported in March, 1932, in a scientific periodical which is seen by practically every scientific worker in America once a week, and which has an appreciable circulation among scientists abroad. The periodical in question is the official organ of publication of the largest scientific society in the United

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States. By these tokens anyone who is sufficiently interested should be able to run down chapter and verse. The miracle is headed "Relations Between Fundamental Physical Constants."

The constants concerned are those already mentioned, including the villainous pi, eight, not seven, in all. Had there been seven, not eight, the story might have been different. This may well arouse the suspicions of numerologists.

Having announced his discovery, the author proceeds as follows. My own remarks are in [], and I have italicized two of the author's observations.

"A NUMERICAL relation has been found between the fundamental physical constants shown below and the velocity of light. This relationship is of such a nature that the constants can be calculated from a single equation [it need not be reproduced here] and the power of the velocity of light shown [never mind what this means; it is perfectly sound mathematical sense, but it does not affect the numerological issue], *provided the decimal point be ignored*. A complete solution of the relation given, enabling the decimal point to be properly placed, has not yet been found. *It is, however, not possible that any merely accidental agreement could produce the numerical agreement shown below. . . .*"

More than one poor devil has laid down his life at the stake because it was "not possible" that numbers, in particular the number 3, could lie.

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Numerology has had its martyrs, no less than science. What are the facts here?

The author produces no fewer than seven astoundingly accurate agreements between the numerical values of the physical constants, as determined by his single, simple equation, and the numerical values of the same constants as found by the most exacting experiments ever performed by physicists.

Let us take the famous example of the electronic charge. For his marvellously ingenious experimental determination of this, Millikan was awarded the Nobel Prize in physics for 1922. An accepted value (if the decimal point be treated as the author sees fit to do) is 4.774, with a "probable error" of plus or minus .005—never mind what this means, it has no bearing on the highly improbable miracle. The author's equation gives him 4.77401.

The agreement is almost incredibly good. It becomes quite immorally good when the equation gives equally close agreements with experiment on every one of the seven devils the equation was invoked to cast out. This matter of the physical constants will turn up again in a somewhat more sophisticated and more mystical disguise toward the end of our story. So it is worth a moment's attention.

The bane of numerology has been the common whole number. It wrecked the higher numerology of Pythagoras and, no doubt, if there is any con-

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tinuity in scientific history, it will sooner or later destroy the modern Pythagoreanism of Eddington.

In the present instance, it is the immediate offspring of the common whole numbers which demolish the harmony between the fundamental physical constants. The harmony is in fact an irresolvable discord.

Where is the trouble? The author stipulated that the decimal point be mistreated, and he used pi in his equation. He based all of his calculations on the number which expresses the velocity of light in the proper units. Now, if he will give any competent mathematician, instead of the number for the velocity of light, the license number of his automobile for any year—say 1930, or his telephone number, or any other almost human or quite inhuman number, and if he will allow the mathematician to use pi and abuse the decimal point, the mathematician will produce, *from the same equation as that used by the author*, agreements to the billionth decimal place (if desired, and if experiment ever gets that far) with the ascertained values of the physical constants. If this is not enough, the mathematician will throw in for good measure the height of the Great Pyramid and the age in seconds of the late Empress of China at the moment of her death.

After this astounding demonstration of the power of abstract thought all scientific experiment seems a waste of time and money. Why rack nature with experiments when she had given away

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the whole show in common arithmetic? As Plato put it, "God ever geometrizes;" as Jacobi recast the idea closer to the numerological heart, "God ever arithmetizes;" as Jeans modernizes the thought, "The Great Architect of the Universe now begins to appear as a pure mathematician." Instead of "experiment answers all," it seems closer to the truth to substitute "numerology" for the subject of the phrase.

To develop this and actually do the trick would require no great ingenuity on the part of the mathematician, but it would demand a yard of dreary arithmetic.

The trick is possible on account of certain simple properties of *irrational* numbers (one of which is pi) which have been a stock in trade of professional mathematicians for many, many years. If anyone is interested in looking into the matter, he may enquire for the theorems of Liouville, Hermite, and Kronecker. In the meantime he will have to take my word for what has been stated.

Anyone familiar with the meaning of units will see another way (less mathematical) of disposing of the miracle, which amounts to saying 6 cows = 5 horses, or something of the sort. But this is not sufficient to do all that can be done with the very interesting equation.

Any expert numerologist will at once see an easier way out. The author got *seven* agreements between *eight* constants. If he had obtained *seven* agreements between *seven* constants, no

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fundamental principle of numerology would have been violated, and *all of the agreements would have been mathematically and physically correct.*

Even the most skeptical mathematician will assent to the last when he reflects that it takes at least two to make any sort of an agreement.

UNIVERSAL NUMEROLOGY

As we go on to the classical numerologists we shall soon perceive an unfair disadvantage under which most of them have labored. To see this, contrast them with the stock broker. The broker sought only to numerologize the New York Stock Exchange. Some of the ablest mathematicians, philosophers, and scientists in the history of rational thought have devoted the best years of their lives to attempts to numerologize the universe, including mankind and mathematics.

The broker could appeal to a court of law to help him out of his difficulties. In that possibility of appeal the broker had his unfair advantage over the universal numerologists. The only court to which they might have appealed for a decision of any sort has handed down none since it delivered ten at once to Moses. Nor is it likely to hand down another so long as scientists continue to usurp the Supreme Bench.

Universal numerology will be the main thread through the following chapters. It may be allowed to define itself.

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CHAPTER II

NUMBERS AND MARRIAGE

AN IMPROVED GAME

There used to be a game of "Authors," in which one player had to guess the authorship of the quotation fired at him by another. The game went out of fashion years ago, when the young players suddenly realized that their sport was merely a disguised cram in literature.

It recently occurred to one of my mathematical friends to resurrect the old game with a perfectly devilish improvement of his own. Suppose the native language of one player is English. Then, in the improved game, no other player is to have English as his mother tongue; but all are to know English fairly well. The player whose native language is English asks each of the others to state whether there is any real difference between several passages of English prose or poetry which are slowly read aloud. In particular, are all the passages sense, or is at least one nonsense?

Not very exciting, some will say. Try it on a group of cultivated Chinese. You will be astonished at the devastation. If you particularly cherish some masterpiece don't submit it to the impartial judgment of unbiased orientals. Philos-

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ophy, religion, and the foundations of mathematics are good subjects to avoid. Ethics also had better be left alone if the game is not to end in a free for all fight.

PLAYING THE GAME

All but one of the following three quotations may be safely tried on almost any group of players. The doubtful specimen gives the game away to those who grew up on English. Does it? Place your own wagers; there may be a catch in it somewhere. My friend included the easily guessed one because it seemed to him to have been directly inspired by one of the others, and he could find no apter example with which to round out the mystic three. Here are the quotations.

AUTHOR A. "For that which, though created, is divine, a recurring period exists, which is embraced by a perfect number. For that which is human, however, by that one for which it first occurs that the increasings of the dominant and the dominated, when they take three spaces and four boundaries making similar and dissimilar and increasing and decreasing, cause all to appear familiar and expressible.

"Whose base, modified, as four to three, and married to five, three times increased, yields two harmonies: one equal multiplied by equal, a hundred times the same: the other equal in length to the former, but oblong, a hundred of the numbers upon expressible diameters of five, each diminished

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by one, or by two if inexpressible, and a hundred cubes of three. This sum now, a geometrical number, is lord over all these affairs, over better and worse births; and when in ignorance of them, the guardians unite the brides and bridegrooms wrongly, the children will not be well-endowed, either in their constitutions or in their fates."

I wish there were space to reproduce the whole of the next. Fair play however demands that one contestant be not allowed to overwhelm another by the mere numerosity of his offering.

AUTHOR B. ". . . . it may be perceived how little dependence may be placed upon algebraic symbolism in ascertaining essential concrete factors such as the foci of the impacting radii of Solar forces, which absolutely demonstrate the superphysical infallibility of Cosmic Energy, as shall be mathematically disclosed herein by spheroidal measurement. . . . .

"Thus the present tentative spheroidal measurement of [the?] physical continuum of but approximately 45 degrees, or of about one-eighth only of the demonstrable universal scope of futurity, is not alone the lax procreating cause of mundane profanation, or transitional criminal imperfections, but it constitutes the provisional introstatic, or perturbed, growing pains of the evoluting Cosmic Paradise, that is now in course of superphysical growth . . . . in accordance with precise mathematical conjecture."

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AUTHOR C.

"... . One, two! One, two! And through and through
The vorpal blade went snicker-snack!
He left it dead, and with its head
He went galumping back. . . . ."

When these three were bowled at me I was stumped. Authors A and B seemed to be identical, on numerological grounds. Thus A with his sly allusion to brides and their usual prospects has undeniable affinities with B and the latter's "lax procreating cause of mundane profanation." But obviously, it seemed to me, C could not have written what B did. And yet the numerological rhythm of the first line of C's contribution is almost identical with that of A's, although one is verse and the other prose.

Balancing all the pros and cons, I concluded that C was the author of A's effort, and that he had tried to make it more intelligible in a subsequent attempt, represented by B's contribution. The difference in rhythm—or vibration—in the prose of A and B could be attributed to the respective states of C's digestion when he composed the two specimens.

How mistaken I was is revealed by the correct answer. Author A is Plato (429–348 B.C.); author B need not be named, for reasons stated in the first chapter; Author C is, of course, Lewis Carroll. The quotation from B is taken from an open

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letter to physicists, dated March 4, 1932. It was received by my friend, who rather scurvily played it off on me. Practically every scientist gets scores of such letters every year. According to my friend, however, no scientific worker of modern times has ever got a letter like Plato's from any individual not enjoying an enforced retreat from the cares of the world.

THE NUPTIAL NUMBER

Numbers have played a tremendous part in marriage—witness monogamy, polyandry, polygyny, polygamy, and concubinage, to cite only five noteworthy manifestations of numerology in human mating. But the completest summary of the nuptial implications of numbers is undoubtedly Plato's.

The "lord over all these affairs, over better and worse births" of the quotation is not unlikely the number $60 \times 60 \times 60 \times 60$, or 12960000. This is staggering. Even King Solomon had all he could manage with a trivial 300 wives of the first kind and 700 of the second. Notice in passing that $300 + 700 = 1000 = 10 \times 10 \times 10$. This numerology is probably at the root of Solomon's harem.

Where did Plato get his famous "nuptial number," 12960000? To go into even a fraction of a percent of the guesses would fill an enormous book. One conjecture is of particular interest to

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those numerologists who like to trace the origins of things fundamental back to the wise and mystic East. Plato got his number at second or third hand from the Babylonian priests. What they were doing with it, God only knows. The number certainly occurs in Babylon at a time when Greece was barely civilized.

There is no point here in going into this in detail, but it may be mentioned that the hint of a Babylonian origin fairly shouts in the arithmetical nature of the number. It is the fourth power of 60, and the Babylonians, who gave us our sexagesimal system of minutes and seconds, reckoned with 60 as a base instead of with our 10. Every time a good numerologist looks at his watch he should think of Babylon with reverential gratitude and raise 60 to the nuptial power.

There is more than conjecture behind this. The number has been found on the clay tablets of the Babylonian mathematicians. A modern numerologist should base his ambitions on the decimal system, and take the fourth power of 10, or 10000, as a more manageable number for our effete race. 12960000 would have deterred even Bluebeard.

To return to Plato. No profundity in all of his works has given his commentators more trouble than this passage in Book VIII of the *Republic* concerning the nuptial number. A too practical critic might say that all of the tremendous brain

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power which has gone into attempts to explain what Plato meant has been a futile waste.

Even Plato's immediate successors could make but little of what he was talking about, and in the succeeding 2400 years but one person—to a mathematician's way of thinking—really has understood what Plato was saying.

If Plato and not some wily mathematician in the dim background *proved* what the obscure passage implies, then Plato was a very great arithmetician indeed, centuries ahead of his time. He would overtop even the mighty Diophantus himself. The same holds, of course, for the unknown X who actually supplied the proof—if X existed.

If Plato's assertion was nothing but a bold guess, it was at least a remarkably interesting and acute one. Let us see what he might have meant.

THE ROPE-STRETCHERS

To get a point of view we shall have to go back to the men who built the pyramids. For religious reasons, which we seem to have inherited, the Egyptians were very particular about the orientation of their temples.

A true north and south base-line could easily be determined by simple astronomical observations, such as have been made by many peoples less advanced than the ancient Egyptians. To get a true east-west line they had to lay down a line exactly at right angles to the first. Moreover, in

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setting out the corner of a building, the east-west line had to be drawn at a specified point of the north-south line. Without surveying instruments, and with only the crudest beginnings of geometry, the problem is less easy than it sounds.

Before 2000 B.C. the Egyptians were using an extremely practical solution of the problem, which

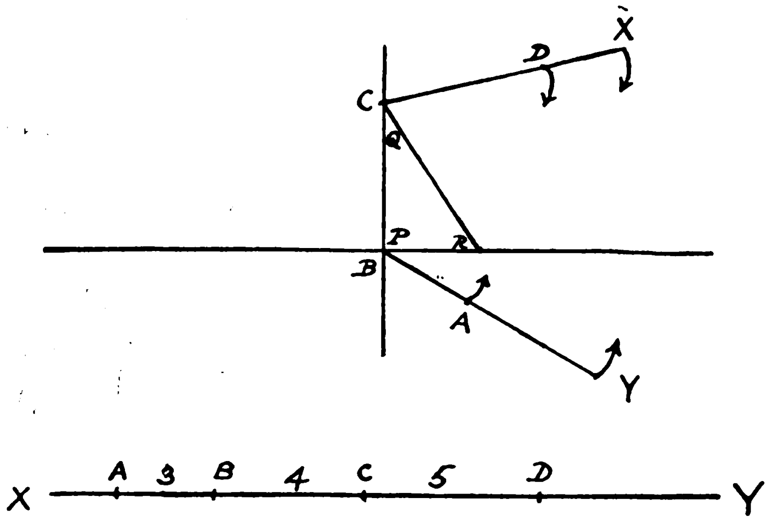

FIG. 1

was also known to the ancient Chinese and the Hindoos. Taking any convenient unit of length, not too short, say a yard, they marked a rope XABCDY at four points A, B, C, D, so that the lengths of AB, BC, CD were 3, 4, 5 yards respectively. Suppose they wanted the exact east-west at the point P of the north-south line NS. They drove pegs at P and Q on NS, four yards apart, and stretched the rope on P, Q, so that the marked

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point B was at P and C at Q. Then, always keeping B, C on P, Q, and keeping the rope taut, one man holding the end X, and another the end Y, walked toward one another till the marks A and D coincided, say at R. The rope now formed a right angled triangle RPQ on the ground, and RP was the required East-West.

This solution was possible of course only because any numbers in the ratios of 3 to 4 to 5 *are* the sides of a right-angled triangle. Who first discovered that useful fact has vanished into oblivion. It started a deluge of numerology and no end of practical mathematics.

As astrology and numerology are first cousins, we note in passing that the earliest Greek to consider the problem of drawing a perpendicular from a given point to a given straight line was Oenopides. According to Proclus, Oenopides (500 B.C.–430 B.C.) interested himself in the problem because he considered it important for astrology. This is almost as queer as the alchemical origin of chemistry. But even illegitimate children are not responsible for the lapses of their parents.

Another way of stating what made the trick of the rope stretchers possible is to say that the sum of the squares of 3 and 4 is equal to the square of 5, or $3^2 + 4^2 = 5^2$, that is, $9 + 16 = 25$.

This set Plato off like a beagle after a red herring. What could be more alluring than this? Here we have *three consecutive numbers*, 3, 4, 5, and

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the sum of the squares of the first two is equal to the square of the third.

The practical Egyptians saw nothing more in this than an easy way of making right angles in their architecture. The unpractical Greeks saw a great deal more, including some of the strangest flights which numerology has ever taken, and including also the germ of much that is of no use whatever in orienting temples, but is indispensable in an age of peace treaties, science, mass production, over-population, and bombing planes. So the score is even, whichever way we look at the board.

THE NUMBERS 3, 4, 5, 6

I have not forgotten Plato's mysterious nuptial number. But before the promised revelation can be given a truly remarkable property of the numbers 3, 4, 5, 6 must be noticed.

A great deal of circumlocution will be avoided hereafter if we use "powers" as we did at school. Take any number, say 10, multiply it by itself, 10×10. Multiply the result again by 10, thus $10 \times 10 \times 10$. Repeat the process, $10 \times 10 \times 10 \times 10$. Instead of writing out these clumsy strings of numbers we condense them as follows, 10, 10^2, 10^3, 10^4, and call them the *first*, *second*, *third*, and *fourth powers* of 10. Similarly for any number and its successive powers. For example, the successive powers of 6 are 6, 6^2, 6^3, 6^4, . . . or 6, 36, 216,

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1296, . . . ; the seventh power of 3 is written 3^7 and is 2187. If n is any number its powers are written $n, n^2, n^3, n^4, \ldots$; thus n^5 means $n \times n \times n \times n \times n$.

Here I have to record two exasperating exceptions to the above simple system of naming powers as the first, second, third, fourth, and so on. The *second* power of a number is called its *square*, the *third* power its *cube*. For instance, the *square* of 10 is 10^2, or 100; the *cube* of 10 is 10^3, or 1000.

In passing I must point out that these irritating exceptions are one of the direct contributions of Pythagorean numerology to mathematics, as we shall see considerably later. They are on a par with the barbarous way of writing ratios and proportions which persists in our school books and which makes mud to the average intelligent child of what it has known ever since it stopped sucking its thumb. Why should the fact that $\frac{3}{4} = \frac{6}{8}$ be disguised as 3:4::6:8? Let some numerologist explain. But we must get back to Plato.

Who can blame him for being fascinated and mystified by the following facts?

$$3^2 + 4^2 = 5^2, \quad 3^3 + 4^3 + 5^3 = 6^3 = 2^3 \times 3^3.$$

It would be a very dull numerologist and a yet duller arithmetician whose curiosity would not be aroused by such an arithmetical miracle. If anyone thinks it commonplace he is beyond arithmetic in this world. The best that can be hoped is that

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he will probably misprove Fermat's "Last Theorem" in the next, for this unsolved mystery, as we shall see in a later chapter, is of the same magic as the miracle which fuddled Plato.

THE MYSTERY UNRAVELLED

It seems singularly and numerologically appropriate that the elusive nuptial number of Plato should have been caught by a woman. In a fascinating article in the *Proceedings of the London Mathematical Society* for 1923, Grace Chisholm Young—a name well known to all mathematicians—elucidated the mystery.

To do justice to Mrs. Young's explanation of what Plato meant would demand the reproduction of her entire article. This being out of the question here, I shall simply give what seems to me to be the central gem revealed by Mrs. Young's penetrating analysis. She concludes that Plato *guessed* and possibly *proved* that the only *whole numbers* x, y, z, w *free of a common factor, for which it is true that*

$$x^2 + y^2 = z^2 \text{ and } x^3 + y^3 + z^3 = w^3,$$

are $x = 3, y = 4, z = 5, w = 6$.

In the preceding section we saw that 3, 4, 5, 6 actually do satisfy the two equations. The clause about "free of a common factor" is inserted to exclude the trivially obvious solutions got by multiplying 3, 4, 5, 6 by the *same* whole number. For

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example 6, 8, 10, 12, or 9, 12, 15, 18, or 12, 16, 20, 24, and so on, also are solutions.

The astonishing thing about Plato's guess is contained in the word *only*. Ingenuity of a very rare kind is required to *prove* that 3, 4, 5, 6 is *the one* solution apart from those described above. The problem is not one suitable for a university examination paper. Few could solve it in a reasonable time, even with all the powerful machinery of modern mathematics.

If Plato proved that part about "only" he deserves a high place among the great pioneers in the theory of numbers. He had no algebra to aid him, and the Greek way of writing numbers was merely an extremely crude sort of shorthand. Moreover, being devoid of algebra, Plato was forced to talk about cubes and squares and actually to think about numbers in these awkward terms.

Mrs. Young's achievement in making strict mathematical sense of everything in the obscure passage—except of course the purely numerological parts, in which she does not claim to be expert—shows that even a philosopher may have to take his hat off to a woman.

Plato was talking not merely about one number, but about several, and all of these were uncovered by Mrs. Young and shown by her to dovetail precisely as the *mathematical* meaning of Plato's words requires. To reach her conclusions, Mrs. Young gave Plato's words the mathematical values

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which scholars in the history of Greek mathematics have ferreted out in many places. All make sense.

One of these dovetailing numbers is the 12960000, or 60^4, already cited; another is 36, which is the square root of 1296. Another is the square of 4500, or 20250000, and this, according to Mrs. Young, is *the* nuptial number. It fits all of the requirements of the second part of the passage. Anyhow, whichever one of all is to be blessed with the coveted title does not matter much. The whole tribe is so inextricably interlocked in the bonds of numerological wedlock that not even the Grand Lama himself could dissolve the marriage.

Why?

To ask why Plato, or his commentators, or his final disentangler have devoted possibly years of their lives and certainly much hard labor to the sort of thing discussed above, is not a silly question, but merely a lack of tact. What would we think of a man or woman who should stride into the middle of a perfectly innocent bridge game and shout, "What are all you idiots playing cards for?" If the hostess were up to her job she would rout the intruder with the unanswerable weapon of perfect courtesy.

The "why" of it all will be plain enough as we proceed. For the moment we may let the hostess speak for herself.

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"Long intercourse with mathematicians has taught me," writes Mrs. Young, "that their covert allusions to mathematics, as throwing light on philosophical or other matters, are usually as profound as their own mathematical knowledge: and some years ago, when I first found myself face to face with the question what mathematical truths Plato was referring to in these oracular utterances, I felt that Plato was here epitomising some part of his own mathematical cogitations, and that, in unravelling this mystery, we should be gaining a clue by which we may be able to trace somewhat of the intellectual biography of one of the most ancient and eminent of Greek mathematicians."

To this I have only one minor dissent, and I think it is justified by the example of Plato and his most baffling excursion into numerology. It is my conviction that not only some eminent Greek mathematicians, but also several eminent English, French, German, Italian, Dutch, and American mathematicians and mathematical physicists have proved themselves to be very queer fish indeed when they have tried to mix numbers with marriage—or with anything else besides more numbers. Monogamy is the best policy.

Keep Out

As Plato's name will recur frequently in our story, I shall dismiss this matter and introduce the next by citing a famous warning of his own.

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"Let no one who is ignorant of geometry enter here."

This warning is reputed to have been posted above the entrance to Plato's Academy. No doubt he used geometry in the broad sense of mathematics just as the French sometimes do today. Now, whenever an apologetic mathematician with an irrepressible inferiority complex deems it necessary —it never is unless there is a better mathematician present—to excuse himself or his trade for existing, he sooner or later drags in this Platonic praise.

I believe that few professional mathematicians who will take the pains to examine for themselves what Plato actually said about numbers and geometry will ever again quote the famous warning. Mathematics is better off without the sort of recommendation that Plato was competent to give. At heart he was a numerologist, so far as his mathematical beliefs went, and this, I think, is true in spite of the great services he rendered mathematics by talking about the work of other men.

All that numerology about the nuptial number was not included idly. It is Exhibit A to lend some color to the contentions of those modern critics who assert that Plato did mathematics far more harm than good.

Is it credible that a mind capable of what Plato said about "better and worse births" could foresee the practically important or intellectually valuable flowering of mathematics which began only when

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irreverent innovators dared to shake off the Platonic incubus after nearly 2000 years of suffocation? Singularly enough it is quite credible, in spite of the critics. As we continue we shall meet more that one intellectual giant, modern as well as ancient, whose numerology is stranger even than Plato's.

Whoever explains these mysteries will discover why our bedevilled race is more receptive to philosophy and numerology than it is to science and arithmetic.

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Chapter III

RECURRENT NIGHTMARES

THAT PERSISTENT THREE

Perhaps a Freudian could suggest a rational explanation of the cosmic numerology which we are now about to explore. Whether such a rationalization of this strangest of all the continents of numerology would be either true or satisfying is another question. All that I know is that a Freudian analysis of it is at once obvious, simple, and rational. But as this book may be sent through the mails the analysis must be left to the ingenuity of the reader.

A very mild form of the "Eternal recurrence" appears as a footnote in James Thompson's (1834–1882) *City of Dreadful Night.* The entire poem may be recalled to the memory of optimists in times of depression. A few lines must be quoted to give the footnote its meaning.

> ". . . . Here Faith died, poisoned by this charnel air.
> I ceased to follow, for the knot of doubt
>   Was severed sharply with a cruel knife:
> He circled thus, for ever tracing out
>   The series of the fraction left of Life;
> Perpetual recurrence in the scope
> Of but three terms, dead Faith, dead Love, dead Hope."

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To the last line is appended the appalling numerological footnote, "Life divided by that persistent three $= \frac{\text{LXX}}{333} = .210$."

Comment seems superfluous. Personally, remembering the Beast, I should have divided by 666 instead of by a mere half Beast.

Whatever truth there may be in the terrible equation must be grasped intuitively. The LXX is 70, the three score and ten prescribed by Holy Writ as the span of a man's sojourn on this earth. The triple trinity 333 is more sinister. But the whole numerological force of Thompson's equation is concentrated in the ghastly decimal .210. As written this is incorrect; it should be the recurring decimal $.\dot{2}1\dot{0}$, or .210210210 . . . ., the digits 210 being *repeated for ever*. If this is not a numerological demonstration that life on this earth is eternal hell, what is it? Thompson intended it to mean this, and only a numerologist can say whether or not he succeeded in expressing what was in his mind.

It will be interesting to glance at one or two more famous instances of this numerological nightmare. Thompson's was not the only good mind which lost its way in black horror while threading the mazes of the infinite, and particularly of the recurrent infinite.

A remarkable and psychologically significant feature of the recurrent nightmare is that nearly

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every man who has experienced it in modern times believes that he is the first to dream the oppressive vision. As a matter of tradition it was known to the ancient Babylonians, and it may be reasonably conjectured that Plato's comparatively mild form of the nightmare was caught from them through Pythagoras.

Numerologists will pardon me for occasionally using the scientific terms recurrence and periodicity instead of the strictly numerological vibrations and harmonies. Either way of speaking suggests waves and wheels, so both may be used.

LONG AGO

One of the few perfect miniatures of the past that time has not yet succeeded in dimming was painted about 325 B.C. The scene is under an olive tree somewhere in Greece. Eudemus, a distinguished disciple of Aristotle (384–322 B.C.) and one of the earliest historians of mathematics and astronomy, is talking with his own students. The sky is as blue as only a Mediterranean sky can be, a lazy breeze just riffles the fresh spring grass, and a swallow lights for a moment on an olive twig. Attention wanders. What the Babylonians may have discovered about the precession of the equinoxes seems a trivial and far off thing in this eternal morning. Eudemus brings his pupils sharply back with a quaint conceit.

"According to the Pythagoreans," he remarks,

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holding up the short baton which he used to emphasize his points, "I shall once more be talking with you, this little rod in my hand, and then you shall be about me, exactly as you are now. And so will it be with all the rest—that sky, this grass and the olive tree; and the bird that just flew away will again take wing, and I shall be saying these same words to you."

To bring out what Eudemus meant I have put some of the words into his dusty mouth, but I feel sure he said it all with a single gesture. He made his pupils *see* what he meant.

It is all to happen again, and again, and again . . . . just like that infernal decimal .210210210 . . . repeating itself without end.

Is it an appalling nightmare of a thought, or is it just meaningless nonsense that need terrify no one? To a gin-free rationalist it is incredible that brooding on this "Eternal recurrence" could ever have driven any educated man mad, but it did.

If the nonsense of it is not obvious without explanation nothing that anyone can say will make it so. You either have a clear head or you have not, and although a clear head can occasionally muddle itself with alcohol or metaphysics or love, nothing in heaven or earth can unmuddle a naturally muddled head. It is a gift of God.

What happens when an irresistible force encounters an immovable body?

In all this I realize that neither rationalists nor

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mathematicians are any better than they should be. To restore the balance I shall quote what seem to me to be drastic but perfectly fair statements of the other side of the case in a later chapter of this book. In the meantime I must get on with the numerologists.

WHILE SHEPHERDS WATCHED

The deeper the excavator's spade digs in Egypt and Mesopotamia, the farther back in time are shoved the beginnings of astronomy, astrology, mathematics and numerology. There are fair reasons for believing that these four "sciences" developed in the order indicated. Certainly universal numerology in the full sense of Pythagoras arose later than the others. Even today it is a bit of a luxury, although the Great Depression cheapened it somewhat.

To a primitive agricultural or nomadic people astronomy was a necessity. A little later, in Egypt, for instance, a crude sort of geometry and a rough but not too ready arithmetic developed as they were required for practical purposes. In the meantime the priests or witch doctors saw to it that astrology did not languish, and considerably later the philosophically minded looked after numerology.

I have not wandered from the recurrent nightmare. As was already remarked the Cosmic Year, which is another name for the first stage of the

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nightmare, was familiar to the Babylonians. How did they get it? Putting aside the Freudian explanation, let us look at a possible astronomical origin of the great dream—for great it is, even if it is mad. Unfortunately what follows must be classed as rank speculation. This does not apply of course to the purely astronomical part.

Years ago I came across many detailed accounts of what follows, which seemed to be based on reliable evidence, and which were put out over the names of reputable scholars. I kept no notes, and cannot now remember a single reference. If any reader of this book knows where the vanished references are to be found, I shall be grateful for the information.

To continue. It is a commonplace that a nomadic, intelligent people use the sky both as their clock and their calendar. Every alert child today knows how to locate the pole star. Suppose some learned priest had been asked to locate the pole star when the pyramids were being built. We saw in the last chapter that it was important to the ancients to be able to do this. Would he have pointed out our pole star? As everyone knows he would not.

About 4000 years ago the star known now as Alpha Draconis was the pole star; about 12000 years hence the pole star will be Alpha Lyrae; about 26000 years hence, whether there is any human eye to see it or not, the full circle will have

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been completed and the pole star will be the one we know. This periodic circling is due to the slow conical rotation of the axis of the earth around the pole of the ecliptic. Its cause is the gravitational tug of the sun and moon on the equatorial bulge of the earth. For reasons we need not go into here, the periodic circling is known as the precession of the equinoxes.

The fact of precession was discovered by Hipparchus about 120 B.C. Quite recently it has been argued that the Babylonians anticipated him. Now here comes the speculative part. At least 400 years before Hipparchus the Babylonian astronomers knew of precession and—what seems quite incredible—*estimated the number of years for the pole star to make the full circle*. The modern estimate, quoted above, is 26000 years; the Babylonians said 36000.

If there is any truth in this it should strengthen the faith of all orthodox numerologists, *for* 36000 is *one 360th of the nuptial number* 12960000. It is little short of a disaster that the nuptial number was not 100 times what it is, for then *the recurrent 36000 would have been its square root*. What could not Plato have done with this?

It is not impossible that the Babylonians could have estimated the 36000, provided they had extended their observations over a long enough period, but it is highly improbable. In those accounts which I cannot locate the number 72000

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also played a part. The full form of the recurrent nightmare, as stated by Eudemus, was also attributed to the Babylonians, or to the Chaldean Shepherds. This seems like stretching it a bit.

Surely it takes no phenomenal imagination to leap from the majestically true recurrence to its ghastly generalization. Gazing out on the night skies for century after century, the shepherds would hand down a slowly changing tradition of the aspect of the sky, until some sufficiently imaginative mind, inspired by a false analogy, should soar to the conception of a Great Year, a Cosmic Year, embracing thousands of common years, in which the life of the whole universe would decline from spring to winter, and again renew itself in a vernal resurrection, only again to die, and so on for ever.

Everywhere the earliest observers looked they had periodicity thrust at their eyes. They were closer to the earth than we are, and the cyclic rhythm of seed time and harvest, the return of the Pleiades, drought and flood, the seasons, even birth, life and death, all these and scores of others were vivid "proofs" of the universality of recurrence. If there is recurrence here, why not everywhere?

In putting this question the way they did those far off watchers of the skies might have been speaking today. Perhaps they are; the recurrence may be starting again where they left off. For it

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seems to be an extremely rare questioner who ever thinks of contenting himself with a modest "how?," or even a timid "why?," instead of committing nature and himself to all sorts of indiscretions by a suggestive "why not?." Consider, for example, the effect of striking the "not" out of Shelley's

"Nothing in the world is single;
All things by a law divine
In one spirit meet and mingle.
Why not I with thine?—"

The dash is Shelley's too. It may be recommended to modern speculators for similar use in popular expositions of current scientific theories.

The particular "why not" of the shepherds ended in this: "When the stars return to their former places, all that has ever happened shall happen again, exactly as it happened before, and so on for ever and for ever."

To give this recurrent nightmare a twentieth century twist, we restate it in the language of general relativity: "All geodesics in the four-dimensional space-time manifold are simple, closed, rectifiable curves." Every word in the preceding sentence is necessary. Needless to say, this great generalization is not due to Einstein. Strictly numerological equivalents can be stated in an infinity of ways. Here is one: "Life and the cosmos are periodic continued fractions, and hence both are irrational."

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I admit that the last is an excruciating mathematical pun, but I was not trying to be funny. Some others do not agree. We shall see toward the end of our story that this very conjecture is seriously debated by obscurantists today as a possible escape from the current mathematicization of God and the universe. If sustained this solution will indeed provide theologians with a happy issue out of all their scientific afflictions. Eristic itself is one phase of the "Eternal recurrence."

THE CENTRAL SUN

In the last century a very popular astronomical speculation—seldom taken seriously by astronomers, however—was that of the Central Sun. Those who purveyed this may conveniently be called centrosols. Finally Mädler replaced the dream of the centrosols by a mathematical point, a much finer conception, because it was true. Unfortunately it was also trivial. Mädler sought to locate the centre of gravity of the stellar universe from insufficient observations, and in doing so put forward a bold claim to be immortalized as a numerologist.

But there is nothing new under even the Central Sun. The centrosols' conception of the Cosmic Year is practically identical with Plato's. Whether he ever got as far as the complete horror of the "Eternal recurrence" I do not know, and I expect

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to remain in ignorance as long as I live, for I have no intention of ploughing through all his works to find out.

What Plato did believe seems to have been roughly as follows. With less imagination than his predecessors, he contented himself with an arbitrarily assigned 10,000 years as the complete period in which the planetary orbits would repeat themselves against the celestial vault containing the "fixed" stars as a background. This 10,000 ordinary years was a year of years, a Cosmic Year, responsible for periodic seasons in the affairs of this world. These seasons might manifest themselves in the rise and fall of empires, age-long depressions, the periodic decline of virtue and piety, the like for vice and impiety, and so on, precisely as any skilled player on the lying lute of words would be likely to improvise. And just as in an ordinary year the familiar sun suffers eclipse, so in the Cosmic Year the Central Sun, or the Godhead, would also be temporarily obscured from the eyes of men by the interposition of some Black Body. The consequences of such a terrible eclipse are incalculable, even by tensors.

Plato was handicapped by his lack of telescopes. The centrosols had more than they knew how to use. The great Greek cosmologist might have done much better had he know that our galaxy (the Milky Way) is a swarm of millions upon millions of suns compacted in an enormous grind-

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stone and not the souls of the dead or superfluous milk from the breasts of whoever it was that suckled the infant godling. The centrosols placed their Central Sun somewhere near the centre of the grindstone, and imagined all the hosts of heaven wheeling in one tremendous year about this gigantic immovable monster.

I have included this to emphasize a point which, it seems to me, contains a hint which we occasionally forget in the joyful exuberance of our modern theorizing. It is true that the galaxy does have a motion of some sort, if not as a whole (although some authorities believe that it all spins like a Catherine wheel), then in the form of slowly drifting star clouds. These things are not asserted here as facts. For a competent opinion the astronomers must be consulted. But the fact of motion of some sort is currently accepted.

Now, the point is this. Neither the controsols' nor any other numerological hypothesis had anything whatever to do with the discovery of the real motion. It did not inspire astronomers to point their telescopes night after night at the starry heavens in an attempt to verify a baseless guess. For all the effect it has had on the advancement of human knowledge the Central Sun might have remained unborn in its conceiver's brain. Wild speculation is not always sustained by science, nor is facile guessing invariably a spur to the hard, painstaking labor by which the enduring things in

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science are achieved. If scientists themselves but seldom take their working hypotheses seriously, why should philosophers, theologians, mystics, and numerologists?

To finish off Plato's encounter with the nightmare, let us recall the opening sentence of the passage concerning the nuptial number.

"For that which, though created, is divine, a recurring period exists, which is embraced by a perfect number."

We shall have something to say about perfect numbers later.

THE SUPERMAN

It is impossible in a mere sketch of the history of the "Recurrent nightmare" even to catalogue its eminent victims. One, however, cannot be overlooked.

In one form or another the nightmare has afflicted scores of the world's leading thinkers. For this reason I am always chary when anyone says "Stop and think." To judge by the history of thought, better advice would be "Run like the devil, and don't think, whatever you do." Had Friedrich Nietzsche (1844–1900) run occasionally he might have died sane. He took to his heels too late, and the "Recurrent nightmare" leapt upon him and smothered what remained of his mind.

In his autobiography, *Ecce Homo* ("Behold the Man"), he tells us that the idea of "Eternal recur-

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rence" overwhelmed him suddenly while he was walking in the woods by the lake of Silva-Plana at Sils-Maria. He thought he was the first.

The Nightmare is an integral part of Nietzsche's philosophy, and he experienced it in its completest, most self-contradictory form. The whole universe is to be shattered or degraded into its ultimate particles (he would have substituted waves for particles if he had lived to 1926), only to come together again with all of its events in precisely the same order as that in which they have happened, not once, but countless millions of times. Man is to remember and not to remember. This sort of thing is to go on repeating for ever and for ever and for ever, like an idiotic recurring decimal. The crucifixion, for example, will recur an infinite number of times. Incidentally, in this connection, note the self-conscious audacity of the title of the autobiography. The superman philosopher, who permitted himself to be driven mad by a childishly obvious psychological contradiction, identified himself at the last with the teacher whom he had reviled for his "slave ethics." Poetic justice, with a vengeance.

Nietzsche is perhaps better known as the leading modern exponent of the doctrine of the superman. Plato held similar but not quite so fully matured views.

The numerological exploits of these great thinkers raise serious doubts in any moderately critical mind

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as to this whole business of leadership in thought. The Nietzschian doctrine, when united in patriotic wedlock with the Treitschkian during the Great War, gave birth to some very skittish cattle. Genius, no doubt, is often just a little mad, although the really first class specimens have been the sanest of the sane, and most of them could have successfully passed a Wassermann test. Just how much of a given carcase can be tainted before all of it is unfit for human consumption?

A HARMLESS VARIANT

Theosophy has not been one of the varieties of my own religious experience, but I have had opportunities for observing the creed in practice. All the Theosophists I have known have been gentle souls, living at peace with their neighbors and attending strictly to their own business. Their version of the Recurrence is as mild as their lives. But I doubt whether all of them realize that it is even older than they believe it to be. There is a lot of it; the following excerpt from *The Ancient Wisdom*, by Annie Besant, will give some idea of the flavor of the whole. Incongruously enough this Wisdom emanated from Hollywood, of all places in the Seven Globes. But that was in 1897.

"On the three lower planes of His evolving realm the planetary Logos establishes seven globes or worlds. . . . . These are the 'Seven small wheels revolving, one giving birth to the other; He builds

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them in the likeness of older wheels, placing them on the imperishable centres.'

"Imperishable, since each wheel not only gives birth to its successor, but is also itself reincarnated at the same centre, as we shall see."

The further gyrations of the wheels may be left to experts. The fraction quoted is sufficient for the full evolution of the "Eternal recurrence." If the Theosophists are not familiar with Pythagoras (sixth century B.C.) and his successors in the Golden Age of Greece they may be interested in finding a confirmation of their vision, almost shadow for shadow, in the cosmogony of the master himself. The closest inspiration of their version is not the famous wheel passage in the Old Testament which immediately comes to mind, but the cosmic numerology of the colossus whose exploits will be considered presently.

Before saying that all eternal recurrence is merely a symptom of wheels in the head, have a look at the next. There may be something in it.

CIRCLES OR PARABOLAS?

Like the final crash of some eternal symphony that has been seeking its lost chord for ages, the latest reverberation of the "Eternal recurrence" stuns us with the sublimity of complete harmony which sums up everything that has gone before. Only the tuning fork of time will decide whether this great chord is science or whether it is a glorious classic of

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numerology. At the present moment it might be either, so it may justly find a place here.

If the reader will glance back at what follows after having read Chapter V, and especially what is said there about a certain touchstone, he will see that the scientists responsible for the following are *not* numerologists.

To cover the theme in a reasonable time it will be necessary to use a few technical terms. These however need not baffle anyone who has profited by any of the excellent current popularizations of mathematical theories of the universe by eminent scientists which are now available. I shall presently speak their language.

Let us glance back first. Those early speculators who invented the "Eternal recurrence" were confronted by many possibilities for their theory of the universe as a whole. Only two of these need detain us. Was the course of eternal time and everything in it a perfect circle, or was it some other curve, possibly open in both directions to infinity? Could it possibly be a parabola—the path, very approximately, of the flight of an arrow? With their obstinate and stultifying predilection for complete roundness in everything the Greeks pitched on the idiotically simple circle.

We moderns have more imagination. Picture a stretch of smooth sandy beach. Toward the receding wave the glistening sand is deliciously cool to feet that love cold and moisture. As the sand

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slopes toward us in the glaring sun it gets hotter and hotter.

An athletic sandflea lifts himself out of his burrow in the wet sand, and gives one first, tentative skip toward us. He comes down on cool territory and takes a more confident skip. This landing is less pleasant. The sand is hotter than he had anticipated. The next leap is tremendous, and the one after that almost beyond the powers of mathematical analysis to calculate. Each graceful

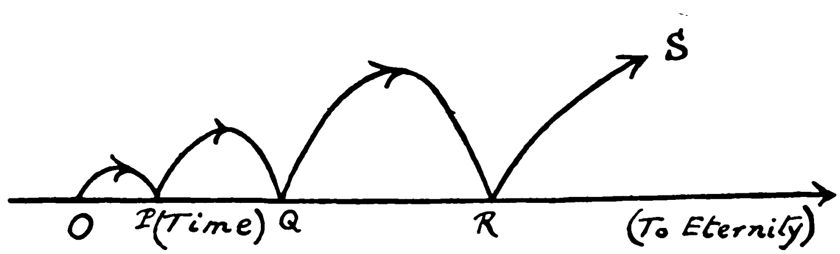

FIG. 2

arch of his recurrent skip is approximately a parabola, and each arch is higher than its predecessor.

To interpret this cosmologically, let S be the sand flea, OR the beach, and O the hole from which S emerged. Then S is the Cosmos, OR the axis of time, or the direction of increasing time from the creation, O (origin). The periodic expansion and contraction of the Cosmos (or Universe) is represented by the sublime rise and fall of the arches and the flight of the flea. At P, Q, R, . . . the Universe contracts to nothing, as it was in the beginning, at O. The steepness of the arches, or the accelera-

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tion and deceleration of the skipper, pictures the rate at which the universe transforms matter into radiation, or vice-versa. This is the Eternal Recurrence with the modern enrichment of an exponential factor, the effect of which is to produce bigger (and possibly better) cycles from creation to annihilation as time speeds into future eternity.

The points P, Q, R, . . . are open to suspicion. It has not yet been satisfactorily settled whether S passes safely through these points and skips more than the single first arch, or whether he explodes at each of the critical points and demands the birth agonies of creation all over again as he did at O. Anyhow, he keeps on skipping, and we have the Exponential Eternal Recurrence. Sir James Jeans seems to favor one and only one explosion; others prefer an eternal barrage.

All this can be restated in cold, hard mathematics. In the earlier cosmology of general relativity one of the hypotheses prescribed a curved space-time. If the curvature were of one sort, half the stars we see in the sky would be mere images of the other half—if we happened to be living after the light of the real stars had gone completely round the universe. Another possibility was that of curvature in the direction of time alone. This however was less enthusiastically received; it was pretty close to the middle variety of the Recurrence. But all of these echoes of the past were obtained from static line elements.

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Non-static line elements were ignored in the early days of relativity partly on account of mathematical difficulties, partly because there was plenty to do in working out the consequences of the simpler variety. In passing, I must emphasize that none of this will in any way affect the classical predictions which Einstein made and which have since been repeatedly verified experimentally. All this belongs to an offshoot of the theory, in which further hypotheses must be adjoined to those of general relativity in order to get a foothold on cosmological problems. Not even a scientist can lug rabbits out of a vacuum, although many think they do. He *must* make hypotheses, in spite of Newton's (false) boast "Hypotheses non fingo."

As early as 1922, Friedmann had obtained non-static solutions of the gravitational equations. They passed almost unnoticed till 1927, when the young Belgian abbé Lemaître independently had a similar and more thoroughgoing success. I think it may be said that a mathematician would not be satisfied with a static solution when others exist.

One consequence of Lemaître's work was his own brilliant hypothesis of the expanding universe; another, the periodically expanding and contracting cosmos described above.

Another is that "in the beginning" the Cosmos was a single, gigantic atom, which looks suspiciously like numerology, because it would seem to be one

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of those fascinating speculations that can be neither proved nor disproved.

Doubtless many more recurrences will be discovered by mathematicians and discussed by physicists, and possibly we shall some day either bury the "Eternal recurrence" forevermore, or we shall establish it on a firm mathematical foundation, checked and tested in its every stone by the most exacting experiments which human ingenuity can invent to torture nature. Neither end is yet in sight.

In the meantime, any numerologist who feels the urge to rush in may be advised that this is a region where Einstein himself treads gingerly.

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Chapter IV

500 B.C.

IF—

"Why, man, he doth bestride this narrow world like a colossus." Who? Surely not the Julius Caesar of whom it was said. To the scientific habit of mind which has made our present attempt at civilization possible and which is rapidly making it impossible, no Roman ever contributed anything. Neither Roman law nor the Pax Romana brought about the industrial revolution; the sort of thing Pythagoras started did.

If Pythagoras (sixth century B.C.) did not take the gigantic stride toward the scientific method that Archimedes (287–212 B.C.) took, he certainly stumbled forward a measurable step. To Pythagoras belongs whatever honor there may be in having made the first recorded discovery of a definite physical law, that of musical intervals.

Let us pause here for a brief numerological speculation. Suppose that Pythagoras, instead of following numerology after making his splendid discovery, had continued along the harsher road of scientific experiment, what then? Instead of having to wait for Galileo to initiate the age of modern science, with his rediscovery of the scientific method,

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in 1581—when he timed the oscillations of the great bronze lamp in the cathedral at Pisa against his pulse—our scientific age might have been well started at Croton in 530 B.C. And what then? With this flying start of 2000 years where should we be now?

Before hazarding our guesses we may notice the other great chance to follow science which the world passed up in 212 B.C.—the year in which the practical Roman legionary butchered the venerable Archimedes. Had the world followed Archimedes and not Plato in its mathematics and in its fact-finding approach to nature it would have become scientific 1800 years before it did. But it preferred the mystic Plato to the scientific Archimedes.

To return to our question. Where would the world be today if Pythagoras had preferred science to numerology, or if mathematics had listened to Archimedes rather than to Plato?

One guess is that Western Civilization would have perished about the year 100 A.D. Julius Caesar would never have terrorized the British Druids. Nor would he have written a line of *De Bello Gallico*. Instead he would have recorded with modest pride the destruction of Athens and the total extinction of all of its bourgeoisie, bankers, captains of industry, communists, mathematicians, philosophers, politicians, proletariat, and scientists by the first and second cruising squadrons of the Roman Air Fleet in a two-hour flight from Ostia

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with incendiary bombs, gas bombs, demolition bombs, cholera germs, anthrax germs, and bubonic plague germs.

The sequel to this unachieved history is obvious. Having destroyed Athens, the squadrons returned to a floral triumph at Rome. For all of a century after this glorious exploit the Roman Republic bloated on reparations from a supine and spineless world, only to creep into an economic paralysis for lack of brains or science to conduct its affairs intelligently. Finally, about the year 300 A.D., universal barbarism having blotted out the very memory of science, what remained of homo sapiens returned to its ancestral trees and lived happily ever after on numerology and nuts.

One answer to the "where" is therefore "up a tree." So much may be credited to numerology. And we should have been spared 666, if not 606, had Pythagoras stuck to science.

A TURNING POINT

Having made his brilliant discovery of the law of musical intervals, Pythagoras did exactly what more than one eminent modern physical scientist has done. He proceeded to indulge in an orgy of mathematical speculations on the nature of the universe as a whole, got numerologically drunk, and died scientifically of intellectual delirium tremens.

The scientific death of Pythagoras is one of the major turning points in the history of mankind.

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If this seems an overstatement we need but reflect on what this world might be if it had got the scientific method in 500 B.C. instead of in 1581 A.D.

From the Pythagorean disaster however the world did get some very useful working mathematics as a practical byproduct. It also inherited the seeds of some truly sublime pure mathematics from all the sorry muddle which the Neo-Pythagoreans made of what the master numerologist left them.

But all of these things were unregarded trifles compared to the vision of universal unity which the master dreamed. Science still dreams it. What will our disillusioned successors 2500 years hence—if any survive that long—have to show as the useful or interesting byproduct of our own orgy of speculation? Only a numerologist knows.

Pythagoras' law of musical intervals is set as a laboratory exercise in school physics. Possibly one of the easiest ways by which boys and girls of today verify the law was that devised by the master himself. This however is not the really vital part of the man which has survived. His spirit goes marching on through theory after theory of the universe, until it seems that modern physical speculation is proof positive of the Pythagorean doctrine of the transmigration of souls. No sooner is one numerological speculation buried than its

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spirit passes into another. The psyche of all is the same.

Any numerological theory of the universe seems queer to a mathematical skeptic only when he persists in staring at the modern aspect of the Pythagorean dogma. Transpose the abstruse metaphors of modern mathematical physics into those of the older mysticism, and the illusion of queerness vanishes. Spectacular novelties take on the familiar form immortalized by Pythagoras when he turned numerologist. Number is at the bottom of everything; the Pythagoreans in the sixth century B.C. said that number *is* everything and that everything *is* number—essentially. They meant that *is* in the fullest sense. More than one modern classic of high physical speculation says exactly the same thing, and numerology says Amen.

Whether the numerologists permit it or not, it must be emphasized that not all scientific speculators go the length indicated. But as this story is about numerology, and not about science, we are interested here only in those who have turned the corner with Pythagoras. Once for all the other side of the case may be stated in the words of one of its leading advocates.

"*The only object of mathematical physics*," according to Dirac, "*is to calculate results that can be compared with experiment*." Italics and all, this is from Dirac's book on *Quantum Mechanics*, which Einstein has characterized as the most scientific

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and logically soundest account of this much speculated about theory which has as yet appeared.

Having given the anti-numerological devils their due, we may return to the saints, whose exclusive company we shall enjoy henceforth.

WATER OR NUMBERS?

Thales, first of the seven sages of Greece (640–550 B.C.), summed up the thought of a lifetime in the elaborately false statement that "Everything is water." His successors today substitute electricity for water.

Although he was wrong, and his lost cause is no longer debated, even at Oxford, Thales occupies a unique niche in the pantheon of universal numerology.

Outside of purely theological speculations on the nature of the universe as a whole, Thales' water theory seems to have been the first attempt to unify nature. More important for our story is the conjecture of some authorities that Pythagoras caught the itch for universal unification from his master and teacher.

Pythagoras went far beyond his master Thales when he declared that everything is number. Before seeing who Pythagoras was, let us look at that last statement. The meat of it is not mine; I neither agree nor disagree with what it implies, for to me it has no meaning whatever. Just why

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is it a great step in advance to say that everything is number, when some things have only a far-fetched connection, if any, with numbers, instead of sticking to the honest lie that everything is water? If we could answer that question satisfactorily we should have plumbed the deepest mystery of numerology. Some believe Pythagoras was greater than Thales because he was abstract. This merely begs the question.

For anyone who is technically inclined, one variant of the question may be given a modern twist. Why is it significant to restate physical laws so that they may be formulated mathematically by varying an integral? Putting aside a purely mathematical convenience, what else can we find in such a formulation?

Some mathematical physicists say they can find nothing. Others repeat in effect the reason given by Maupertuis. According to this Eighteenth Century philosopher and mathematician we find a "principle of least action" in nature because God put it there. With this, I suppose, no devout person will disagree. But when Maupertuis went on to insist that God put least action into nature because God hates waste, we are at once on shaky ground. The exact opposite is true in biology and evolution. Even Pythagoras never went so far, although some of his modern disciples have gone much farther. There we may leave them for the present, to catch up with them later.

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THE COLOSSUS

The life of Pythagoras, colossus of numerology, fits the giant like tights. No two legends agree in detail, but all ascribe certain characteristics to the man which, presumably, have some foundation in fact. No human being whose name is counted among the great in mathematics—unless possibly it be Cardan, who was miles below Pythagoras—has had so colorful a life or so curious a character.

Pythagoras was at once a very considerable traveller, an eager student of all the lore of his time, a mathematician of the first rank, a rhapsodical mystic, a successful lecturer, a great showman, and a founder of lodges and secret societies. Here we have incidentally the ingredients for a charlatan of the first water. But Pythagoras was never a quack, even when he expounded medicine. He believed what he said, or at least had the genius for making others believe that he believed what he said. There can be no doubt that he deceived himself honestly and completely. But how any mind with the intelligence of his could have taken itself in as Pythagoras' did is a mystery.

The legend of his life covers the span from 584 to 495 B.C. The dates, of course, are as doubtful as the rest. He was born in Samos, and one very dubious legend makes him a Phoenician. As a youth he accompanied his father on trading trips to the shores of Greece and Asia Minor. About

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550 B.C. he left Samos and went to Lesbos, "where burning Sappho loved and sang," but probably she was as cold as a potato when he arrived. However, he did hear Phericides, who seems to have been something of a numerologist on his own account. Proceeding to Miletus, Pythagoras took in the discourses of Anaximander and Thales. The numerological mischief was done then and there.

Thales had learned much from the Egyptians. He persuaded his young follower to go and do likewise. All legends agree that Pythagoras travelled extensively in Egypt, most assert that he commerced freely in Geometry, Astronomy, and other matters with the priests at Thebes and Memphis, and at least one states that he resided twenty-seven years in Egypt—which seems excessive for a man of his acumen. Iamblichus, in his *Life of Pythagoras*, is my authority for this and the next; it is in the fourth chapter of his book. The same legend says Pythagoras was among the Egyptian captives taken by Cambyses to Babylon, where he spent twelve years with the Magi. From them he "drank in" Numbers, Music, and other disciplines.

If this is true it may have a bearing on Plato's 60^4. It is said that while in Babylon the captive delved into the mysteries of the Chaldeans and the tangled theogonies of India. The numerological importance of all this—if it happened—is immense. Not all of our own aberrations can be blamed on the Greeks.

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The dates at this point become a little more hazy, and I shall leave them to the imaginations of historians. Having been freed from wherever he was, Pythagoras returned to Samos and his aged parents. According to Iamblichus, the rover was now sixty. Before long he was off again, to Crete, to Elis, to Sparta, and to Delphi. Scholars may decide whether the oracle was doing business when Pythagoras visited Delphi. If it was, it probaby learned more from him than he from it. He returned to Samos, where he founded his first school. It closed for lack of students. The reasons for this failure would doubtless be interesting, in view of the great teacher's unbounded success later. It almost equalled Bergson's as a lecturer, in New York.

We next hear of him at Sybaris in Southern Italy. There he acquired none of the exquisite laxity which we usually associate with that resort. All his mature life Pythagoras was a somewhat austere sort of person, with his mind constantly turned toward what he considered the higher things of life. Next there is a rumor of him at Tarentum in Sicily. The legend declares that he taught medicine at Tarentum, but omits to state whether he knew anything about it. Anyhow his teachings infected Hippocrates, "the father of medicine," and from him the Arabian medicos contracted some of the wierdest numerological quackery in the history of medicine. To find its equal we have

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to step right into the present and read the advertisements of patent radium cures.

At the Greek colony of Croton in Southern Italy Pythagoras finally settled down. There he founded his great school, his Brotherhood, and one of the first successful attempts at communal living. Incidentally he married Theano, the young and beautiful daughter of his host. The wide disparity in their ages proved no obstacle to happiness.

Theano survived Pythagoras and wrote his life. Unfortunately her account has been lost, so we do not know what she really thought of her distinguished old husband.

The episode of his marriage emphasizes a fine trait of Pythagoras' character. Theano was but one of many women whom he admitted to his lectures on an equality with men. The others were no more of the hetaerae than was she. The contrast with the classical Greece of Socrates, Aristotle, and Plato is a striking tribute to the civilized liberality of Pythagoras. Greek love as a respectable institution was a later development.

His liberality of mind and life was in fact the poor old man's undoing. Not content with preaching or lecturing about what he considered the good life, he applied his theories to practical politics, just as Bertrand Russell did in the Great War. The populace knew better than he did what it wanted.

One tradition asserts that Pythagoras escaped

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to Tarentum, where he died in peace; another states that he perished with his disciples in the flames. Croton would have none of him or of his enlightenment. The outraged citizens made an example of his school by burning it to the ground. Whether Pythagoras escaped or not the outcome for Croton was the same. Normalcy returned with the incineration of the school and its contents, human and material, and brooded peacefully once more on the arcadian scene. There it has sat like a dumb dodo ever since.

This then is the picture 500 B.C. leaves on the record—a single torch flaring against a pitch black night of ignorance, bigotry, and prejudice. It would be a pity to spoil such a picture by pointing out the smokiness of the light, so we shall leave it here, and examine the light by itself next.

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Chapter V

IN THE BEGINNING

HIS MONUMENT

"If you seek his monument, look about you," directs the epitaph of Sir Christopher Wren in St. Paul's Cathedral. It might be applied today to Pythagoras but for one gross blemish on the modern scene which will obtrude itself as we journey down to the present. The enduring monument of the colossus is a certain mathematical habit of speculative scientific and mystic thought.

As the fabulous figure of the giant recedes into the mists of legend his numerology steadily brightens. Everywhere it survives and shines, even in the severely scientific theories of many who consider numerology slightly disreputable. The taint, if it is a taint, seems all but inescapable.

Pythagoras was a pioneer. From the main business of this chapter it will appear that Pythagoras was first in each of three fields which we still cultivate, philosophy, science, and numerology.

Like many pioneers when all the world was fresh, Pythagoras hit upon a generalization which even the most critical will admit is at once simple and great. But it does not follow that one iota of his generalization, for all of its greatness and

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simplicity, is true, or even that it makes sense. Nevertheless it survives, just as the fame of Ananias does.

To lead up to the great generalization, which will be restated later in a modern form, we shall presently have to examine a few of the thousands of strange things Pythagoras and his followers actually said about the part played by numbers in the scheme of nature—if there is a scheme. Instead of asking whether any of this is true, which is a silly sort of unscientific question in the twentieth century, it is sufficient to see whether it hangs together. Then whoever cares to be inquisitive can ask why on earth rational human beings ever thought such things, and what came out of their cogitations.

Although it may surprise some who have not already noticed it, an instructed mind today is not concerned with the search for truth. To a sophisticated seeker the search would be like looking for the Holy Grail in a mare's nest. Our chastened age has outgrown that sort of research. We are modestly indifferent to truth in the sense in which Pythagoras and his successors thought they were seeking it. Convenience is sufficient. Even consistency is somewhat of a luxury.

It makes not one particle of difference today whether a particular mathematical theory of the universe is fantastic nonsense, provided only the theory is of some use for a week or more in guiding scientific work. Because some theory makes cor-

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rect predictions in three or three hundred instances is no evidence that it is more than a fictitious scaffolding of imaginary and unnecessary lumber. In so far as anyone believes the contrary, to that extent is he a numerologist in the traditional sense of the Pythagoreans.

The touchstone is belief, in the fullest sense of the word, theological, mystical, or other. Whoever believes mathematical theories of the universe to be anything more than convenient maps that may be radically revised or torn up at any moment is a numerologist. A scientific worker who holds no such belief is a scientist. There is of course no stigma on either term. The distinction is self-evident to anyone who undertakes a dispassionate comparison of ancient and modern mathematical theories of the cosmos.

A sinister feature of numerological theories is their persistence long after they have usefully fulfilled their purpose. Scientists toss them aside as carelessly as they discard an old pair of shoes; others pick them up, cherish them, fight for them, die for them, or force others to die for them.

> "The evil that a theory does lives after it;
> The good is oft interred with its bones,"

as Bacon would certainly have observed had he written Shakespeare's works. In passing, the fact that Shakespeare said something even truer is a sufficient refutation of the numerological

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Baconians—a numerous and hairy tribe which we cannot pursue here.

With the baser sort of numerology, beloved of Pythagoras and certain of the Christian Fathers, which puns with numbers, we shall have but little to do in its modern phases. The greatest difficulty in appraising some of the latest universal numerology is just this point of numerical punning. Is a particular speculation of the baser sort, or does it rise to the dignity of the Pythagorean universal numerology? No general rule for deciding has been proposed.

In the end the personal equation settles the matter. Numerology after all is an art rather than a science. What one critic thinks sublime another, not so green, sees through immediately as a tawdry fraud on the real thing. But so long as a Christmas tree makes some child happy I see no occasion for scoffing because the innocent lamb prefers gaudy glass balls and fireproof snow to the chaste whiteness of the Hermes of Praxiteles.

FIRST IN PHILOSOPHY

Only a mind like that of Pythagoras could have given numerology sufficient momentum to keep it moving in a straight line from 500 B.C. to the present. No mere mathematician could have budged it. Nor could a philosopher, a mystic, or a scientist have set it moving. It required a union of all four to give the initial shove, and Pythagoras

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was the union. Possibly this is the ultimate numerological reason for Pythagoras' peculiar veneration for the number four.

To Pythagoras is ascribed the invention of the word philosophy. If the following sentiments usually attributed to him are indeed authentic they give a matchless summary of the man's life, his character, and his ideals. Whether or not Pythagoras ever said these things he certainly lived them, and some of us might do worse than follow his example.

"I have no trade," he declared; "I am a philosopher."

"And what may that be?" he was asked.

"This life," Pythagoras explained, "may be compared to the Olympic games. For in that concourse some seek glory or strive for wreaths; others, peddling goods, pursue profit; others again, less base than either, go to the games neither for applause nor for gain, but merely to enjoy the sport and keep abreast of the times.

"In the same way we men quitted our celestial home and came into this world, where many toil for honor and the majority for gain, and where but a few, despising greed and vanity, study nature for its own sake. These last I call philosophers."

He goes on to say that just as at the games the part of a spectator without any personal interest in the outcome of the contests is the noblest, so in this life the contemplation and patient study of

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nature for their own sake are infinitely nobler than any other pursuit.

We have put a universe between ourselves and this ideal. The study of nature is science, and science is more hotly pursued today for the power which it bestows than for its own sake.

The very things which Pythagoras despised are the raison d'être of many of the greater scientific research institutes endowed today by a host of opulent and generous benefactors. Perhaps if these canny patrons of science had ever played any game for its own sake they would have been less reckless in their generosity, and philosophy—as Pythagoras defined it—would not have to cringe, blush and equivocate every time it dares to hold its head up.

But there is another side to this story, which was overlooked not only by Pythagoras, but also by Archimedes and Plato. Human slavery is no longer considered decent by decent people as it was in the aristocratic Greek state. Our pursuit of scientific power is not wholly base—if it be not ignoble to lighten human burdens and make men something more than brute beasts. Like everything else human even the purest of pure science is healthier for an occasional dash of impurity.

THE PURITY OF NUMBERS

Had Pythagoras been invited to define numerology after disposing so neatly of philosophy he might

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have described it as the love of numbers for their own sake. That, in brief, seems to have been his somewhat bemused attitude toward life and the universe. Pythagoras was the first and greatest of the great pure mathematicians, and to such geniuses all things are pure mathematics.

This phase of his numerological doctrine is still a cardinal article of the creed. Why those who specialize in the theory of numbers should glory in their purity is another of the unsolved riddles of numerology. One of them even went so far as publicly to thank God that the greatest charm of one singularly beautiful discovery he had made was its scandalous uselessness for any scientific purpose whatever. But alas for mathematical vanity! This very discovery proved useful in the study of crystal structure, and the latter, unfortunately for mathematical purity, is at the bottom of the theory of metals, fine steels, big guns, and the devil only knows what else.

In fairness it should be added however that the arithmetician (H. J. S. Smith, 1826–1883) who thanked God was only joking, and knew that he was joking. I rub this in because the joke has frequently been used despitefully by intensely earnest scientists as an argument against the study of mathematics for its own sake. Smith little dreamed how disastrous a sense of humor can sometimes be. He died in time to escape decapitation by his own boomerang.

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In the true Pythagorean vein is a more recent remark by another mathematician (G. H. Hardy, 1878–). The theory of numbers, Hardy pointed out about the time the treaty makers were busy at Versailles, has never robbed a single black man of one yard of his territory in the advancement of civilization. If this was not intended ironically Pythagoras would have approved of it to the echo, for he could not possibly have forseen that the advancement of civilization must inevitably disprove it to the last yard.

If mathematics is the handmaiden of science, and if science is the servant of civilization, and if further mathematics is built on numbers, it would seem to follow that numbers are not so inhumanly pure as Pythagoras imagined.

FIRST IN SCIENCE

There can be little doubt that Pythagoras was the first to discover a definite physical law. The traditional story of his epochal discovery brings out many significant facts in the career of numerology. Nowhere is the oppressive authority of the numerological way of looking at nature better illustrated than in this absurd fable.

For centuries after Pythagoras abandoned the scientific method the world followed him in his numerology without once doubting his creed. It seems incredible that for century after century not a single human being had the curiosity to

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repeat his fabulous experiment. It could have been done in five minutes in the poorest hovel of the squalidest village of the Middle Ages. But nobody had the gumption to doubt. Belief was so much more comfortable and respectable. Just like most of us those lazy louts preferred to take their science sitting. At that they were no lazier than those of our contemporaries who have devised theories of the universe from mediaeval theology, sloppy mathematics, and tobacco smoke.

This is the fable. Passing a blacksmith's shop one day Pythagoras noticed that the clang of all hammers but one pounding the anvil successively produced harmonious chords—the octave, fifth, and third. The clang between the fifth and the third was discordant. The differences of the weights of the hammers accounted for everything, as Pythagoras found on performing the first recorded physical experiment in the history of science.

According to the legend he suspended four weights, equal respectively to the weights of the hammers, by four equal strings. On striking the strings he discovered that the sounds corresponded exactly to those of the hammers. From this he devised the musical scale.

This was swallowed whole for centuries.

A not unreasonable conjecture states that he discovered the law of musical intervals by experimenting with a stretched string, SPT, and a moveable bridge, B, as in the figure. By slipping

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the bridge along the sounding board ST, and observing the pitch of the note emitted when SP was plucked for various positions of B, he found that the pitch depended upon the length of the vibrating string SP. In particular he discovered that the lengths SP which emit a given note, its fifth, and its octave are in the ratios of 6 to 4 to 3.

These numbers 6, 4, 3 are three terms of what Pythagoras called a *musical progression*. Notice

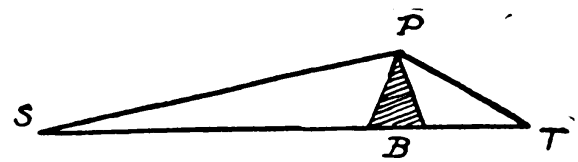

FIG. 3

in passing, for reference when we come to the tougher numerology, that

$$\tfrac{1}{4} - \tfrac{1}{6} = \tfrac{1}{3} - \tfrac{1}{4}, \quad 4 = \frac{2 \times 6 \times 3}{6 + 3}$$

Today we would say that 6, 4, 3 are in *harmonic progression*, without any musical afterthought.

As we have started a little serious arithmetic, we may as well get it out of the way once for all by giving the general definition: A sequence of numbers a, b, c, d, e, f, ... is said to form a *harmonic progression*, if the numbers $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \frac{1}{e}, \frac{1}{f}, \ldots$ form an *arithmetic progression*. Now a sequence of numbers x, y, z, u, v, w, ... is said to form an

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arithmetic progression if the successive differences, y − x, z − y, u − z, v − u, w − u, ... are *equal*.

For example, 4, 6, 8, 10, ... are in *arithmetic progression*, because each of 6 − 4, 8 − 6, 10 − 8, ... is 2. Looking at the first definition now we see that

$\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, ... are *in harmonic progression.*

As another example, for the reader, one of the following is an arithmetic progression, the other is harmonic,

$2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, ...
$\frac{2}{5}$, $\frac{2}{7}$, $\frac{2}{9}$, $\frac{2}{11}$, ...

After this slight lapse we can continue. But I thought it might interest some harassed high school boy or girl to see where the silly A. P. and H. P. came from—old Pythagoras. These progressions are indeed silly unless they are used for something. Otherwise the hours squandered juggling them had much better be spent on something less Greek. But tradition dies hard, if it ever dies.

The details of Pythagoras' great discovery matter little. The fact that a beautifully simple relation exists between *small whole numbers* and *musical sounds* astonished and mystified the Pythagoreans, as well it might have done. Few human beings ever find anything so beautiful.

If it seems an easy thing to do, try to find a numerical law which is true and universal for any simple natural fact of everyday life. Father

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Mendel, growing peas, turned a similar trick when he linked numbers with heredity. Philosophers and numerologists had been growing peas ever since Adam and Eve were forced to work for their living, and all had been content to break their backs until it occurred to Mendel a little before 1866 to use his eyes and his head.

Why, Pythagoras, asked, *have the first six whole numbers any connection with musical consonances?* For example, strings of the same material but of different lengths when subjected to the same strain give the perfect consonances of the octave, fifth, or fourth respectively when their respective lengths are as 1 to 2, 2 to 3, or 3 to 4. Here was a genuine mystery.

Compared to any of the mysteries confronting physical science today the one which puzzled Pythagoras seems childishly simple. But that, I suspect, is only because his is old and familiar, while ours are new in form if not in substance. Anyhow, the "why" of Pythagoras was answered satisfactorily only in the Nineteenth Century, when it was shown that the ear analyzes all compound vibrations (enter, numerology!) into simple vibrations. Without going into mathematical technicalities it is impossible to be precise, but for any who are interested it may be recalled that the "why" is answered thus. By Fourier's theorem a given periodic function is resolved into a sum of simple periodic functions whose periods are sub-

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multiples ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ...) of the period of the given function. But this has little to do with numerology.

My luck may have been poorer than the ordinary, but I have heard of only half a dozen young people—say from fifteen to twenty—who expressed astonishment when they rediscovered for themselves the Pythagorean relation between pitch and length. Of course they were all coaxed into repeating the experiment; only geniuses, like Mendel for instance, ever actually do an experiment on their own. Of the half dozen three became electrical engineers, one a mathematician, one a chemist, and I cannot make out yet whether the sixth is a great mathematical physicist or just a plain nut. All the hundreds of others who saw nothing wonderful in the relation between pitch and length are happily following gainful pursuits, including numerology.

THE BIRTH OF NUMEROLOGY

Numerology was born the hour Pythagoras discovered the law of musical intervals. Before seeing how his brilliant discovery tricked him into universal numerology let us try to think ourselves back to 500 B.C. Then perhaps we shall appreciate from our blasé indifference to the marvels which science crowds into our hands how tremendous was the step which Pythagoras took.

Without being too numerological, it seems moderate to say that Pythagoras in his direct question-

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ing of nature by experiment strode as far ahead of his contemporaries as they had advanced beyond their simian ancestors. What Pythagoras did showed men that they might hope to discover order in the apparent anarchy of nature, even if they have to put it there themselves. One short step more and Pythagoras might have anticipated what some now believe, namely that it is within human powers to subdue brute nature and make her one-tenth decent. Only human nature remains to be civilized. Pythagoras erred partly in attempting the harder job first. The result was the most tragic and the most lasting failure in history.

Who can blame the enthusiast for being blinded by the brilliance of his discovery? Music, sound, aesthetic values, all were revealed in at least some of their aspects as things *measurable in space.* Might not virtue, friendship, love, justice, beauty, truth, and other human values be similarly amenable to the beautiful discipline of numbers? Pythagoras thought so, and turned his back on experiment. The world followed him.

Instead of the fact-finding approach to human affairs which the world might have got 2500 years ago with the scientific method it discarded, it still enjoys the rhetorical approach of abstract metaphysics. But there is this to be said in favor of the existing fact: we have survived, anyhow, whereas our suicide by science might have been

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consummated centuries ago. Whether the gain is worth the price is another question.

Even if Pythagoras did forsake the proved path to knowledge he yet towers above the great pretender to the discovery of the scientific method. Literary folk sometimes take the claims of Sir Francis Bacon (1561–1626) with undue seriousness. Bacon's proposals for scientific discovery are sonorous sermons, and about as effective for their intended purpose as most sermons usually are. If we experimented by Bacon's program we should still be ignorant of the fact that a kettle of cold water placed on a red hot stove will almost always boil and not freeze. Sir Francis preached beautifully about experiments; Pythagoras did one.

Equally just so far as Western civilization is concerned is the claim of Pythagoras to be first in numerology. Inspired by his scientific discovery he soared to the blue heaven of metaphysical mysticism, where he and his numerology exploded in the rarified atmosphere like a toy baloon. We shall witness the explosion later; for the moment let us follow the master into the blue.

We are in excellent company. That glistening white sphere on our left and a mile or two higher is Plato; the irridescent one beside him is Aristotle; the blue one below us is Hippocrates, the father of medicine; that bright green one directly overhead that seems to have reached the bursting point is—but it wouldn't do to tell, as he has not yet

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definitely exploded. The faint, thin pop however is expected at any moment now. The air is full of the things, all shooting heavenward and expanding like Lemaître universes as they rise. Up here we drink in the music of the balloons. Listen to them "still choiring to the young-eyed cherubim."

The Master Pythagoras himself leads off with this: "All things are fittingly ordered according to the nature of numbers; number is the eternal essence; God is number; number is God."

A Satellite from the Middle Ages, drowsy with incense and numerology, takes up the litany in the response "Omnia conveniunt numero."

As if he had not made his point sufficiently clear Pythagoras booms out again: "All things are numbers."

A faithful acolyte amplifies the chant: "All things which can be known have number. For it is not possible that without number anything can either be conceived or known. I am Philolaus, scrupulous reporter of the master. Ipse dixit."

A voice which declares itself to be Aristotle's drones out a discordant note: "Because you Pythagoras *assume* that all things *are* numbers, you prove easily that justice is an attribute of numbers, which it is not, and in the same way you prove that music and the soul are begotten by numbers on numbers. Opportunity, you say, is caused by numbers, but you have lost yours. You saw the properties of ratios and musical scales reflected

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on the distorting mirror of numbers, and from that you inferred that numbers are the first and last things in the whole of nature."

A sonorous interruption cuts short Aristotle's dissonance. "All numbers are sensible, mathematical, or ideal. The sensible are entangled in matter, and hence delivered up to the flux of generation and corruption. But above these the reason soars to the motionless mathematical numbers. There the contradictions of the senses are reconciled numerically with the simplicities of the intellectual ether. Yet above these, at the apex of the intellectual world, reign the eternal essences of the sensible and the mathematical numbers. These are the ideal numbers, the only real, the begetters of dialectic. Ideal numbers can be neither added nor subtracted, multiplied nor divided. And over these God ever geometrizes." It is Plato.

"He does nothing of the kind," a positive modern note asserts. "God arithmetizes."

"You are both wrong," a suave tone informs the cocksure balloons. "God is a great architect. He builds. How, we shall hear presently."

Undismayed by these minor thirds, Aristotle continues. "Because you Pythagoreans think you have proved that all things are numbers, you think you have proved that the elements of all things are numbers and that the whole heavens are a musical scale and a number. Your logic is

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faultless, for a false hypothesis implies any proposition you like, true or false."

Pythagoras himself comes to the rescue. "The whole heaven is a number and a harmony. The supreme ruler of the universe is the number which is the One and the Many. For the One is the Even-Odd; the Even is the imperfect; the Odd, the perfect; their union is the perfect imperfect, or the imperfect perfect."

"But they surely can't be the same thing?" comes a plaintive query from the direction of the earth, and another puzzled voice breaks through with something about it being exceedingly odd that God should be God, or doubts to that effect, only to be drowned in the general clamor. The few distinct snatches of this that ring out clear and true above the universal din give some conception of the whole.

"I say, Aristotle old chap, all that about the mirror of numbers isn't yours, you know. Alice said it from behind the looking glass, and I've got a patent on what she said. Of course if some of my rivals see fit to drag the Dormouse into their mathematical cosmogonies by the tail, I can't stop them. But I do wish they would give the poor sleepy creature a rest. Now I believe—"

"Oh?" It is the architectural silencer of Plato and Jacobi. "My dear fellow, it really doesn't matter a damn *what* you believe."

As if to back this somewhat testy declaration of

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fact, a cool, measured voice enunciates slowly: "Thank God I have my feet on the ground and have the use of a hundred inch telescope on a solid concrete base."

This seems to end it. But no; Plato insists on his prerogative of having the last word. "The true astronomer should dispense with the starry heavens."

To this the false astronomer has no reply. He might get on without his telescope, but to ask him to give up the sky is going a little too far.

And so numerology is born again and again and again—the "Eternal recurrence."

Before closing this part of the account, I should like to correct an erroneous impression which has been scandalously sensationalized by the press. It is not true, as was rumored, that a certain eminent mathematical astronomer threatened to blow up the hundred-inch telescope at Mount Wilson, when a handful of observations obtained by its use exploded his most daring numerological speculation on the nature of the universe as a whole. He was merely heard to mutter that he would jolly well like to.

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Chapter VI

ANCIENT AND MODERN

FULL FLOWER

Anyone dipping at random into what follows might conclude that we had returned to Hollywood or New York. Such is not the case. Although the fragrance is modern the flower itself is more ancient than some of the giant redwoods of California, and it is still as fresh as it ever was.

Presently we shall take a very considerable step in time. But as we must start from somewhere we shall take off from Greece, and carry with us a nosegay of the precious things Pythagoras said about numbers.

Millions of dollars have been made out of these or similar truths, creeds have been founded on them, and blood has been shed or flesh burned in defense of the sanctity of at least one of them. My sparse anthology is culled almost at random. To exhibit the full luxuriance of the flowering would tax the acreage of all the city parks in Europe and America. Here goes.

"What is justice? Obviously a square number. For is not justice the equal multiplied by the equal? Justice therefore is 4 or 9, since all numbers greater than 10 but repeat eternally the perfect properties of the first ten. But 4 and 9 are the

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only squares less than 10, as we Pythagoreans have our suspicions about 1 being a number at all. As 4 is closer to our oath than 9, it follows that 4, not 9, is justice.

"Or again, is not justice a proportion? Shall not the judge do to the offender what the offender did to the offended? Hence, to us, the proportion A:B = B:C is the numerological expression of Babylonian justice, 'an eye for an eye, and a tooth for a tooth.' Any numbers A, B, C such that A is to B as B is to C *are* this justice of the Babylonians." As a modern would say, the judge is a *mean* proportional between plaintiff and defendant.

An echo of 4 is heard in the snappy decisions of high-powered personnel officers. A man is judged to be on the square because he has a square jaw, square feet or square head.

Does the following have a familiar ring to those who have listened to lectures on philosophy or metaphysics?

"All things are divided into two categories, one of which falls on the side of the Limited, the other on that of the Unlimited.

"Now 10 is the perfect number (not in the technical Euclidean sense, noticed presently, but in the philosophic). *Therefore* there are precisely 10 fundamental pairs of contraries in the universe, as follows:

(1) Limited—Unlimited.
(2) Odd—Even.

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(3) One—Many.
(4) Right—Left.
(5) Masculine—Feminine.
(6) Rest—Motion.
(7) Straight—Crooked.
(8) Light—Darkness.
(9) Good—Evil.
(10) Square—oblong.

"Thus the Universe is generated from the Odd, or the Limited, and the Even, or the Unlimited.

"From the Odd and the Even all numbers are generated. Number therefore is the ruler and essence of gods and men, and numbers are both the substance and the attributes of any thing and of all things."

In the science of acoustics pitch, harmony and other qualities of sound are ultimately resolved into something or another concerning vibrations, and vibrations have what is technically known as frequency. From the frequency is derived a number. Finally, then, harmony has precise relations to numbers. But it was not from these, I suspect, that vibrations wriggled their way into modern numerology. The Pythagorean burrow is shorter. In passing, I wonder whether those old pioneers of numerology knew that wherever there is sound there is something vibrating not very far away. If they did, I cannot find that they thought it worth recording. But let them speak for themselves.

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"All is number and all is harmony, because every number is a definite union of the odd and even. And what is union but harmony?"

When they said "because" they meant exactly what they said, namely *because*, just as a schoolboy today says the angle A of some triangle ABC is greater than the angle B *because* the side CB is greater than the side CA. *All this numerology was logic and proof to the pioneers*. But they soon snapped out of it, as we shall see in the next chapter, and stopped using "because" so recklessly. Some of our contemporaries have still to snap. They also are pretty free with "therefore."

"The unity of opposites is number, and therefore harmony."

"Unity is the Cause, the Active; Duality, the Inactive, or Matter; God is Unity (1), or All-Good; Matter is Duality (2), or Evil."

Developing this thought for his feminine listeners, Pythagoras deduced that One is a decent, orderly, masculine number, and Two, which is constantly opposed to One, is feminine and therefore disorderly.

I trust that Theano gave him what-for for this when she got him home. Possibly he was properly attended to before he left the lecture hall. But probably the ladies took it as a compliment. After outrageously insulting the intelligence of every woman in his audience, a distinguished foreign philosopher in Los Angeles recently was handed an

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ovation and a double honorarium by the ladies who had hired him. So there is no telling what happened long ago in Croton; eggs may have been cheaper than ovations.

"Unity is the Good, Reason, or Deity; Duality is Evil, Matter, Daemons."

This sounds like too much Theano; duality, or 2, is feminine. Let us hear what he has to say on some of the bigger numbers. Three, as might be expected after Pythagoras' exploits in Egypt and Babylon, was tremendously important. With the Homeric trinity (supplicated as One Person) of Zeus, Athene, and Apollo, the Indian trinity of Brahma, Vishnu, and Siva, the Egyptian trinity of Horus, Isis, and Osiris, and heaven only knows how many more which the avid traveller swallowed from Thebes to Tarentum, the ambitious pioneer of numerology had more than he could digest at first. The final Pythagorean Three was really a very judicious cutting of all the inextricable knots of the others.

I do not know whether scholars have considered a possible origin of one part of Aristotle's Poetics in the Pythagorean doctrine of Three. Anyhow, Pythagoras set great store by Three because it is the *first* number which has a Beginning, a Middle, and an End, and it is the *only* number which has *precisely one* of each of these highly desirable possessions.

Is this the origin of tragedy? If so, just imagine

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the possibilities for the dramas of the future. Why not construct them on 5, which has three middles, or 7, which has five? Instead of one Strange Interlude, we might have a million. To do so would be not one whit more absurd than some of the recent attempts to numerologize art—if they *are* absurd. I don't profess to understand them. But I never heard of an artist who tried to paint a picture or write music to the accompaniment of soft numerology in the background. In the later history of numerology it was denied that 3 has either a beginning or an end, and people were burned at the stake for disagreeing.

Seven must be handled with tongs. The associations of this number with our own sacred literature are so intimate that a full statement of what Pythagoras and his successors said about it might give offense where none is intended.

The famous seven ages of man, which Shakespeare platitudinized in a passage which every urchin memorizes, go back at least to Pythagoras. Seven, the colossus declared, is the critical time—a very curious idea. So far as I can make out, he meant just this: the *number* seven *is* a *time*. This beats Minkowski by 2500 years. His reason was that seven marks off the climacterics, real or imagined, of human life. Thus seven came also to indicate the lucky time. We have rather changed this, for we are expressly forbidden to do anything on the seventh day of the week.

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Five is more promising. To understand why 5 is marriage, we must remember that Pythagoras either ignored 1, as not being a number at all, or assigned it the highest place of all, just as it happened to suit his purpose. Ignoring 1 for the conveniences of marriage, which seems reasonable and legitimate, we have 3 as the first odd number. Clearly 2 is the first even number. But 3 is odd, and therefore masculine; 2 is even, and hence feminine. Note the prehistoric phallicism of it all. Now $5 = 2 + 3$. Therefore, since one masculine united to one feminine is a lawful marriage, it follows that 5 *is* marriage, since 2 and 3 combined according to legitimate arithmetic give 5 and nothing else. How would Pythagoras have interpreted the equally true statements $2^3 = 8$, $3^2 = 9$, $3 \times 2 = 6$ sexually? They will tell you in Hollywood or New York, but I cannot repeat the answers here.

To return to 2. In identifying this with opinion, Pythagoras must have been having a row with Theano. Anyhow, it is proverbial even with us that it is a woman's special privilege to change her mind for any reason or for no reason at all. Because 2 is even, it is feminine (see the table of contraries stated earlier); for the same reason it is unlimited, variable, and indeterminate. In short 2 is a shilly-shally sort of she-number, and hence it is opinion, which has all the same characteristics.

It would be worth a hundred dollars of anyone's

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money to know what Theano wrote about her husband in that lost biography of hers. Remember, she was little more than a flapper when she married the old man.

PERFECTION AND MASSES AT INFINITY

The number 10 deserves a book to itself, as also does 6. To do full 4 to either, we should have to trace the development of scientific and mathematical thought from 500 B.C. to today. The numerology which Pythagoras evolved from 6 is not all dead; much of it survives in vigorous mathematical research, in the sense that the purely arithmetical questions which were first suggested numerologically by the pioneers are as yet unanswered.

I do not mean to imply that any of these questions—those concerning perfect numbers, for instance—are of any mathematical or scientific significance in our age. But I do assert that, without the impetus given by numerology over 2200 years ago, these questions would probably never have arisen to torment us. They are clear off the main track, it seems to many, of reasonable progress. But merely because these questions resist all our modern mathematical skill scores of men have racked their brains in our century alone to answer them.

What can be said in defense of this futility, if it is futility and if it needs any defense? First, there

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is the human pride which refuses to ignore any challenge to human intelligence, no matter how fantastic. If some silly idiot breaks his neck by jumping off the roof of his garage because some sillier idiot dares him to, why shouldn't a pure mathematician break his head by butting it against the riddle of odd perfect numbers?

Second, there is the historical fact, which can be checked in detail, that some of the most powerful engines of modern mathematical research, without which much of practical physical science as we know it today would not be possible, had their beginnings in attempts to settle these very questions or others which have grown naturally out of them. This is one of those rare struggles where the means justify the end.

I said that to do justice to 6 and 10 we should have to consider scientific thought, as well as mathematical, and I meant it. The pill may be unpalatable without sugar, but it is no less efficacious. All the inspiring nonsense of the past fifteen years that has been written about the fourth dimension in connection with popular expositions of relativity is pure Pythagorean numerology, and it originated in the mystical spatial properties which Pythagoras ascribed to the numbers 6 and 10. Pythagoras would have understood every word of the wildest popular explanation of "dimensions," for he held closely similar ideas. The sober and entirely unromantic fourth dimension of the mathe-

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maticians would not have appealed to the old boy at all, for it makes the strictest and driest kind of common sense.

While I am on the fourth dimension, I should like to remind some that it was not a mathematician but a physicist who started all that enchanting spiritualism about the fourth dimension. He was quite a good man, too, in his day. He died insane.

To get back to 6 and 10, let us define first the *proper divisors* of a number. The proper divisors of 6 are 1, 2, 3; those of 10 are 1, 2, 5; those of 12 are 1, 2, 3, 4, 6. A proper divisor of n is a number which divides n (exactly, without remainder), and which is less than n.

Observe that 6 *is equal to the sum of its proper divisors*, $6 = 1 + 2 + 3$. The like holds for 28, of which the proper divisors are 1, 2, 4, 7, 14. A number which equals the sum of its proper divisors is called *perfect*. Notice that both 6 and 28 are *even*. Now, if anyone wishes to immortalize himself, let him *prove or disprove* that an odd perfect number exists.

I may say that the method of trial and error is not likely to succeed. If there is such a thing as an odd perfect number it will be very large. Although it is usually disastrous to make predictions about arithmetical theorems the weight of current guesses is against the existence of odd perfect numbers. If this is the right hunch, some amateur is as likely to prove it as the most ac-

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complished mathematician. This is enough of arithmetic for a while; we must see why Pythagoras was delighted with 6.

The clue is $6 = 1 + 2 + 3$. With their unworkable conception of dimensions, the Pythagoreans said that a point has *one* dimension in space, a line *two*, a plane *three*, and a solid *four* dimensions.

This is less absurd than it seems at first sight when we know how they arrived at it. The point has no parts; it is indivisible, and therefore must be 1, which is not divisible by any other number. The

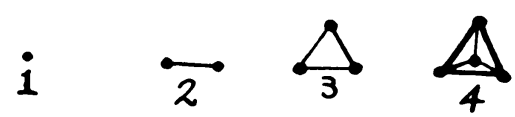

FIG. 4

line is determined by 2 points; the simplest regular figure in a plane, namely the equiangular triangle, by 3 points, and the simplest equiangular solid (the equilateral tetrahedron), by 4 points. Instead of 1, 2, 3, 4 we have 0, 1, 2, 3 for the corresponding numbers today. Eudoxus and Euclid set all this right.

From this it is easy to see why the tetrahedron is sometimes identified with the four ancient elements. I cannot see offhand why it should be fire; the Greek for fire has only three letters. Possibly the tetrahedron caused the English language with its four-lettered fire.

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Now consider the miraculous and mystical equalities

$$6 = 1 + 2 + 3, \quad 10 = 1 + 2 + 3 + 4.$$

In the first are summarized all the *plane* figures of geometry; in the second, all the *plane and solid*, in fact *all*. Moreover 1, 2, 3, 4 have been seen to have important numerological properties themselves. Therefore we should not be surprised to find that geometry as well as arithmetic is full of justice, truth, beauty, masculinity, femininity, goodness, and theology. The proof may be left to the reader.

Now here comes the mystical geometry which has survived till today in some attempts to make the fourth dimension intelligible. The attempts, to a mathematician at any rate, are less comprehensible than the n plus oneth dimension, where n is any integer you please. According to some authorities, the ancient numerologists said that the *point generates the line by motion; the line generates the square by moving parallel to itself; the square generates the cube by moving parallel to itself* and—why did they stop there? Let the cube move parallel to itself, and generate—but Schemhamphoras only knows what it would bring forth. This is *not* the plain, simple way mathematicians pass to space of higher dimensions, but we cannot go into such things in an account of numerology.

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Out of this came in particular our exasperating squares and cubes of numbers, instead of second and third powers. Anyone with a particle of imagination can mysticize the entire cosmos from these hints of the numerology of space. The Greeks, including Plato, went far in this direction. Aristotle was not far behind.

For instance, if we listen attentively enough, we hear the first musical chords of the harmony of the spheres. The crystalline spheres of the first astronomy are already gyrating in the harmonious spatial numerology of $10 = 1 + 2 + 3 + 4$. Look at the right-hand side. The possible ratios 3:4, 2:3, 1:2 leap to the eyes, as the French say. But these are the mysterious ratios for the musical intervals of the fourth, the fifth, and the octave.

There is more in it than this. It is easy to show numerologically that the universe is a sphere. At its centre is the Central Fire. Around this revolve precisely *ten* celestial bodies. There must be no more nor less than ten from what has been proved. The outermost of all is the heaven of the "fixed" stars. Next, the five planets. There were only five known when Pythagoras theorized. Next, the sun, moon and earth. So far we have $1 + 5 + 3$. What on earth is the matter? There should be 10, not 9, because 10 is philosophically perfect. But we have already reached the earth. The solution is obvious: there must be a *Counter-Earth*, to balance ours, and this is the tenth celestial

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body. Where is it? Really it does not matter; it *must* exist.

The ingeniously conceived Counter-Earth illustrates beautifully the distinction between numerology and science which was proposed in the preceding chapter. The pioneers were well within their rights in inventing this mystical body to balance their equations. Where they differed from modern *scientists* was in *believing* that the Counter-Earth was more than a mathematical fiction. The *fact* that it *fitted* their *mathematical* theory was *proof* of its *existence*.

I do not wish to labor the obvious, but this does seem to me to be the crux of the whole matter. The point is an important one, I believe, on account of the numerological flavor of many otherwise scientific theories, or popular expositions of such theories, especially the largely mathematical ones. Of course any one who wishes may take the numerological point of view instead of the scientific. It cannot kill anyone today.

Fortunately for my obvious point, there is a beautiful and famous illustration of it in current cosmological speculations. For detailed expositions of Einstein's applications of general relativity to the possible structure of the universe numerous clear articles and books are readily available in any good library, so I need not attempt to explain the Einstein universe. In one of his models of the universe, Einstein found it necessary to postulate

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the existence of vast quantities of matter beyond what the hundred-inch at Mt. Wilson records or theoretical astronomers guess at by extrapolation (a potent scientific term which will be explained later).

Where are these tremendous masses? There are at least two possibilities. They may be finely distributed throughout space in the form of cosmic dust. This, I believe, has been ruled out by astrophysical observations. Another possibility is that they may be concentrated in swarms of spiral nebulae far beyond reach of our telescopic vision. By the time the two-hundred inch is finished theory may have pushed the masses yet farther out into the inaccessible unknown. Whether these masses exist or not, they were *postulated for purely mathematical reasons.*

Was this essentially different from what the Pythagoreans did when they invented the Counter-Earth? It was, and therein lies the whole difference between numerology and science. If this is not obvious, I fear I cannot make it any clearer.

BENZENE

From masses at infinity to benzene may seem like a far cry, but numerology can make itself heard no matter how far-fetched its echoes are. Benzene is the highly inflammable stuff that rash housewives sometimes use for cleaning gloves. I once knew a poor woman who cremated herself that way. But if I understand the following cor-

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rectly it was not benzene which burned her up, but the Pythagorean 6.

In his wisely conservative lecture on Science and the Unseen World (religion?) delivered at Swarthmore College in 1929 Professor Eddington called attention to a profound property of 6 which numerologists had overlooked. As his audience was nine-tenths good solid American Quaker the lecturer was no doubt sober enough.

Benzene, it may be recalled, is the simplest of the so-called aromatic compounds, and Kekulé's formula for it is a bracelet of 6 carbon atoms with 6 hydrogen atoms dangling like jewels from the carbons.

Eddington first pointed out, exactly as Pythagoras did, that it can scarcely be called an accident that 6 is one of the integers. He goes on to say that if Nature's arithmetic had overlooked the number 6, organic life would never have started. ("Organic" to an orthodox chemist means "carbon compound." Such compounds were first called organic because they were found in matter which had once been alive, or had been synthesized in a living animal.)

So far all is clear. Kekulé would have been forced to use some other number than 6 if, instead of the aromatic benzene, the beginning of things organic had been some stinking silicon compound. Silicon is a close relative of carbon, but the smells—. Carbon is bad enough.

It is then remarked by the lecturer that the 92

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chemical elements could conceivably beget a world of very considerable but *limited* diversity.

Finally he reaches the striking conclusion that we have the terrific number of known organic compounds (and hence also the infinite variety of living things), *because it is the peculiarity of the number 6 to rebel against limits.*

I suppose if we had been silicon monsters and water-glass gels instead of the staid carbon compounds we are, life would have been less numerological. But it has not been proved.

In some of his severely scientific writings Professor Eddington puts forward a striking speculation which turns Pythagoras inside out. Why has no one thought of this before? Eddington's reasons are based on quantum mechanics, but surely the material for them must have been lying about for ages. Nature is not built out of numbers; it *exudes* them.

FRIENDSHIP

Pythagoras was such a decent old sort that we feel like shaking hands with him before parting, instead of kicking him in the pants for all the numerology he dumped on our over-burdened race. So we shall say goodbye to him by recalling one very fine thing he said, and then giving one of his quaintest inventions in connection with it.

Asked what a friend is, Pythagoras replied, "Another I." Aristotle copied this in his Ethics.

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From this developed the amicability of numbers. What could depict a closer friendship than the *amicable number pair* 220 and 284? The proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, and the sum of these is 284; the proper divisors of 284 are 1, 2, 4, 71, 142, and the sum of these is 220. Each of 220, 284 has the power to generate the other, and surely nothing could be more intimate than this. It is almost indecent.

I regret that space does not permit any account of the triangular and polygonal numbers of the Pythagoreans. Much less can I go into the vast history of what grew out of these. At least one boundless tract of the modern theory of numbers is an elaboration of the few *arithmetical* theorems discovered by the Pythagoreans and their immediate successors. So numerology can be credited with something which is serious enough, God knows.

But, as a student of the theory of numbers, I have often wondered whether that great theory might not have followed an entirely different course, and perhaps one closer to reason and nature, if Pythagoras had never lived. Being unable to suggest any more rational thing to do than what is now being done, I merely throw out the suggestion to any curious mind which may, possibly, penetrate to a deeper stratum of the relations between numbers than any we have yet struck. My own feeling is that the theory of forms is a beautiful bypath, and that it is such because

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arithmetic has not yet had its Descartes, let alone its Newton or Einstein. So much for my own numerological confession of unfaith.

There remain two brief matters to close this part of the account. It was promised that Plato's equation would be placed beside Fermat's. Almost three centuries ago Fermat asserted that no whole numbers x, y, z, n (none zero) exist such that $x^n + y^n = z^n$ if n is greater than 2. This has not been proved. For $n = 2$ we have (among an infinity) $3^2 + 4^2 = 5^2$, and 3, 4, 5, are the Cosmic Triangle of Pythagoras.

A guess which is first cousin to Plato's is this, also unproved: If n is any integer greater than 1, the sum of *less than n* n th powers of integers is not an n th power, except in the trivial case of exactly one power in the sum. For example, if $n = 3$, the sum of two cubes is not a cube (true); if $n = 4$, the sum of two fourth powers is not a fourth power (true), the sum of three fourth powers is not a fourth power (unknown; may be false).

The "most perfect proportion," the "musical proportion" which Pythagoras brought to Greece from Babylon may well be our parting salute to the great teacher. If the numbers a, b, c are in arithmetic progression, b is called the *arithmetic mean* of a and b; if x, y, z are in harmonic progression, y is the *harmonic mean* of x and z. Now it was considered profoundly beautiful by Plato no less than by Pythagoras that

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$$a : \text{A} :: \text{H} : b,$$

where *a*, *b* are any numbers, A is the arithmetic mean, and H the harmonic mean, of *a*, *b*. In the Timaeus of Plato occurs the following instance of this musical proportion

$$12 : 9 :: 8 : 6.$$

This contains space, number, and harmony.

And so, Ave atque vale, magister Pythagoras, alter ego! Our science is your shadow stripped of its numerology.

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CHAPTER VII

SACRED AND PROFANE NUMEROLOGY

A REQUEST

Most men who are not professional artists experience a difficulty, which I share, in deciding which of the two ladies in Titian's famous painting of Sacred and Profane Love is sacred. To me they look equally good. Can it possibly be that the artist set out deliberately to pull posterity's leg? However that may be, I feel sure that many readers will sympathize with my inability to credit some of the mediaeval numerology with the sacredness it may deserve, in spite of its august source.

For the past week all my leisure, which might have been more pleasantly wasted, has gone to a vain attempt to numerologize my own name into 666. I undertook this in response to an unequivocal challenge from the greatest living anatomist of mystical beasts. This authority stated that *any name whatever, or any collection of letters whatever, can be numerologized so as to give 666 as the number corresponding to the name.*

So far I have failed completely, although I have tried it in English, French, German, Italian, Dutch, Russian, Latin, and Greek—with the scholarly

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help of sympathetic friends. Last night we all gave up when Chinese, Sanskrit, and Hebrew failed. We got 66 repeatedly, and even 6666 once, but never 666. Thinking something must be faulty with our technique, we checked it against the names of the rest of the party. No difficulty appeared. Then we began picking names at random from *American Men of Science*. Every last one of them we tested was a 666.

There must have been something wrong with my birthday date and my christening. Will any numerologist who can do a 666 for me please send me the demonstration? Until I get it I shall have no peace of mind.

MEDIAEVAL BEASTING

There is an opportunity for some serious historical research into the origins of the numerology of 666. The number itself is not very promising; it is $2 \times 3 \times 3 \times 37$. The 3×3 is suggestive enough, but the 37 is incomprehensible, in spite of the fact that it is $6 \times 6 + 1$. Now 37 is a prime (a number divisible only by itself and 1). Primes do not seem to have played much part as such in numerology till the very positive social philosopher Auguste Comte (1798–1857) went wild over them. Certainly the Babylonians never blundered into this rich field. Possibly St. John himself was the first mortal to use 666. As it is part of his revelation this may not be unreasonable. He undoubtedly

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had access to sources denied the Babylonians. John said some very hard things about Babylon.

The number appears in the Revelation of St. John the Divine, numerologically enough in Chapter 13. There are two Beasts, a seven-headed Sea Beast and a two-horned Land Beast. The number 666 belongs to the second of these, as stated in verses 11 and 18.

11. "And I beheld another beast coming up out of the earth; and he had two horns like a lamb, and he spake as a dragon.

18. "Here is wisdom. Let him that hath understanding count the number of the beast: for it is the number of a man; and his number is six hundred threescore and six."

That is, the number of the Beast is 666, and this is the number of a man. But *what* man? Ask another.

For nearly 2000 years numerologists have been torturing men's names to fit them to the procrustean 666. Not a week goes by but that some unfortunate is branded on the forehead with the flaming number. But, like the monks of Rheims with their wicked jackdaw, nobody seems one penny the worse. Not so long ago it was no light matter to be beasted a heretic.

The art of beasting a man is and always was practically the same as one of the commonest numerological devices by which many a marriage or less temporary union is contracted today in

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Hollywood. It is not necessary to use the birthday date; no genuine expert any longer takes this fallacious step. Numbers are assigned to the letters in some suitably chosen or invented alphabet; the letters of an intended victim's name are given their numbers, and these are added. If the sum does not come out the desired 666, the name is appropriately mispelled and, if necessary, more than one alphabet is used. If this fails, the meaning of the name, if it seems to have any, is translated bodily into some foreign language. If this doesn't work, plain jargon is used. Greek and Hebrew give particularly pleasing results; Latin also is good. The superiority of Greek and Latin is due to the fact that numbers in these languages were written with the same letters as those used in spelling names or other words.

Beasting as a serious or dangerous science blew up in the sixteenth century, when two mathematicians fought one of the fiercest numerological duels in history. This decisive encounter is sometimes referred to as the Battle of the Windmills. The combatants were Michael Stifel and Peter Bung.

The fight was furious beyond all decency, for Michael and Peter were good haters. Neither saw exactly what it was that he was fighting, but the outcome was the same as if they had joined forces and slashed into the common enemy together with all their bodkins. They succeeded in

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murdering the sacred mediaeval numerology which each of them thought he was defending with his last breath.

Stifel was probably the foremost German algebraist of the Sixteenth Century. His first love however was numerology. Inspired by the Book of Daniel in the Old Testament, and Revelations in the New, he started well, but soon degenerated into a mathematician. But the old Adam would not down, and he remained a very great numerologist to the end of his life.

Peter Bung on the other hand was the encyclopaedist of numerology but not much of a mathematician. The long-distance, all-time record for amassing of numerological data undoubtedly belongs to Peter. With indefatigable love and energy he swept together everything, dust, wheat and chaff, that had ever been said, thought, or imagined about the mystical properties of numbers from Pythagoras to Stifel. The result was an enormous tome that would have choked all seven gullets of the Sea Beast. There were in fact 700 pages of the *Numerorum Mysterium* ("The Mystery of Numbers"), and the pages were of the noble quarto size. Truly Peter was a formidable opponent for all the heretical mathematicians in Christendom.

Stifel delivered the first thrust. By a short and extremely elegant—but also extremely crooked—piece of numerology he fastened 666 on the Pope

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himself. It was a daring thing to do, and if Leo X had been vindictive he might easily have made things pretty hot for the audacious Michael.

But Leo was having troubles elsewhere, with all the heresies stirred up by the brazen, reforming Martin Luther. Peter rushed to the defense. The Pope's self-appointed champion girded up his loins, tucked his beard into his bosom, buttoned his shirt, took a deep breath, drew his cutlass, and charged the blasphemous Michael like a Dutch windmill in a cyclone.

When the dust cleared it was seen that Peter had firmly affixed 666 to the forehead of Martin Luther —not, however, without mangling Luther's name almost beyond recognition in the process. Those were the good old days when you not only called a man a beast but proved it.

When the clamor died serious numerological beasting died also. It had proved too much. If the Pope was the Beast, reformer Luther could not possibly claim the distinction, and vice-versa. The populace openly declared that sacred numerology was tripe.

MODERN BEASTING

Has the reader ever taken an intelligence test? If not, don't; it is not an intelligent thing to do.

An inspector running a boiler test, or a geodetic surveyor doing a triangulation, knows almost by instinct what is meant by probable error. Any-

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how, he knew once what it meant, and he knows now that if he abuses it something unpleasant is likely to happen. The like applies to biometricians and correlation coefficients.

Does it apply to professors of pedagogy specializing in educational measurements? Does it apply to teachers in the elementary grades giving intelligence tests to weanlings and striplings and beasting them with an indelible I. Q.? I am sure I don't know, but I hope it does. Anyway, if we demand of a physician that he know the rudiments of materia medica before letting him physic our children—if they ever do that nowadays—it would seem to be not unreasonable to ask the corresponding thing of those who minister to our feeble and ailing intelligence.

I have alluded to these practices because they offer a fair example of a second description of numerology. Is it unfair to say that anyone who vigorously applies mathematical formulas to human affairs, without a critical insight into the meanings and limitations of the mathematical methods used in deriving the formulas, or without a precise understanding of the assumptions on which their derivations are based, is a numerologist?

Perhaps no great harm has come out of this modern variant of beasting, and perhaps none will, beyond a possible confusion of docile Babbitry with intelligence.

The whole matter is on the borderline between

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science and numerology. For example, those who follow the sanguinary fluctuations of the battle over the meaning or existence of Spearman's g (general intelligence) with a suspended judgment are scientists. Those who believe or disbelieve in g at this time may justly align themselves with Peter or Michael, for the experimental evidence is not yet conclusive one way or the other.

WHITEWASH

The hectic brilliance of the Greek numerology dimmed as suddenly as it had flared up. Like a gifted consumptive who toils with feverish haste to keep ahead of his disease, get his work done, and be quit of the world as quickly as possible, the Greek fire burned itself out in the first hot flush of youth. Thereafter stagnation, darkness and triviality for a thousand years.

Half a century ago it was quite the thing to damn the Middle Ages (300–1500 A.D.) up one side and down the other for their squalid dirt, superstitious credulity and perverted cruelties. Today the fashion has changed, and the general tendency seems to be to apply a liberal coat of whitewash, if not chloride of lime.

I have neither the proper brush nor the bucket to aid in this worthy sanitation, so I shall not try. Nevertheless it is a fact that certain aspects of research today in the foundations of mathematics are vivid reflections of the sort of thing the hair-

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splitting scholastics spent centuries over. But as this belongs to the strictly modern outgrowths of numerology, discussion of it must be deferred for the moment. I shall try merely to whitewash my prejudice that the numerology of the European successors of the Greeks was on a par with their mathematics, and a smashing comedown after the pioneers. Just as in mathematics their chief concern was to remember enough arithmetic to compute the date of Easter, so in numerology their orbit seems to have been prescribed by fanciful embroideries of the poetical arithmetic of Holy Writ. There were exceptions of course, and some of them had a pretty hot time.

Their provocation, it must be admitted, was great. Consider, for example, the following fistful of fours, fives, tens, and twelves: the 10 commandments; tithing; 7 × 10 elders; 7 × 10 years of the captivity; 5 kinds of punishment (a sixth was added later, the severest and most lasting of all, to complete the perfect number); 4 winds; 4 corners of the earth; 4 guardians, each with 4 wings and 4 faces; 4 rivers of Paradise; Daniel's 4 beasts; 4 square sides of the temple chamber; 3 × 4 tribes of Israel; 3 × 4 jewels in the high priest's breastplate; 3 × 4 apostles; 3 × 4 foundation stones and gates of Heaven; 10 × 4, or 4 × 10 days of Moses, years in the wilderness, or days and nights in the same. All of these and many more were lovingly tormented by the ingenious nu-

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merological predecessors of the encyclopaedic Peter. He amassed their labors, added thereto, and dumped the lot on us.

Not so many years ago I enjoyed a stirring sermon by an English bishop which was the purest kind of pure Bung from beginning to end. If I were to reproduce any of the numerology of that profound discourse some might be shocked, so I shan't. I was too, but possibly for a different reason. The bishop had been trained as a mathematician, and he was reconciling science and religion. It was news to me that they were still not on speaking terms.

But to return to the Middle Ages—or just before. We start conservatively enough with a few staid reminders of Pythagoras. The colossus was not yet forgotten. Thus St. Jerome tells us that God omitted to pronounce the work of the second day of creation good, not by an oversight, but because 2 is fundamentally evil. Poor Theano! But probably Jerome never heard of her.

The perfection of 6 ($6 = 1 + 2 + 3$) led to much that was less perfect. As late as 1493 it was *proved* that the creation took precisely 6 days *because* 6 is the first perfect number. If Columbus in 1492 had steered his cockleshell by such arithmetic he would have gone on the rocks somewhere south of Suez. No such practical disaster deterred the sacred numerologists in their blasphemous reckoning.

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The great tradition lingers on in the *proofs* given by Philo Judaeus (20 B.C.–54 A.D.). *Because* 6 is the most productive of all numbers, generating at least part of space, *therefore* the perfect 6 as the limit for the work of creation could not have been surpassed by the Creator even if he had wished. The 4 beloved of the Pythagorean Brotherhood, *because* of its harmony and justice, numerologically *caused* the creation of the heavenly bodies on the fourth day, as duly recorded in Genesis. Animals with their five senses got that way because 5 naturally enough saw fit to generate them on the fifth day. Man being the *perfect* crown of it all could enter no earlier than day 6, and as 6 was the working limit, we have what we have.

Had St. Augustine (354–430) lived eight centuries earlier than he did he might have rivalled the colossus himself. As it was he did pretty well with the somewhat uninspiring numbers of his chosen profession. Pythagoras had the whole universe to select from; Augustine was compelled to draw almost entirely from Palestine.

His first exploit was the generation of Genesis from 6. As this has already been glanced at, we may go on to a more spectacular demonstration of his prowess as a numerologist. This feat is suggestive in the history of mathematics; it indicates that Augustine may have known enough Greek arithmetic to sum an arithmetical progression. Anyhow, whether neatly or by brute force,

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he summed one. The sum of the first 17 numbers 1, 2, ... , 17 is 153.

Where did 153 turn up in Palestine, and why? This double question, it seems to me, brings out the whole distinction between sacred and profane numerology. The "where" is a decent enough query; the "why" is just plain silly. The double solution follows.

St. Peter and others, it will be remembered, once caught 153 fish. By summing the 17 numbers from 1 to 17 according to the Greek rule, we get one half of 17 times 18, or 17 times 9. Why the 17? It is very simple; 17 is 10 plus 7. Now 10 is the law (in the Old Testament sense), and 7 is the sacred number of the gifts of the spirit. But the law without the spirit is death. Hence to 10 must be added 7, to give the sum of the old and new dispensations. In the same way the other factor 9 is accounted for, but I shall leave the details of the proof to experts. They are not all dead yet.

I do not wish to give an unfair picture of patristic thought. That, to my way of thinking, would be sheerly impossible so far as numerology is concerned. No matter how bizarre a chop suey of numbers and nonsense you dish up to me from the numerology of today, I will undertake to produce something as crazy from the Middle Ages or just before, given a reasonable time to thumb over the classics.

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Why did human beings ever *reason* that way? Schemhamphoras only knows. Will our own efforts at serious thinking seem as queer to the curious 1500 years hence? Let us hope so, or the world will have stagnated worse than it did from Augustine to Galileo.

One hopeful sign marks us off from the old fellows. Whatever others may do, scientists no longer cherish or respect their obsolescent theories. One reason for this free and easy irreverence of true scientists toward the children of their fancy is that science is not sacred in the sense in which Augustine numerology was. Nor, for that matter, is it particularly profane. It is neither; it is just science. Of course an occasional mystic can rush in and upset the applecart when scientists' backs are turned, but the mess is soon cleaned up, and the business of selling clean apples for a penny apiece to eager children goes on as before.

I have just this moment made an astounding discovery! Inspired by Augustine's analysis of 153, I returned to the Beast. My discovery marries the Beast to 36, one of the many wedded integers in the nuptial number of Plato. If you add up all the first 36 numbers 1, 2, 3, ... , 36, you get 666! After this I can go back with sympathy and an enlightened understanding to Augustine for just one more of his feats.

What great lesson can the fair and fat learn from 40? This number is 4×10. Now 4 rep-

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resents time, *because* the hour, day, month, and year are divided into 4 periods. The other factor is even more significant; 10 is 3 plus 7. But 3 comes from the 3 persons of the Trinity, and 7 from the 7 elements which make up the creature created by the creator 3. What are these elements? There are three spiritual elements, namely, heart, soul, and mind, and four corporeal elements, namely earth, air, fire, and water from which all material things are fashioned; total, 7 elements. I should think 3 plus the 7 deadly sins would do as well; gluttony is the least hygienic of the lot.

But again, the perfect 10 is knowledge. We have just seen that it contains both the creator and the creature. Therefore 10 is knowledge of both of these beings.

Now follows the inevitable conclusion, on multiplying the *time* 4 by 10, the knowing creature: 40 instructs us to *live in time according to knowledge*.

But this is not all. The final step of the proof, I confess, baffles me. *We must fast for 40 days*. That is what 40 teaches and commands. Augustine probably allowed fish, after all the good things he proved about 153.

Before tossing this mystical demonstration aside as an outgrown and profitless playing with numbers, tune in on your radio when the seers are seeing aloud and, what is more, raking in the shekels by the barrelful. For a self-addressed, stamped envelope and one dollar in currency mailed to

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Station ——— (name your own), you will receive a booklet by the eminent Swami ——— (they are all eminent, and most of them come from the East Side, if not from the East) which will numerologically and astrologically solve all your problems, financial, marital, and gastronomic.

PLATO'S PIGS

The depths of the Middle Ages may seem a peculiar sort of hole from which to tender Plato an apology for anything that may have been not quite fair in what was said about his numerology. Nevertheless the apology is due now, if it ever was. The time, the place, and the spirit are united here as nowhere else in the long stretch from 500 B.C. to the present.

The point is really the outstanding peak in the history of numerology. Looking back, as we shall do toward the end of the whole story, we shall see that our own age in one of its most feverish pursuits is linked to the sixth century B.C. by the tenuous mediaeval thread of hair-splitting, logic-chopping dialectic which most scientists have been taught to look upon with contempt.

When numerology exploded about 410 B.C. it created a disturbance which has not yet quieted down, and which is not likely to stop reverberating so long as our generation has ears to hear. Plato heard it in his time, and his comment is the only

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recorded evidence that he, habitually so even-tempered, knew how to lose his temper when it was the only sensible thing to do. By that one outburst of temper Plato redeemed himself, and proved after all that he had the mind of a human mathematician, even if his philosophy did betray him into talking like a numerologist.

Numerology, we remarked, exploded about 410 B.C., and we shall see how and why in a moment. Disregarding the great blow-up, Fathers Augustine, Jerome, Origen, and their successors all down the Middle Ages went on numerologizing precisely as if mathematics, life, and the universe were built out of countable collections of whole numbers, as the pristine numerology of Pythagoras demanded before the explosion. But at the same time a more subtle scheme of things had taken root in the fertile minds of the scholastics. We cannot go into the evidence here for believing that they were somehow familiar with the history of the great explosion and were fully alive to all it meant for the numerological outlook on the universe, but it seems that they were. If they never heard of it, their exploits are only the more remarkable.

Whoever speaks today with contempt of the work of the schoolmen should look into the developments of the past fifty years in the foundations of mathematics. There he will find all the fuel he needs to keep his wrath hot. And, in the opinion of those who should know, not all of it is rubbish.

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It may yet burn up and abolish every existing scientific speculation.

So all down the despised thousand or twelve hundred years we have two parallel but antagonistic tendencies. One carried on the Pythagorean tradition in its most extravagant form; the other undermined the true numerological faith and laid the foundations of the great heresies of today. The heresy of today of course is in general the orthodoxy of tomorrow, as has often been observed and emphasized by bishops.

More remarkable still, as the dark ages draw to a close, we frequently find a single mind—Kepler's for instance—divided against itself. No modern conflict of a neurotically split personality can compare with the complete anarchy which must have torn those devout believers asunder. At all costs they must find the truth, of sacred numerology no less than of pagan mathematics, for their faith in eternal damnation was lively and personal. Today the truth or falsity of any scientific or mathematical theory has no bearing whatever on anyone's prospects of eternal salvation, and only patients in insane asylums allow themselves to be tortured by their inability to believe patent absurdities. But to the numerologists of the dark ages it was a very serious matter indeed to tamper with the mysteries of 72.

To take just a couple of numerological horrors before we recount the explosion, let us remember

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Nicolas Cusa and the thundering Schemhamphoras. Cusa was a very good mathematician and a numerologist of the first rank. He left a fine mark on the mathematics and circle-squaring of his time, and on the numerology he made an impression that was not appreciably deepened till the dialectical philosopher Hegel (1770–1831) went him several thousand better. Carried away by the pure Pythagorean numerology, Nicolas summed up all of its evenesses and oddnesses in the astounding principle of the *maximum*. This he *identified* with the *minimum*. Thus *the greatest and the least are identical*. Could unification go farther?

Schemhamphoras went infinitely beyond Cusa's faltering two-step, for this terrific word is nothing less than the knotted numerological sum of all 72 ($72 = 2^3 \times 3^2$) of the mystical names of God. It is literally true that to curse a man in the name of Schemhamphoras by exhibiting a numerical dissonance between the letters of his name and those of 72 was frequently equivalent to signing the wretched man's death warrant, and it wasn't a particularly pleasant death at that.

Now for that explosion of about 410 B.C. Imagine two rods A, B of different lengths. Say A is 10 feet long, and B 12 feet long. The *ratio* of the length of A to that of B is 10 to 12, or 10:12, or $\frac{10}{12}$, or finally $\frac{5}{6}$. We must consider with great care one thing that this means. Going back to the rods, we see that A is 10 *feet*, and B 12 *feet* long.

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That is (and this is the crucial point), there is a *common measure*, namely *one foot*, which can be laid down *exact whole numbers of times on* A *and on* B, and *which will completely measure both of them.*

We do not have to choose 1 foot as our *common measure;* in this instance 2 feet would do.

But suppose A were 10 feet 7 inches long, and B 12 feet long. Will the 1 foot rule measure both of them exactly? Obviously it will not. *Have the rods any common measure?* Try turning their lengths into inches: A is then 127, and B 144 *inches* long, and A, B have the common measure 1 *inch.*

Please be patient. I know this is childishly simple. Then why am I boring you with it? Because a far better mind that yours or mine went hopelessly astray over it. Not only one mind better than ours, but scores of them.

Is it not clear from the example of the rods that *by taking a sufficiently small length we can always find a common measure for any two rods, no matter what their lengths are?*

For example, suppose A is 1.2794 inches long, and B 2.38806 inches long. Then A is 127940 *hundred-thousandths* of an inch long, and B is 238806 *hundred-thousandths.* This means that we have found a *common measure*, namely one one-hundred thousandth of an inch for A, B.

From this to the universal conclusion is but a short, easy step, and Pythagoras took it. Is it

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not clear that *by taking a very small common measure if necessary*, but still a common measure which is not nothing at all, *we can always find such a measure for* A *and* B, *no matter what their lengths?*

If you were one of those who thought everything was childishly simple, you doubtless will have answered Yes to the last question. I sincerely hope you did, for I baited the trap with some care.

The correct answer is No. This diabolical No was the explosion which shattered Pythagorean numerology.

Had the correct answer been Yes, we could have said *that the ratio of the lengths of any two lines whatever is expressible as a/b, where a, b are whole numbers.* This is what Pythagoras first thought. It is false. When he learned the truth he suppressed it—a curious thing for a seeker after truth to do. The custom however persists.

In particular, about 410 B.C., one of the Brotherhood divulged the fact that a diagonal and a side of any square have no common measure. Some authorities put the discovery earlier, and assert that Pythagoras himself was the discoverer. Whatever the truth of this, the discovery escaped from the secrecy of the Brotherhood about 410 B.C.

The truth about the diagonal is proved by a little arithmetical reasoning, which I shall leave to the reader. Everything is settled by showing that if a, b are whole numbers without a common divisor greater than 1, then $a^2 = 2b^2$ is a false

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statement. If it were true, Pythagorean numerology would probably never have blown up. To prove the falsity, notice that a must be an even number, and go ahead. Presently you will arrive at the contradiction that a is both even and odd. This of course would not have deterred Nicholas Cusa in his numerological moods, but it is too much for any mathematician in his right mind.

Now, why did this destroy Pythagorean numerology? Simply because the Brotherhood had believed that *all things in the universe are exactly measurable by common whole numbers*. To state this more fully: if a number n is of the form a/b, where a, b are whole numbers, then n is called *rational;* if n can not be expressed in this form, n is called *irrational*. For example, .75 is rational, because $.75 = 3/4$; the square root of 2 is irrational, because no whole numbers a, b exist such that a/b is equal to the square root of 2.

The universal numerologists believed that the universe is *entirely rational*—in the arithmetical sense, of course. The existence of numerology is conclusive proof of irrationality in the medical sense.

Numerology blew up because the universe, including mathematics, contains infinitely more irrationals than rationals. The rationals are as rare as miracles compared to the irrationals. The exact opposite of what numerology asserts is closer to the truth of observation. But—and it is a

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tremendous but—there is the important reservation that the logic of irrationals is not yet completely cleared up. Nor, for that matter, is the logic of rationals. We shall glance at this later, and see how the ousted rational is coming back like seven devils.

What has all this to do with apologizing to Plato? Everything. In the Laws, the Athenian stranger says it is "scandalous" that so few Greeks are aware of the fact (possibly at the time of rather recent discovery) that not all geometrical magnitudes are commensurables, or, as we should say, ignorant that irrational numbers (like the square root of 2) do exist. With charming frankness he remarks that he himself heard of this devastating fact only a short time before. For it *was* devastating. It destroyed—but we need not go into that again.

At this point Plato blew up. He declared that those who do not know that incommensurables (irrationals) exist are not men but swine. The last word is his own. Personally I think it a little harsh; "numerologists" would be inoffensive and exact.

Schemhamphoras!

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Chapter VIII

A GREAT DREAM

MATHEMATICIANS AND CHRISTMAS TREES

The ancients and the men of the Middle Ages having shown us samples of their numerological wares, let us see in this and the final chapter what the moderns have to offer. If we look with reasonable closeness we shall gradually see the great dream of Pythagoras materializing in the universal fog of speculation.

In an earlier chapter I promised to restore the numerological balance by quoting what seem to me to be fair estimates of the limitations of the mathematical mind and of the severely rational. As much of what we shall have to look at requires something more than mere rationality to appreciate this is a good place to fulfill the promise.

I have never implied that one type of mind is superior to another, nor that the conclusions reached by one are necessarily inferior to those attained by another, and I hope that anyone who disagrees with anything I have said will be reasonable enough to see that this is the only consistent attitude any rationalist could possibly take. For, rationally, it is nonsense to talk of superiority in this connection. There is no defined scale of

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values, so comparisons are impossible. Mystics of course can consistently claim superiority.

The following two quotations favor the anti-mathematical side of the case by pointing out the weakness of the mathematical. One quotation is from an eminent (deceased) historian of philosophy; the other is from an equally eminent circle-squarer, still living. As the name of the latter cannot be given, it would be unfair to betray the former. The reader should have no difficulty in deciding which is which.

"The mathematician's attitude, when he is confronted with mere probabilities and plausibilities incapable of demonstration, will depend in a remarkable degree on the accidents of temperament and training. In religion and folklore as a whole he will be completely at a loss. At one time he will reject them root and branch with the impatience of reason toward nonsense; at another time he will willingly bow his head under the yoke of tradition."

To this I say Amen. The same author points out that the very natures of mathematical reasoning and of the mathematical disciplines in general are such that "those who cultivate these branches of knowledge are but too frequently apt to mistake the firm concatenation of a doctrine as an adequate substitute for its defects on the side of outward proof. The rigor of deduction is often compatible in their minds with an arbitrary and subjective

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looseness in the premises." Having said Amen once, I feel backward about confessing again in public, but I think some mathematical and theoretical physicists and astrophysicists might respond without impropriety.

The second critic is equally sour in his remarks. "Because a mathematician fails to find what tradition has instructed him to observe in a demonstration, he arrogantly refuses to look at any matter ab initio and on its own merits. Narrow devotion to a prescribed technique is not the way of discovery, nor is it the path of science. Only a rash or unbalanced mind ventures to dispute the conclusions reached by a concensus of competent mathematicians. Granted the hypotheses, the conclusion is rigidly unique and inevitable. But if the hypotheses be absurd, the conclusion, for all of its rigor of deduction, has no status either in the realm of metaphysical reality or in the world of objective fact."

It is not my turn to speak, but let me cap these by another, a short one, from an impartial judge who was neither philosopher, circle-squarer, nor mathematician. He was not even a numerologist. "Mathematical knowledge adds vigor to the mind, frees it from prejudice, credulity and superstition."

Now, *does* it? How many mathematical dons have masculine intellects? The most bigoted man I know is a mathematician; the most gullible soul in America is a mathematician; the shrewdest

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mathematician I ever heard of never started any important undertaking on Friday the 13th. After all this I think the scales are pretty nearly even.

Anyone who tries to follow the physical speculations of today, with all their strange metaphysics of free will and determinism, may be helped by reflecting on the following true parable of the Christmas tree.

Last Christmas two painters, a man and a woman, dropped into a Spanish-Mexican restaurant for lunch. As they sat waiting to be served they took in the festive decorations. The central attraction was a glittering Christmas tree sheltering a snowy Mexican farm scene. As usual in Americanized versions of Mexican art, the United States, Spain and Mexico had suffered horribly to bring forth an incredibly hideous and meaningless monstrosity. As the pair sat silently taking in the abomination, the man's face grew glummer and glummer. The woman's brightened. Finally she said, "If you live long enough, *all* art will look like that Christmas tree to you."

That is how some feel about numerological theories of life and the universe, and how some are beginning to feel about scientific speculations, especially those of physics and astrophysics. In the end all scientific theories of the cosmos will hang like brilliant red glass balls in the evergreen foliage of the mystical tree planted by Pythagoras 2500 years ago, and we children will clap our hands

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in ecstasy every time a new speculation is hung up for our mystification. In the meantime imaginative but sober men with balances, telescopes, interferometers and the thousand and one other tools of the toymakers, will work away unobtrusively in well-aired shops, getting ready for the next Christmas.

ISOMORPHISM

This terrifying word really is worth numerologizing. What Schemhamphoras was to the cabalistic numerology of the Middle Ages, isomorphism is to the crudest brand of current fortune telling or the ethereally refined numerology of modern mathematical speculations on the natures of God and the universe. No mere 72 mystical names are summarized in this modern variant of omnipotent omniscience but, literally, an infinity.

Roughly, if two "things" have the "same form," they are said to be isomorphic. This must be made more precise before we can see its bearing on numerology, ancient as well as modern, but even a close description is extremely easy to grasp.

There are two main kinds of isomorphism, *simple* and *multiple*. The simple kind is the one which has bred numerology. Multiple isomorphism offers a boundless field to the numerologists of tomorrow.

The technical sense in which we shall use isomorphism first appeared, about seventy years ago, in modern algebra. It has but little to do in any

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exact way with the isomorphisms of biology and other sciences. Simple isomorphism is the only kind we need consider.

One of the commonest instances of isomorphism is seen on looking at any faithful map. If the map tells us that town B lies between towns A and C on a particular road R, and if we wish to go from A to C and pass through B on the way, we can follow the road R. The *real* towns A, B, C and the *real* road R are *related* in precisely the same way as are the *points* A, B, C and the *line* R on the map. The story told by the spots and lines of printers ink is *isomorphic* (it is *not* the same) with that told by the real countryside in its purely geometrical relations of connection and betweeness.

Now did anyone ever hear of even the drunkenest driver mistaking his map for the countryside, and trying to get anywhere by crawling like a fly from A to B on the paper? I do not say that it cannot be done, by slipping into the fourth dimension at A, for instance, and oozing out to the third again at B and C, but I have never seen it done.

Numerology is just this sort of magic when it is applied to scientific speculations. *The thing mapped is identified with the map.* For example, if someone says that things *are* numbers, he probably has a four-dimensional vision denied alike to mathematicians, scientists, and people of reasonable common sense. This however does not prove that the mystics may not be right and all the rest wrong. Proof in these matters is impossible.

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If anyone dubs me a numerologist for that last statement, I return the compliment, with the addition that we disagree on the meanings of proof and evidence. The whole question is ultimately one of belief, or faith. Either you have faith, or you haven't. Many haven't, at least of that kind. Nor, for example, have they faith in Berkeley's idealism (*Essay toward a New Theory of Vision*), although they are unable to find a flaw in it, or to see that its attackers have seriously damaged it. To them it is simply untrue, and they feel this, not in their heads, but in their viscera. Similarly with universal numerology. Some of the ancients must have felt the same way when they located the seat of the soul where they did.

But to get back on the road after this bad spill into the ditch of metaphysics. Isomorphism applies the idea of mapping to other things besides country roads. Suppose we have a collection of "things" A, B, C, ..., Z, before us for contemplation. These things may be any whatever—words, mathematical symbols, human beings, physical "facts," bricks, historical occurrences, or anything else imaginable. The problem is to construct some sort of intelligible description of the "relations" between them. This presupposes that there *are* relations between them.

To be concrete for a moment, suppose A, B, ... are men all of different ages, and we wish to *map* the

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relation of seniority in age on *something*, preferably on *numbers*. Strictly, of course, A, B, ... are *not* men; they are signs, pictures, or symbols for men; but we need not be as precise as this yet.

If A is older than B, let us picture this by writing (A, B) *where the elder of* A, B *is put first in the picture* (A, B). Then if (A, B) and (B, C), it follows that (A, C), by the meaning of "older than."

Where have we seen anything like this before? Any numerologist will cite scores of numerical relations which map this exactly. A simple one is the relation "greater than." *If the number a is greater than the number b, let us write* $[a, b]$. We can now construct our map.

As A, B, ..., Z are all of different ages, one must be older than all the rest, and it must be possible to arrange them in a row, with the oldest at the extreme left, the youngest at the extreme right, and each man, except the last, older than his right-hand neighbor. Suppose this ordered row places the men alphabetically,

A, B, C, ..., Z.

Now, to each man give a number, not at random, but as follows. Give A the number a, B the number b, and so on, giving to Z finally the number z, and write these numbers in another row,

$$a, b, c, \ldots, z.$$

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What condition must these numbers fulfil to make our map true? Obviously a must be the greatest, z the least, and every number, except z, must be greater than its right-hand neighbor. For example, if there are exactly 26 men, we could take $a = 126$, $b = 125$, ..., $z = 101$. Whatever numbers satisfying the stated condition be chosen, we have a true map.

The significance of this, obvious as it is, must be labored a bit, as we shall come presently to something just as simple but by no means so easy to assent to.

What has been done? We have made a *one-one correspondence* between the *men* A, B, ..., Z and the *numbers* a, b, ..., z of the following kind: if P, Q are *any* two of the men for which it is true that (P, Q) (P is *older* than Q), and if p, q are their corresponding numbers, *then it is true that* $[p, q]$ (p is *greater than* q). Conversely, *if* $[p, q]$, *then* (P, Q). Having made this one-one correspondence, we can study the relation of "seniority in age" by attending only to the relation of "greater than" for the corresponding numbers.

But who would say that *because* $[p, q]$, *therefore* P is a number, not a man, and that he is greater than the man (or the number, if you prefer) Q?

The map is not the thing mapped. When the map is identified with the thing mapped we have one of the vast melting pots of numerology. Notice also that the relation considered in the map

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does not (in this instance at least) make sense if it is supposed to hold for the things mapped.

Instead of *numbers* in this example, we might have used *colors* arranged according to tints, or to the solar spectrum. To call a man blue merely because he is older than one who is green may be true in a sense, but it is messy mapping, as we see immediately if we substitute yellow for green.

Let us make a fresh start and now give the inclusive descriptions for this kind of mapping. If (X, Y) represents a specific relation between any two members, X Y of a certain collection, and similarly for [x, y] and another collection, and if we can pair off the things in the two collections, say X with x, Y with y, ..., in such a way that [x, y] holds if (X, Y) does, and likewise for all paired-off couples, then we say that the X, Y, ... collection is *isomorphic*, for the relation ( ), to the x, y, ... collection for the [ ] relation.

That is, we set up a one-one correspondence (if we can) between the collections and map one on the other so that the relations between the things in the first are imaged exactly in relations between the things in the second. In this narrow and extremely special kind of mapping either collection can be viewed as a map of the other. But neither *is* the other, unless they are the same collection to start with, and the relations ( ), [ ] are the same.

The advantages of carrying a map rather than a county are so well known that the object of the

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highly practical kind of mapping we have just described should be plain enough.

I said in an earlier chapter that Pythagoras reached a generalization which was both simple and great, but which was not therefore necessarily true: *The Cosmos can be mapped on the integers 1, 2, 3, ...*

I think it fair to say that Pythagoras believed the map works both ways; namely, given *any* relation between the integers, we can go out in the actual world, or universe, and find real things reflecting exactly the integers and the relation between them; and, conversely, anything in the universe can be reflected in the integers. Finally, *he identified his map with what he was mapping.*

He taught much more than this, but to give any idea of it I should have to describe not only simple isomorphism, but multiple. Without a few symbols this is too much of a job, and I shall content myself with the customary mathematical lie "it is easy to see."

THE GREAT DREAM

The most comprehensive dream ever dreamed by our race is short: *The Cosmos is isomorphic with pure mathematics.*

Doubt it, if you like, but don't turn your back on it. It may be true. At present we do not know whether it is a great and simple truth or whether it is just nonsense.

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MODERN HIGHER NUMEROLOGY

Glancing back at what Pythagoras said about things being numbers and numbers being things, we easily recognize the following generalization of some modern speculators as just what he would have said had he known all the mathematics we know today: *The Cosmos is pure mathematics, and pure mathematics is the Cosmos.*

To see how this complete and final confusion between maps and things mapped has arisen, we must glance at a few of the speculations of recent times. This will be done (with a disgraceful inadequacy for which I apologize) in the concluding chapter. Any sufficient account would demand a heavy treatise and the hard labor of a small army of philosophers, mathematicians, logicians, scientists, and historians of science, with perhaps a numerologist or two to maintain a just balance. The task of making a thorough analysis is probably beyond the powers of any three men. But it should be done, or we shall never know whether we are standing on our feet or our heads. Perhaps it doesn't make any real difference.

Let us look over our maps for a moment first, to see whether we can find a possible reason why the great dream may not be true. If modern higher numerology is the truth, then everything the numerologists do in Hollywood, New York, Suckers' Pool, and elsewhere is beyond criticism. Still, we don't have to believe it, even if it is true.

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Anyone can disbelieve a lie; it takes intellect and independence to doubt the truth.

THE NEEDLE'S EYE

All that has been said, and all that will be said, does not plug up the needle's eye through which those who wish may squeeze their way into the lowest of the seven heavens of numerology. This lowest heaven is the one they advertise over the radio and sell for a dollar a front foot. It is the numerology which takes your number and your dollar and harmonizes your vibrations with those of the stock exchange or the divorce court. Nothing in the whole realm of reason can ever stop up that tiny hole which is the one-dollar-rich suckers' entrance to the paradise of poor fish.

What is the needle's eye? Simply this: all the explosions of Pythagorean or universal numerology can not budge the hypothesis that things of the spirit and purely human values *are* mapped on the integers.

Disproof is impossible. Believe it or not.

Why should anyone who is not a sucker point out the needle's eye to others? Merely to make that good man who said all those nice things about mathematics adding vigor to the mind, etc., not an utter fibber. If I omitted to point out the loophole in the rationalists' destruction of *universal* numerology, namely, that the destruction in question can not possibly affect the sucker rackets

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of numerology, I should be doing exactly what Pythagoras did when he suppressed his knowledge of the explosion. And although I admire the old man tremendously, I cannot see eye to eye with him in this matter. The cheapest way of letting an error correct itself is to give it plenty of rope. Millions for rope; not one dollar for tribute!

EXTRAPOLATION

If isomorphism is the Schemhamphoras of numerology, extrapolation is its Mesopotamia—that blessed word which gave unspeakable comfort to the poor tired charwoman.

Imagine any good map you like. No matter how detailed the map is there will be certain features of the countryside which it does not portray; not every pebble can be charted on the map. But if the map is really good, we can guess from what it does give. This process of guessing is called *interpolation*, if we apply it to forecast some detail of the landscape which, if it had been mapped, would have been—we *believe*—mapped between two points *on the map*. This sounds much more mystical than it is, but anyone who has used a map knows intuitively what is meant.

Suppose now that the cartographer had been careless, and had omitted to mark a precipice crossing the highway on his map. Anyone interpolating over this precipice would probably break his neck if he drove the road in the dark.

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Extrapolation almost always, sooner or later, leads to broken theories, if not broken bones. For it is practically all driving in the dark.

Suppose our map is that of a "white" (known) spot on a "dark" (not wholly explored) continent. A plateau on the white looks as if it should continue uniformly into the dark. There is a river down the middle of the plateau, so we confidently head our canoes downstream toward the dark. More than one daring man has lost his life that way by going down the rapids or over the falls, swept along between precipitous cliffs. It almost happened to Major Powell several times in the Grand Canyon. But he had better luck than more scientists, even if he did have only one arm while most of them have two.

Extrapolation is guessing *beyond* the map.

Now, in mapping the cosmos, or any part of it, on pure mathematics, the things mapped have to be ideally simplified before any reasonably intelligible map can be drawn at all. The colorful rocks, the shapely hillocks, the winding gullies and hundreds of other more interesting details of the real landscape are omitted entirely from the map. One thing at a time is the rule.

Thus, by sufficiently *abstracting*, or *idealizing* the reality, we may be able to make a serviceable map of the physics of electricity, or of gravitation, or of the behaviour of atoms, or of the metaphysics of radiation. If it is painstakingly drawn the

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map will give more than was put into it, but sooner or later it will need revision. Finally so many corrections have been smeared over the map that it is no longer legible and it is discarded, to be snatched up eagerly by—never mind whom.

I do not wish to push this analogy beyond all limits, but one more aspect of it must be glanced at. Many maps are made for one and the same territory—road maps, geological maps, sociological maps, and dozens more. Each has its special purpose, for which the others are useless. Who but a fanatical unitarian would ever dream of plastering all the geology, sociology, topography, and the rest onto one and the same piece of paper? It could be done, but what use would it be? If a *simple* map for all could be made, the story would be different.

Here now comes a curious thing. The dream of physical science is just this, to unify electromagnetism, gravitation, radiation, in fact all physical phenomena, by mapping them on one grand, unified theory. At present fashion favors a purely mathematical unification. This is the vision which Pythagoras saw, only he went farther. He would have included humanity and all that human nature means. Then he would have joined some of the moderns by saying the map *is* the cosmos.

Is the dream absurd? There are no grounds for answering one way or the other. At least that is the conservative opinion of many. Whether the

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dream is sublime or not is purely a matter of taste. But it is a fact that every approach to the dream has frightened it farther back into the shadows. Every attempt to grasp it has destroyed whatever consistency there may be in the tenuous stuff of which it is made. Then why pursue it? The history of science is the answer. It pays, intellectually and practically.

All this may seem a long way from numerology, but it is not. The maps we have been talking about are mathematical theories of the cosmos. The interpolations and extrapolations are the fascinating speculations of modern theorizers. One extrapolation—it is little short of scientific blasphemy to say this—is the great generalization called the second law of thermodynamics, from which some deduce that our end is to be a universal "heat death," a dreary sort of eternal stagnation in a lukewarm soup of everlasting idleness. This must have been what the washerwoman had in mind when she looked forward to heaven as the place where you sit and do nothing for ever and ever.

Anyone who is inclined to confuse prediction and prophecy may be helped by the following neat genealogy which a great engineer gave me. "Interpolation is the mother of prediction, extrapolation is the father of prophecy."

Where do the numerologists come in? Neglecting those who squeeze through the needle's eye,

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we need look about only for the universal numerologists. These are the pickers up of discarded maps and the mystics who believe that the dream is a reality and the reality a dream. I do not pretend to know what reality is, but this does not affect the preceding statement.

To any mathematician who is not antediluvian in his mathematical creed the most curious thing about the beliefs of the universal numerologists is their shadowy foundation. When this is not purely emotional it is severely mathematical. *Because* it is mathematical—when it is—*therefore* it is true in fact and not merely on the map. That "because" and "therefore" are of exactly the same kind as those used by Pythagoras in his numerological moods, and by Origen, Augustine, Jerome, Peter Bung, and Nicolas Cusa in theirs. This does not necessarily make nonsense of them, but we needn't go into the why of this again.

In the next and concluding chapter we shall incidentally glance at the numerology of mathematics itself. Then, if we care to look back over what has been said above, I think we shall agree that it is fair enough. However, every human being has a right to stew in his own prejudices.

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CHAPTER IX

ULYSSES

UNATTAINED

The quest of the modern Pythagoreans for a mathematical unity in the scheme of things is strangely like that of Tennyson's Ulysses:

> "Yet all experience is an arch wherethro'
> Gleams that untravelled world, whose margin fades
> For ever and for ever when I move . . . .
> . . . .
> . . . . yearning in desire
> To follow knowledge like a sinking star
> Beyond the utmost bound of human thought."

And with the gray pessimism of old age he presently voices what is today the wildest optimism of science:

> "Death closes all; but something ere the end,
> Some work of noble note, may yet be done
> Not unbecoming men that strove with gods."

Finally,

> ". . . . Come, my friends,
> 'Tis not too late to seek a newer world."

It would be difficult to find a juster summary of the great dream of universal numerology from

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Pythagoras to the present day. If Ulysses had kept on sailing he would eventually have found himself home again, without once having touched the horizon. No doubt he would have sailed again, just as we do today, confident that *this* time some work of noble note may yet be done. But we haven't done it, and the margin fades for ever and for ever when we move.

Shall we ever cross the horizon and bring back a complete mathematical map of the cosmos? To some it seems entirely unimportant whether we do or not. Those who need the pot of gold may continue to chase the rainbow if they believe the fable. The exercise will do them good and keep them out of practical mischief. Without some such stimulus most human beings would never have left their native villages, and would have remained bigoted, superstitious ignoramuses from cradle to coffin.

UNATTAINABLE?

Disinterested exploration by experiment and the unappeasable hobby of making ever better mathematical maps have advanced physical science as nothing else has, not even the very human desire for what the world can give in the way of rewards, or the insistent demands of our machine civilization for more efficient machines to throw more human beings into the discard.

Whatever the explanation, the scientific mind

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has found it both stimulating and intellectually profitable to behave *as if* the universe *is* mappable on pure mathematics. Is it? Does the question even make sense? Again nobody knows.

What difference does it make one way or the other whether the dream is or is not true, if those who find inspiration in it continue to work harmlessly as if it were true? Today it can make no difference, for in most civilized communities a man's beliefs are his own affair provided he does not try to ram them down his unbelieving neighbor's throat.

The dream has not been attained, although there have been some noble failures to grasp it. Is it unattainable? The majority seem to believe not. The dream is always waiting behind the next shadow, and next time, surely, they will succeed in creeping up on it before it has time to slip away.

Recently however Heisenberg's principle of indeterminacy has shaken the faith of some believers, and they are so violently awake that they refuse to believe in the existence of the dream, much less in the hope of ever falling suddenly on it and squashing it flat. So much has been written on indetermination that there is no need to go over any of the ground again here. The exploring devices by their very nature chase the object sought beyond reach of the devices, like shooting rubber elephants with tennis balls. But I have promised not to thicken the prevailing fog. If you have ever watched a researching baby trying

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to pick up a nice fresh slippery cherry stone you will see all I am trying not to say.

The dream then *may* be unattainable. Putting aside the contributions of physical science to this doubt, let us glance at a far deeper reason why the dream of a mathematically unified cosmos *may* be nothing but a baseless illusion. To report adequately on this matter would take more space than I have, but any interested reader can easily locate the authoritative sources of my meagre account. Only patience and a clear head are required to follow the arguments. I have no opinion one way or the other; to my mind nothing has been settled, but the approach is interesting and seems promising. Scientific speculators and numerologists have entirely overlooked this revolutionary work.

NUMEROLOGIZED REASON

To make my sketch as simple as possible, I shall confine it to a single trait of the movement I am trying to call to the attention of numerologists who may not have run across any of this sort of thing.

How do we ever know that a particular set of assumptions will not lead to self-contradictory results when developed according to the formal laws of logic, such as are habitually used by mathematicians, lawyers, scientists, and in fact by practically all human beings who ever reason at all outside of lunatic asylums?

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For instance, what is the basis of our belief that the assumptions (axioms, postulates) of common school geometry as presented in any carefully thought-out book will not sooner or later produce something like this: the angle A (in a certain figure) *is* equal to the angle B, *and* the angle A *is not* equal to the angle B? Mistakes like that do happen if we are careless. When they happen we feel we have been careless, and we go back and seek the mistake.

We feel, but do we know? We do not, until it has been proved that the assumptions can never lead to a contradiction. It is at least conceivable that we had done all the steps correctly from the assumptions, using common logic, and that still we got a contradiction.

The natural way of facing such a difficulty would be to give our assumptions a thorough overhauling. We should expect to find something wrong lurking at the very roots. Few would think of suspecting common logic itself as being the serpent in the grass, but some have.

Whatever way we might take of setting things right we should be in a more receptive mood than before for the following program: *Prove once for all that the assumptions will never produce a contradiction under the rules of logic.*

It is easier asked than answered. For about half a century there has been a terrific struggle going on to do just this thing, namely to construct a

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set of assumptions for common geometry for which it can be *proved* that they are *consistent* as described above.

The question of interest for all speculators and numerologists is *how has this proof of consistency been gone about?* The latest (1932) bulletin from the front is stimulating if nothing else. I shall get to it as quickly as possible. For those who are unfamiliar with this sort of thing I may emphasize that it is the work neither of numerologists nor cranks, but the serious effort of some of the ablest logicians and profoundest mathematicians the world has known.

The attempted proof of consistency proceeds as follows. Without a great expenditure of ingenuity, *common geometry* is first *mapped* (in the sense of one-one correspondence, see preceding chapter) on *common arithmetic*. This part has been done exactly, according to the unanimous opinion of those competent to judge; it really is very simple. The next step is then to *prove* that *common arithmetic* is self-consistent. Such a simple problem as that, some might think, should be easy, but it isn't. *It has not been done.*

If that were the end of the story we might hopefully ascribe our failure to our lack of skill, and say that the next generation will do in its sleep what we can't do with all of our senses. But it isn't the end of the story. Quite recently it has been proved that it is impossible to prove the

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consistency of any set of assumptions which can be mapped on common arithmetic. This would seem to prove the impossibility of proving the consistency of arithmetic itself. We are all at sea.

If the proof cited stands the drastic criticisms it is certain to get, well—to elaborate the consequences would only obscure the significance of what has been said, and everyone must think it over for himself.

What immediately follows is independent of the preceding, but it suggests a possible reason why we should expect such a disconcerting conclusion.

George Boole published his epochmaking treatise on the Laws of Thought in 1854. Since then his pioneering work on the algebra of logic (symbolic logic) has been pushed far in all directions.

In Boole's algebra, for instance, $a \times b$ means the class of all things common to the classes *a and b*, while $a + b$ means the class of all things that are in *a or b*, and we have the true statements $a \times b = b \times a$, $a + b = b + a$, $a \times (b + c) = a \times b + a \times c$. This simple algebra when fully developed allows us to do *algebra* instead of trying to keep our heads from swimming in seas of words swarming with squirming sea-serpents of sentences.

To sum up: Boole *algebraized logic*; he *mapped logic on a very simple algebra*, and his numerous successors have extended this map in many directions, including the foundations of mathematics. They have not yet attempted to map the founda-

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tions of science. Possibly it has none. An attempt to map the foundations of physics, for example, might perhaps create them.

Now, it is not very difficult to *map* Boole's algebra of *logic* on *common arithmetic*. From this, common logic is mapped on arithmetic. This suggests that any attempt to prove logical consistency by mapping on common arithmetic (as they have done with geometry) can itself be mapped on a kitten chasing its tail.

So the dream *may* be unattainable. This is the strictly "irrational" straw at which the obscurantists, mentioned in a previous chapter, clutch desperately to avoid drowning in the seething welter of free-will and determinism that has gushed like a Niagara from the rock of causality which Heisenberg dealt such a resounding crack. Few of them however really know where they are in all the maelstroms of doubt, any more than the rest of us do.

Numerologists, I should think, would resent any tampering with the foundations of arithmetic, so I shall say no more about it.

NUMEROLOGIZED MATHEMATICS

About fifty years ago the mathematician Kronecker asserted that "God made the integers; all the rest is the work of man." Behind this joke was a deadly earnestness and much bitter controversy. As this is not an account of modern mathematics I

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cannot go into the fascinating story of what preceded Kronecker's joke, or the wry mathematical sarcasm it has proved to be in the past twenty years. All we are interested in here is the return to Pythagoras which Kronecker half jokingly proclaimed. He did not go back alone. Today he has numerous followers.

I mention this for two reasons. First, the modern mathematical Pythagoreans find their way home to 500 B.C. by many roads, one of which traverses the tenuous bridge built by the men of the Middle Ages. Many of the apparently futile riddles, like the angels on the needle point, which the scholastics tortured endlessly reappear in strange new guises in modern mathematics.

Partly to escape these, and partly because they believed it profitless to attempt answers, the Kronecker secessionists resolved to go back to the whole numbers and rebuild all mathematics with them alone. Second, the dispute between the two mathematical factions is singularly like those of physical science over waves against particles. Science may have settled its difficulties; mathematics hopes to do likewise before the next ice age, after which perhaps it won't matter.

The difficulties came in with contradictions produced by analyzing the notion of continuity—which is the antithesis of what can be mapped on the integers alone. Try to analyze the idea of motion, as the Eleatics did and the moderns are

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doing, and you will see how easy it is to get into a muddle. Now continuity has proved a powerful and unifying concept in the mathematical mapping of the physical world. Instead of looking at a charged particle as the important thing, it is more profitable to study the surrounding space, the so-called field. Doing so, we run full tilt into all the paradoxes of the infinite, some of which worried the Greeks. These can be ignored for practical purposes, as a rule. But if the dream of a mathematical map of nature is to be realized the teasing puzzles must be convincingly solved. Prehistoric logic no longer satisfies the rational. The puzzles have not yet been solved.

To give a specific example of the riddles which have driven some to arithmetizing mathematics is impossible without technical language or long explanations, but a fair picture of one of them involves only plain English. It is due to Weyl, one of the constructive living critics in this field.

"Some adjectives have meanings which are predicates of the adjective word itself; thus the word 'short' is short, but the word 'long' is not long. Let us call adjectives whose meanings are predicates of them, like 'short,' autological; others heterological. Now is 'heterological' heterological? If it is, its meaning is not a predicate of it, that is, it is not heterological. But if it is not heterological, its meaning is a predicate of it, and therefore

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it is heterological. So we have a complete contradiction."

Where are we, 500 B.C., the Middle Ages, or in the Great Depression? Fight it out with yourself. By evading such brain-twisters the return to Pythagoras would save many a splitting headache.

But, as always, there are several other sides to the story. Even the classical analysis of mathematical continuity, which is the very stuff out of which most mathematical maps of the cosmos are made, is based ultimately upon the common whole numbers. So, whichever way we turn, we find the colossus blocking the path.

Now what if all those doubts about arithmetic hanging by nothing in midair should be sustained? I suppose we should have to take refuge with the irrationalists and the fortune-telling numerologists. However, it is my expectation, for what it may be worth, that science will have solved all our problems for us long before we all go crazy or become dollar-a-head suckers. The solution may be unpleasant to swallow, but we shall never need another dose till the end of eternity. Nevertheless I should prefer to keep on living, even if I have to turn numerologist. If only all this sort of thing led to none of its damnable applications it could be ignored or classed with neurotic fiction. Ulysses seems all set to sail "beyond the utmost bound of human thought" on his own torpedo.

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POTS OF GOLD

While Kronecker was turning back to Pythagoras, physical speculation was revelling in an orgy of gears, smokerings, and gyrostats that would have delighted Archimedes. Inspired by the overwhelming successes of all the machinery it had brought forth, physics filled its accommodating ether with the most amazing complex of mechanical models the ingenuity of wheel-intoxicated enthusiasts has ever invented.

At least one of the great mechanical engineers of the cosmos seems to have convinced himself that the models were real things. It was that persistent confusion of the map with the thing mapped, and it was a perfect, hideous picture of the age which brought it forth.

Optimism hurdled all previous records; the riddle of the universe was about to be solved—by machinery. One more gadget here, a nut or two there, and the gigantic machine would continue to function till God or the second law of thermodynamics stopped it.

Then some careless mechanic dropped a monkey wrench. There was a stripping of gears, a last terrific threshing of broken piston rods, a mad scurrying of teetering wheels into the infinite, and the whole machine went to complete and final smash shortly after the year 1900.

What made them do it? Exactly the same im-

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pulse that drives our own generation to forget its troubles in a debauch of mathematical speculations. They were looking for the end of the rainbow, just as we are. Shall we ever find the pot of gold? If history is the eternal recurrence some say it is, we shan't.

One short anecdote about a very great dreamer sums up the history of the search by machinery. All his life Lord Kelvin held before him the great vision of Pythagoras, to paint one grand inclusive picture of the physical universe, one sublimely simple masterpiece which would tell the whole story for ever. The only occasion when he ever said anything that shocked his listeners was toward the end of his life. Describing the effort of his long search he summed it all up in one word, failure. It took something—I don't know what—to say that.

So complete was the wreck that few bothered to attempt salvaging anything. Instead of trying to construct dynamical or other not utterly intangible models of the universe, the younger generation contented itself with purely mathematical maps. If a set of differential equations correctly describes the electromagnetic field, why look further?

Thus the mathematicization of the universe set in, and is still going strong. This should please pure mathematicians, but somehow it doesn't seem to captivate all of those who have learned

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their trade in historic times. To be told that God is a pure mathematician drives some of them to profanity. But it would be untrue to say that all, or perhaps even the majority, feel that way. A mathematician can be as mystical as the most speculative scientist living when he really puts his mind to it. So long as the speculators stop short of saying that the cosmos *is* pure mathematics, nobody need get excited. Possibly it is, as one speculator seems to assert, and he may have found the pot which held the gold. Some disgruntled mathematicians would like to crown the discoverer with his discovery.

ONE-THIRTY-SEVEN

Closer to the strict Pythagorean tradition as it was before the explosion of 410 B.C., is the recent work of one of the most daring thinkers of our generation. Even his severest critics admire his courage in sticking to 137—it was 136 a year or two ago—and his cosmological guns. If this work of Eddington's on the "fine structure constant" of spectroscopy, as it is called, stands the test of experiment, probably few will deny it a high place among the great scientific achievements of any age. The indications are that a conclusive experimental answer will be given before the end of 1933. This 137 is without doubt the most Pythagorean thing that has been done since the old man himself died.

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To give some faint idea of what it is all about, recall that Planck quantized energy about 1900: energy is shot out, not in a continuous stream but in definite, separate "packages." Connected with these is a certain pure number "Planck's constant." It, the velocity of light, the ratio of the mass of the electron to that of the proton, the radius of the relativistic universe, and the charge of the electron, are the aristocrats of the physical family. These (and some less noble constants) seem to rule the cosmos. Some of them are tied together to make the fine structure constant mentioned above. Now, if almost any physicist had been told five years ago that this constant is an integer, he would probably have thought it unlikely. Eddington however *deduces* for it the number 137.

What is more remarkable, by purely mathematical reasoning which proceeds from the postulates of relativity and the wave equation of quantum mechanics (largely; there is more to it than this), he calculates the total number of electrons in the universe. There are 129 followed by 77 ciphers of them. This however is not all he gets.

According to Lemaître's theory of the expanding universe, itself a consequence of relativity, the whole swarm of spiral nebulae is spreading out as if the universe were in the midst of enjoying a considerable explosion. The "explosion" produces a doubling of the radius of the universe once about every 1300 million years—about the length of time

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geology needs to account for what is found in the rocks of the earth. So, it would seem, the whole universe blew up when the earth was created. Is it any wonder?

Now the *rate* at which the nebulae are receding increases with their distance from us according to a certain law which, however, has not been fully confirmed by the Supreme Bench. At least there is the possibility of an appeal. Anyhow, Eddington's calculations check what has been observed on this rate with an astonishingly good agreement.

There is more yet; the ratio of the mass of the proton to that of the electron also drops out of the mathematics. As if this were not enough, it is proved that there could be only 92 chemical elements. There *are* 92, but—. There is more yet, but I must stop.

Now, if only a little of this stands up under the experimental and rational criticism it is certain to get it will be the greatest—but I needn't say it again. Eddington himself declares that he can find no flaw either in his hypotheses or in his mathematics. It seems to be up to the mathematicians and the toymakers in the laboratories and the observatories. Which way do you hope it will come out? Or are you entirely objective?

If 137 should be wrong, it will not be so for any merely arithmetical reason, as was the case with the modern miracle in the first chapter. The trouble will be in nature itself.

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I have chosen 137 as the crown of the dream because the spirit of it sums up the aspirations of the thousands of Sir Launfals who followed the dream of a whole-number universe through one maze after another, only to look up and see an irrational staring them in the face on their death-beds. Still, they discovered great things, even if the things they found were not what they sought. He who would save his life must lose it, and it seems to be the same with numbers.

MATERIALIST OR—?

The prim age which dowered lovely ladies with hoop skirts and the luminiferous ether with fly-wheels used the word "materialist" as a reproach for all sorts of infamous conduct. The antithesis of an idealist is not, of course, an atheist, or even an agnostic, but a materialist. For, strictly, an idealist is one who disbelieves in the existence of matter. At least that is one species of idealist.

In view of all that has been said, would it not be more fitting in our age to say "numerologist" instead of "idealist?" That granted, what are we to call mathematicians?

Call them anything you like; you will never make them numerologists.

As my parting shot, I believe that our generation is at the threshold of a revolution in the very rudiments of thinking, and that not even a numerologist can predict what will be standing when peace is restored.

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APPENDIX

I. ODDS AND ENDS

One of the Editors has kindly called my attention to an inexcusable oversight in the preceding account of numerology in the large. As was frequently pointed out, ten, not nine, is the philosophically perfect number. It follows that if the present introduction to a vast and lucrative field of philosophy is to approach perfection, however distantly, there should be ten chapters and not a mere nine. Hence this Appendix. It will be devoted to a very few of the less reputable tricks of the number game, including current monkeyshines with the mathematical infinite, and to two of the more alluring aspects of this fascinating vice in general.

Any numerologist who thoroughly understands what Fermat's Theorem means—it requires no more than seventh grade arithmetic—should be able to apply it with devastating effect in his or her practice, with a consequent doubling of income within a week. I have tried it myself (gratis), and I know what I am talking about.

To keep the somewhat miscellaneous items from jostling one another too closely, I have boxed them up in separate, numbered compartments, each with its own label. I shall give first a few notes on the preceding chapters. Citations of De Morgan refer to his famous "Budget of Paradoxes" (London, 1872). It is hoped that these few allusions will induce all good numerologists to make the mathematician De Morgan's acquaintance; others may be interested in seeing where his novelist son ("Alice For Short," and others) got his whimsical talent. References to other works will be given as we go.

1. The Lion's Skin

There is nothing quite so good for breaking the ice and starting the party off right as a friendly little intelligence test (see chapter

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VII). Although I have never taken one myself, I believe even the flattest of them would go down better than some of the cocktails I have been handed and expected to swallow.

The rules of the game are three. (A) If you write an incorrect answer on the dotted line, your score is zero. (B) If you refuse to answer, and write nothing at all, your score is zero. (C) If you put the correct answer, your score is 1. These rules apply to each and every question; your total score is got by adding the scores on all the questions. If your total score comes out zero you are a low grade imbecile—I believe that is the correct technical term.

The following two questions are taken from an actual test paper. They are typical of many such questions from many such papers. In fact the type appears to be a favourite with the more scientific testers.

QUESTION: *What number immediately follows 5 in the series* 1, 2, 3, 4, 5, ...?

ANSWER: . . . . . .

QUESTION: *What number immediately follows 10 in the series* 2, 4, 6, 8, 10, ...?

ANSWER: . . . . . .

And so we might go on for a hundred or more questions. Your possible scores in the above test are two perfect, both answered correctly, one, and zero. If there were a hundred such questions, your possible scores could range from a hundred down to zero. Thus you might be shown up as anything from a genius to an imbecile.

Without bothering to look at your answers, no matter how many questions you attempted, any mathematician who is not fast asleep can give you your exact score. Your score is zero. Does that prove you a low grade imbecile? Not necessarily. The test is in fact fool proof, in the sense that it necessarily proves everyone a fool.

The joker is in the rules (A), (B), (C), which leave no loophole for that decent common sense which refuses to answer improper or meaningless questions. If some gentle soul is bullied to answer

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yes or no, but to answer, to "Have you stopped beating your wife?" when he never in his life raised his hand against man, woman, or beast, what is he to do?

Here, if you refuse to answer, you are scored zero. *If you put down any number at all, that number is wrong.* So, both ways you lose on this academic shell game.

I really hate to spoil this by explaining why any number whatever is wrong. It should be as plain as day, and it is in fact far plainer to anyone who has had the advantage of a second course in high school algebra, which the testers seem not to have enjoyed, or who knows the meaning of the mathematical word *function.*

On second thought, I am not going to spoil it. I shall leave the questions in their virginal absurdity for my testing friends to do with what they please. A sufficient hint to others is contained in the next.

QUESTION: *Today is Thursday. It rained on Monday and Tuesday of this week in Timbuctoo. Did it rain there yesterday?*

ANSWER: . . . .

If an intelligence test is good for anything, it should leave a loophole for uncommon sense and reasonably close observation. It should catch sharp intelligence as well as dull stupidity. A banker who jumped to conclusions as the specimen questions tempt us to do would hang himself in a week. Respectable intelligence does not jump as the questions would try to force it to jump.

Not all the questions are as ridiculous as the above specimens. Occasionally an innocent looking piece of silliness brings unexpected and fatal results. Some years ago the following glittering bait caught a gilded fish who had correctly (according to the testers' ideas of arithmetic) answered all the 1, 2, 3, 4, 5,. . . kind. "If you were lost in a dense forest, which would you do: Observe the Sun and the direction of flow of the streams, or: Go to the nearest house and ask your way?"

One of the prettiest blondes I ever knew plumped for the second alternative, and not all the eloquence of the testers could convince her that her answer was silly almost beyond belief. Before the

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test, the tester had been planning to propose to her; after the test he was never seen in the company of any but the swarthiest brunettes. However, he was magnanimous. He said a girl as beautiful as his lost love needed no inside to her head; she could do all the thinking necessary for her manifest destiny with the outside only. And in fact she did make a brilliant and paying match. She is now a widow—her husband drank himself to death before the end of the first year. If he had seen her test paper he might still be living. So the tests after all are capable of much good—for the cause of temperance.

Testers and numerologists may pull the lion's skin well up over their ears, but the instant they begin trying to roar the multiplication table at frightened children their voices betray them.

2. *Russell's Barber*

The Honorable but irrepressible Bertrand Russell (saluted in chapter IV) has devised a much simpler paradox than Weyl's (stated in chapter IX). I do not know what Russell's motive was in polishing up this luscious apple of discord, but I have found it ideal for presentation at dull dinner parties. It should be carelessly tossed onto the table just when the conversation begins to bog down hopelessly in social service, the depression, communism, prohibition, humanism, or the good life. Having tossed the apple, the host should slip quietly away at the first hint of massacre, leaving his guests to murder one another in peace. Otherwise he too may have his throat cut with the carving knife, in lieu of a grislier shave by the non-existent barber.

If Russell can be believed, there exist a certain village V, and a certain barber B, who lives in V. The barber B shaves *all* those, *and only those*, who live in V and who do *not* shave themselves. Now, does the barber B shave himself?

If he does, he doesn't. If he doesn't, he does. Therefore . . . .

I see no possible conclusion, except that the barber B is Aristotle's long-sought and Excluded Middle, who has returned to muddle our fatted heads after years of riotous prodigalizing with Plato's pigs (chap. VII).

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3. *The Bung-Stifel Brawl* (*Chapter VII*)

What follows is based on De Morgan's moving history of this bloody encounter. It is to be proved numerologically that Pope Leo the Tenth and his arch-enemy Martin Luther were simultaneously Beasts. De Morgan favours the Hebrew proof, but as I cannot distinguish a jot from a tittle I reproduce the snappy and very convincing Latin demonstration.

The Roman numeral letters with their Arabic equivalents are

I,	V,	X,	L,	C,	D,	M,
1,	5,	10,	50,	100,	500,	1000.

As some of us remember to our cost, the Romans seem to have had no U in their alphabet, but used V for our U; Heaven knows how they pronounced Lucifer. This explains the handsome spelling we see chisled into the granite lintels of PVBLIC BVILDINGS in small towns which point with pride to the large pork chop—usually a Post Office—which their own wearer of the imperial purple snapped up and made off with from under the very nose of the VVATCHDOG OF THE TREASVRY.

Applied to Leo Tenth, or Leo Decimus, or finally LEO X, this legitimate Latinizing made Leo Decimvs of him, thus exposing his rear and both flanks to Stifel's savage attack:

LEO DECIMVS = LEO X

The sum of all the numeral letters in this true equation is

$$L+D+C+I+M+V+X, \quad = \quad 50+500+100+1+1000+5+10$$

This gives 1666. Drat! It should be 666. ... Ah! all is clear: the unwanted extra 1000 was contributed by M. But M signifies also Mysterium, or mystery, and if Leo X is indeed the Beast 666, would not this be well hidden from the eyes of all but God's elect? To reveal this sacred mystery we must strip all M from it and expose it in all its beastly nudity as the 666 which it really is: $1666 - 1000 = 666$.

Now for Luther. Poor Peter at first blundered into the fray as

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naked as a worm. Latin gave him no comfort; Hebrew was a tinkling cymbal. Driven to extremeties to preserve his own reputation, no less than to save whatever Stifel had left of the Pope's fair name, Peter was forced to improve on both Latin and Hebrew. This he did by inventing his own scheme of numbering the letters in the common alphabet, after shabbily Latinizing only the second part of Luther's name:

MARTIN LUTERA

He did not even make the U a V. From A to I the letters of the alphabet were then numbered 1 to 10; from K to S, 10 to 90, by tens; from T to Z, 100 to 500, by hundreds. Thus he got

$$M = 30, A = 1, R = 80, T = 100, I = 9, N = 40;$$
$$L = 20, U = 200, T = 100, E = 5, R = 80, A = 1;$$

total, 666. Two words to a sap are sufficient; the proof is complete.

This is exactly the same brand of numerology that has made radio numerologists rich and poor suckers poorer. More kilowatts to the Federal Radio Commission! The Divine Economy of Nature must not be unduly paternalized: for the efficient functioning of our civilization a large part of any population *must* be poor fish.

Before the last election numerologists all over the country were predicting the outcome by the Bung-Stifel method. Should anyone be tempted to hold a post mortem by the same or similar means, I advise him to exercise extreme caution in broadcasting his findings. Half an hour with $a = 1, b = 2, \ldots, z = 26$ may well provide libels enough for half a century in the penitentiary. I know, because while preparing this Appendix I presided at just such an autopsy.

4. Sacré Bleu!

This well known French swear word may fittingly label the latest in Sacred Numerology. Since Chapter VII was written

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much water has flowed under the Pons Asinorum, and most of it is muddier than all the torrents that have rushed under that historic Bridge of the Asses in the past nineteen centuries. But happily the floods have at last been damned effectively. This great feat of mental sanitation was perfected in 1932.

We must first swallow a bucketful of the older water which De Morgan has piously preserved for us. This dates back to the deluge of 1839. More recent samples—some no older than last year—might have been offered, but for reasons stated in the first chapter the staler specimen is to be preferred.

The mathematical symbol for infinity is an 8 lying on its side, thus ∞. If infinity be multiplied by itself, $\infty \times \infty$, or ∞^2, the result is merely ∞ again. This is no more mysterious than $0 \times 0 = 0$; zero and infinity are much alike, and if one is open to suspicion, probably the other is also. I will show presently that zero is not to be trusted too far. Proceeding with ∞, we raise it to its successive powers (see chap. II), ∞, ∞^2, ∞^3, ..., each of which is again ∞. Instead of 2, 3, ... in this we may put any other finite number (positive, non-zero integer) we please, say f, g or s.

Fractions of infinity will also be required. Divide infinity by 2. As infinity surpasses all finite numbers, half of it must also exceed any finite number, so we have $\frac{1}{2} \infty = \infty$. In the same way one third of infinity is infinity, or $\frac{1}{3} \infty = \infty$, and so on: $m \infty = \infty$, where m is any finite proper fraction.

Following Revilo (*The Creed of St. Athanasius proved by a mathematical parallel*), let us denote the three Persons of the Trinity by ∞^f, $(m \infty)^s$, ∞^g respectively,—Father, Son, and Holy Ghost—where $(m \infty)^s$ means that we first multiply ∞ by m and then raise the result to the s th power. The fraction m represents insignificant and sinful Man, as opposed to ∞^f. Then, following De Morgan, we reproduce two pillars of the Creed with their mathematical parallels in Revilo's revised but as yet unauthorized version.

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But the Godhead of the Father, of the Son and of the Holy Ghost, is all one: the glory equal, the Majesty co-eternal.

It has been shown that ∞^f, ∞^g and $(m\ \infty)^s$, together, are but ∞, and that each is ∞, and any magnitude in existence represented by ∞ always was and always will be: for it cannot be made, or destroyed, and yet exists.

Equal to the Father, as touching his Godhead: and inferior to the Father, as touching his manhood.

$(m\ \infty)^s$ is equal to ∞^f as touching ∞, but inferior to ∞^f as touching m: because m is not infinite.

"I might have passed this over," De Morgan continues, "as beneath even my present subject"—Mathematical Theology. Had De Morgan lived to 1932, or even 1912, he might have tarried long and unlovingly, for in the past twenty years there has been an almost continuous and unprecedented flood of exactly this same kind of Mathematical Theology. All of it has gushed from one and the same source. No matter in what highbrow theological Journal, or under the imprint of what great University Press this Mathematical Theology has appeared, all of it is the purest Bungian mathematics of the infinite—Infinite Numerology. But it must be stated in all fairness that although many have deduced the redemption of mankind from the equation $(m\ \infty)^s = \infty$, no one yet has had the temerity to prove the omnipotence and omnipresence of the devil (theologically represented by zero) from the equally correct equations $m0 = 0$, $0^m = 0$, $(m0)^m = 0$.

One of my clerical friends objected to the last on the ground that it is blasphemous. It is not that; it is just silly. I retaliated by rubbing my friend's nose in some of the modern gems of Infinite Numerology, all taken from reputable journals and books issued with university imprints, with which he is accumstomed to dazzle the un-mathematical eyes of his admiring congregation. But as he lost his temper and began to yowl so outrageously that the

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neighbors interfered, I had to let him go before the job was half done.

I remarked that this flood was effectively dammed in 1932, and I think many will be interested in viewing the great work itself. Professor Hermann Weyl, author of the paradox in Chapter IX, and Professor of Mathematics at the University of Göttingen, Germany, was the engineer in chief. By a masterly but perhaps rather unkind use of profound nonsense and subtle irony, Professor Weyl disposed of Theological Numerology for all time—and after if there is any after. His three lectures on "The Open World," delivered at Yale University in 1931, and since published in book form, did the trick, and it will never need to be done again. The whole delicious skit has a rich flavor of Revilo and Bung that only a master mathematician with a divine gift for satire could hope to concoct. It must be a dismal satisfaction—but still a satisfaction, I should judge—to the lecturer to reflect that many of his auditors and more of his readers have taken his inimitable, learned fooling on the theological meaning and importance of the mathematical infinite at is face value. Weyl almost gives his game away completely when he hands our old friend Nicolas (Chapter VII) several Dutch compliments, for Nicolas was the great pioneer in Infinite Numerology.

Weyl's luck has been but little better than that of the indignant Hippolytus mentioned in Chapter I. Among other doubtful triumphs of the lectures was their effect on my clerical friend. He let out the particular howl that caused the neighbors to rush in when I insisted that Weyl was trying to dam the flood and not endeavoring to swell it. This was too much; some of my friend's most dazzling gems had been borrowed only the preceding Sunday from "The Open World." After all this I trust that no reader of this Appendix will do the distinguished mathematician of Göttingen a similar injustice. Weyl deserves more of our generation than Hippolytus did of his.

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II. SIMPLE TRICKS

5. *Tricks*

Turning to something less disreputable, let us recall one or two simple tricks with numbers that have interested or amused children old and young for many centuries. Most of the simple kind are very ancient and quite easily seen through with a little algebra. But lest some potentially good player be scared off before the real game begins, I shall abjure algebra and leave all explanations of these hoary classics to the wit of the players.

6. *Even or Odd?*

Write any odd number, say 35, on one card, and any even number, say 46, on another. Ask someone to give one of the cards to A and the other to B, but not to tell you who has which. You undertake to tell A which number he was given. Ask A to multiply the number on his card by any even number, and B to multiply his by any odd number. Ask A and B to add their results and tell you what the sum is. If the sum is even, A was given the odd number; if the sum is odd, A was given the even number.

7. *Nim*

This game is frequently played for money, but I do not advise any professional gambler G to try his skill at it against any professional mathematician M. One luckless gambler lost $250.00 at one crack to a greenish looking mathematician on a trans-Atlantic liner this way only last summer.

Any number of matches are divided into any (necessarily not greater) number of piles; the piles do not need to contain the same number of matches. G and M play alternately. Suppose it is G's turn. He may choose any *one* pile and take from it all or only some of the matches, but he must take at least one. Then it is M's turn, and he does likewise. Then it is G's turn again, and so on.

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There are two ways of playing. It is agreed before starting *either* that the player who is forced to pick up the last match *loses*, *or* that he *wins*. No matter which way is agreed upon, M, if he keeps his wits about him, can nearly always beat any G on earth—or even on a transAtlantic liner, which G will probably prefer. There is an exceptional position, where G may win; but it is against all the laws of probability that this position will turn up twice in three consecutive games. So M may safely bet his barrel; G will walk home without his.

I am not going to explain how M can make his fortune so long as the supply of G's holds out, because it would be not quite fair. G's must live as well as M's. Instead I shall give the reference to the easily understood paper by an American M where the technique for skinning G's is explained in detail: *The Annals of Mathematics*, Second series, volume 3, 1901–2, pages 35–39. This can be consulted in any large public library or in any university library. It is a wonder that all our university graduates are not millionaires.

The arithmetic of the technique is quite simple, but requires practice to do it mentally, as must be done in an actual game, with reasonable speed. My own mental arithmetic being very shaky, I have remained poor. The first step is to express the number of matches in each pile in the *binary scale*. Suppose for example there are 234 matches in a particular pile. Now 234, since we usually count by *tens*, means 2 times 100, plus 3 times 10, plus 4, or in the notation of powers (chapter II), $234 = 2 \times 10^2 + 3 \times 10 + 4$. But if we counted by *twos* instead of by tens, this same number would be written 11101010, since it is

$$2^7 + 2^6 + 2^5 + 2^3 + 2 + 0$$

when expressed in powers of 2. In this *binary* scale, 111 means $2^2 + 2 + 1$, or what is ordinarily written 7; binary 1010101 is ordinary 85; binary 1000 is ordinary 8. The simple rule for expressing any number in the binary scale is given in the arithmetics of forty or fifty years ago, also in most of the so-called higher algebras (there is usually nothing higher about them except

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the price) under the heading Scales of Notation. The reader who is interested can easily figure it out for himself. Anyone who is planning a trip abroad next summer might do well to look into this matter. A stinging, $250.00 rebuke is a more effective deterrent to the evils of gambling than a whole library of sermons or laws; it is also more profitable.

8. Missing Digits

This type of teaser is somewhat on the order of crossword puzzles, but requires more thought both for its manufacture and for its solution. Some of these puzzles are probably hundreds of years old; others go back only ten years or less. Almost any of them will keep a noisy nuisance quiet for at least half an hour.

The first is the "Five fives" of W. E. H. Berwick:

```
****)*55**5*(*5*
     **5**
     -----
      *****
      *****
      ------
        ****
        ****
        ----
```

It is required to put a digit (1, 2, 3, 4, 5, 6, 7, 8, 9 or 0) in place of each star, so that the result is a correct long division. There is only one solution, which can be got by trial and error, or semi-algebraically. Anyhow, it beats an intelligence test. To save the reader's sanity I give the answer: divide 2559752 by 3926.

The "Four fours," also by Berwick, is much worse, as there are four solutions:

```
***)******4(*4**
    ***
    ---
    **4*
    ****
    ----
     ****
      *4*
     ----
      ****
      ****
      ----
```

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Put digits for the stars so as to make a correct long division. Not to give the game away completely, I state merely that each of the following numbers is one of those occurring in the four solutions, 846, 1418, 943, 1416.

Probably the worst of all is the similar puzzle manufactured from the long division of 7752341 by 66734. This yields a quotient with two digits before the decimal point; after the point there is one digit followed by a *recurring* string of nine digits. Now, if the long division is carried out, *all* the digits in the whole process can be replaced by stars, and from this starry skeleton it is possible to find dividend, divisor and quotient *uniquely*—there is but *one* divisor (66734) and *one* dividend (7752341) which will make a correct long division of the starred skeleton. If anyone wishes to see how it is done, he may look up the problem in the *American Mathematical Monthly*, vol. 28, 1921, page 61. This will be found in large public or university libraries.

In passing, I may state that mathematicians do not spend all their time making or solving puzzles that lead nowhere in particular. In fact, to call a mathematician a problem solver is about the deadliest insult you can hurl at him, and if the insult is undeserved you have only yourself to thank if you are shot on the spot. Problem solving, to a mathematician, for the sake of solving problems is a vice on a par with the lowest debaucheries of numerology. So let us go to something else.

III. A CORNERSTONE

Having just called problem solvers black, I find myself in the hot and dirty position of the pot that objected to the complexion of the kettle. What some arithmeticians point out as a cornerstone of their vast building, a casual observer might well pass by as an inconspicous brick. Nevertheless Fermat's theorem bears the authentic stamp of simplicity and generality which mark it as great mathematics.

Like several of the really powerful and far reaching generalizations of arithmetic, Fermat's theorem can be easily grasped by

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anyone, although to see completely through it to a proof with no more equipment than is necessary requires talent. Once the proof is pointed out, anyone can remember how it goes and readily reproduce it. But to invent any proof—there are several—on one's own is a certificate of high intelligence. Nothing beyond seventh grade arithmetic is needed. The like holds for its discovery. Anyone who had converted several fractions of the kind $1/p$, where p is a prime, into recurring decimals might easily guess the general truth of what he was doing.

If some doubter who has been placed in the top one per cent of human beings by the usual intelligence tests wishes to check up on his certified genius, let him try his hand at a proof of the theorems of Fermat and Wilson—both stated presently. This does not apply, of course, to those who have had training in higher arithmetic. Of the others, those who succeed in getting a proof without looking at the hints given presently had better make up their minds at once to go into mathemetics. They will be unhappy in anything else.

9. Primes

To get the taste of all that foul Mathematical Theology out of our mouths, let us try to get some clean arithmetic into our systems.

A *prime*, we recall, is a number greater than 1 whose only divisors are 1 and the number itself. The primes less than 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, or twenty-five in all. A *composite* is a number having at least three divisors. Thus 111 is composite; $111 = 3 \times 37$, so the divisors of 111 are 1, 3, 37, 111, or four in all; 9 is composite, for it has exactly three divisors, 1, 3, 9. As an alternative definition, a composite is a number which is not prime. A prime has exactly two divisors, 1 and itself. This leaves 1 in a muddle, which we ignore.

Given any number, how can we tell with a reasonable amount of labor whether it is prime or composite? There is practically

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no way but the good old caveman method of brute force. We try to crack the number apart into its prime factors. This is done by trying in succession all the primes less than the square root of the number as divisors. If none of these does divide the number, the number is prime. Practice will suggest a few short cuts, but for a number of only ten digits the labor is prohibitive. If such a number happened to be prime, it is quite possible that this interesting fact might remain unknown to anyone who spent his entire life trying to factor the number. There are somewhat more refined methods than brute force, in arithmetic as in courtship, and recently machines have been invented for handling fairly large numbers. But none of these offers any civilized, scientific way of finding the prime factors (divisors) of a number.

To take an example, is 279 prime? The square root of 279 lies between 16 and 17, since $16^2 = 256$, $17^2 = 289$. So we need try only the primes less than 16, namely 2, 3, 5, 7, 11, 13. A moment's inspection rejects 2, 5 as possible divisors of 279. Try 7; it won't work; neither will 11, 13. Thus 279 is prime. What about 8593? The square root of 8593 lies between 92 and 93, since $92^2 = 8364$ and $93^2 = 8649$. To get this much, take the square root of 8593 as you did at school (if you did), and stop at the decimal point. This gives 92. Here then we need only try the primes less than 92. From the list of primes given above, we see that we can end our trials at 89. Proceeding as before, we find 8593 composite; $8593 = 89 \times 97$.

Such questions as the following about primes seem hopeless in the present state of arithmetic. What is the next prime after a given one? How many primes are there between two given numbers? Is a given number prime? What are the divisors of a given number? The primes in each of the pairs 41, 43 and 71, 73 differ by 2, and there are thousands of other such prime pairs; is the number of such pairs infinite? If n is an *even* number (more sharply, a power of 2), many numbers of the form $2^n + 1$ are primes. For example $2^2 + 1 = 5$, $2^4 + 1 = 17$, $2^8 + 1 = 257$, $2^{16} + 1 = 65537$, are all primes, but $2^{32} + 1$ is composite, its

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smallest divisor greater than 1 being 641. Is there an infinity of primes of this form? If p is a prime, many numbers of the form $2^p - 1$ are primes; for example $2^2 - 1 = 3$, $2^3 - 1 = 7$, $2^5 - 1 = 31$, $2^7 - 1 = 127$. Does this form $2^p - 1$ furnish an infinity of primes?

The last two questions are of considerable mathematical importance, and we shall refer to them again. A *proved* yes or no to either would most likely preserve the answerer's name for at least 2000 years, if anyone cares to be remembered that long.

I have given these easily asked questions to emphasize how difficult it is to say anything true and universal about primes at all. *Any* such generalization would be of the highest interest in the present state of mathematics. Here is a field where amateurs might make discoveries of genuine interest while amusing themselves. But if anyone prefers numerology to arithmetic it is his own funeral, and I am no missionary to raise him from the dead. Let him rest in peace.

Note: If all the world loves a lover, every sport loves a world's record. Just after this appendix was sent to the publishers, I received a telephone call to go to a local machine shop to see a brand new arithmetical machine perform. As a result, I take back what I said about lack of civilized methods for handling large numbers. At its very first real test, the machine set a world's record that will stand some beating. What would take, on a conservative estimate, years, possibly a lifetime, for a human being to do with all available short cuts and ordinary calculating machines, *was done by this machine in three seconds.* For certain reasons the factors of

1,537,228,672,093,301,419

were required. The machine produced them, 529,510,939 and 2,903,110,321, both of them prime, in three seconds, startling even the young inventor, Dr. D. H. Lehmer, and his assistant, Mrs. Lehmer. They had expected it to take eight hours, and had brought their lunch when they ran the test. It is too bad that this marvellous machine was not completed in time to compete at the

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last Olympic Games— it would never have been necessary to hold another Olympiad.

10. Wilson's Theorem

This is one of the extremely few known general facts about primes; and it is probably the simplest. It was discovered by Sir John Wilson (1741–1793) about 1770. A proof was published in 1773 by Lagrange (1736–1813), the greatest mathematician of the Eighteenth Century.

Take any number, say m, subtract 1, thus $m - 1$; multiply together all the numbers from 1 up to $m - 1$, and add 1 to the result. The final result is divisible by *m if, and only if, m is prime.*

For example, if $m = 6$, we have $m - 1 = 5$. Following the directions we get

$$1 \times 2 \times 3 \times 4 \times 5 + 1, = 120 + 1, = 121,$$

and 121 is not divisible by 6. So 6 is *not* prime.

Take $m = 11$. Then, if the theorem is true we shall have

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 + 1,$$

that is 3628801, divisible by 11. The quotient is exactly 329891. So 11 *is* prime.

Notice that we have in Wilson's theorem a decisive test for the primality of a number. Suppose for example we wish to know whether 97332819111 is prime. It is sufficient only (!) to multiply together all the numbers from 1 up to 97332819110, add 1 to the result, and divide by 97332819111. If the division comes out exact (without remainder), our number is prime; if there is a remainder, the number is composite, although we might not be able to find its factors in a hundred years. The last may be a rash statement, for there are experts in this field as in every other except statesmanship. But a thousand years is probably a closer estimate for an ordinary human being, provided one could live that long.

Now, although Wilson's theorem is useless as a practical test for primality, it is of extraordinary interest on other grounds

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One of these is its non-tentative character. At no stage of the process do we have to resort to trial and error. Everything is straightforward multiplication, subtraction, and division. Again, we do not have to know a single prime in order to apply the theorem. This is in striking contrast to the other way of trying as divisors all primes less than the square root of the number tested. If anyone could find another test with these desirable characteristics, but with the added advantage that human mathematicians could actually use it to test the primality of a large number, say one of 10,000,000 digits in a year, he would make a greater advance in arithmetic than has been made in centuries. It is just conceivable however that no such test is possible. We know next to nothing about numbers.

As the proof of Wilson's theorem proceeds by reasoning of the same kind as that used for Fermat's, which will be outlined, I leave it to the reader. If anyone manages to do it entirely on his own, without looking at any book, almost any arithmetician would be interested in seeing the proof. More than one exists, but at bottom all rest on the same fact. I have never known anyone who gave a proof under the prescribed conditions, although scores of boys and girls under eighteen have gone through the motions correctly in examination papers.

11. Fermat's Theorem

Take any ***prime***, say p, and any number, say n, ***which is not divisible by*** p. We shall use as an example $p = 7, n = 10$. Then (refer if necessary to chapter II for powers) raise n to its successive powers,

$$n, n^2, n^3, n^4, \ldots;$$

divide these by p, and keep only the remainders. In our example we have

$$10, 10^2, 10^3, 10^4, 10^5, 10^6, \ldots,$$

that is,

10, 100, 1000, 10000, 100000, 1000000, . . . , and the remainders 3, 2, 6, 4, 5, 1 are all different, and the last, namely the *sixth*,

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is 1. But $6 = 7 - 1$. Try the same with other primes p and other numbers n not divisible by p. *One* of the same phenomena always appears; namely, the $(p - 1)$th remainder is 1. As another example, for $n = 5$, $p = 11$,

$$5, 5^2, 5^3, \ldots, 5^{10}$$

on division by 11 leave the remainders

$$5, 3, 4, 9, 1, 5, 3, 4, 9, 1;$$

the 10th $(10 = 11 - 1)$ remainder is 1. But here 1 has turned up earlier than place 10, (in place 5), and the remainders are not all different, but each appears twice. Trial with several examples would soon lead us to suspect that the following is always true.

If p is prime, and if n is not divisible by p, then the $(p - 1)$th power of n leaves the remainder 1 when divided by p.

This is Fermat's theorem, stated and proved by him in 1640. The first published proof was by Euler in 1736. Euler incidentally was the most prolific mathematician that ever lived. The ancient Chinese knew the theorem for $n = 2$ as early as 500 B.C.

Experiment would suggest much more. For example, the remainder 1 appears first in the kth place, where k is *some* divisor of $p - 1$, and after the remainder 1 has appeared, the remainders *recur in the same order indefinitely*. Thus for $n = 5$, $p = 11$, we have the remainder cycle

$$5, 3, 4, 9, 1, 5, 3, 4, 9, 1, \ldots$$

and so on forever. So we can say at once what the remainder is on dividing, say, $5^{2000003}$ by 11. It is 4. No human being could raise 5 to 2000003 power. Yet we can state the remainder 4 on observing that 2000003 when divided by *five*, the *place* where 1 *first* appears in the remainder cycle, gives the remainder *three*. So we need merely see what number occurs in place 3 in the cycle.

I promised to outline a proof of Fermat's Theorem. We write rs for the result of multiplying the numbers r and s. With p any prime, and m any number not divisible by p, we divide each of the $p - 1$ numbers $m, 2m, 3m, \ldots, (p - 1)m$ by p. The first

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step is to prove that all the remainders are different (none is zero), and hence, since the divisor is p, and each remainder must be less than p, these $p - 1$ remainders, *in some order*, are the numbers $1, 2, 3, \ldots, p - 1$. This much is proved by supposing the contrary true and getting a contradiction. Next, we can assert from this step that

$$m \times 2m \times 3m \times \ldots \times (p - 1)m$$

and $1 \times 2 \times 3 \times \ldots \times (p - 1)$
differ by a multiple of p. Hence their difference is divisible by p. But this difference is

$$m^{p-1} - 1 \text{ times } 1 \times 2 \times 3 \times \ldots \times (p - 1)$$

Since this *is* divisible by p, and $1 \times 2 \times 3 \times \ldots \times (p - 1)$ is *not* divisible by p (as can be proved), it follows that $m^{p-1} - 1$ is divisible by p; which is the theorem.

The above is a mere sketch of a proof. In particular, both of the following theorems have been assumed. If a *prime* p divides $r \times s$, then p divides at least one of r, s. If a prime p divides neither of r, s, then p does not divide $r \times s$. The proofs of these, although easily followed, I consider difficult, and for a very good reason: when we go on a step to the next kind of "numbers" beyond 1, 2, 3 ..., namely the "algebraic" numbers, "primes" do not necessarily have these common sense properties which are taken for granted by many. To invent a proof for either might easily cause a good mind to stretch itself. Here I shall merely refer to any first rate schoolbook on arithmetic (there is not a decent one published today in America). The French used to excel in the teaching of real arithmetic. Probably they do still, for ever since Voltaire they have been rational sorts of devils.

As some may wish to experiment, I state how the remainders on dividing m, m^2, m^3, ... by p can be quickly calculated without first calculating the powers. Take $m = 6$, $p = 13$. The first remainder is 6; multiply this by 6, get 36, divide by 13, get the remainder 10; multiply *this remainder* by 6, get 60, divide by 13,

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get the remainder 8, and so on: next, 48 and 9; next, 54 and 2; next 12; next 72 and 7; and so on, till 1 appears, when the remainders repeat. The reasons for this short cut are obvious on working a few examples. More short cuts are suggested by experiments.

12. Recurring Decimals

Converting 1/7 into a decimal by short division, we get $1/7 = .\dot{1}4285\dot{7}$, namely the digits in the *period* indicated by the dots over 1 and 7 recur endlessly in the same order. How many digits will there be in the period of $1/p$, where p is any given prime? Try it with a few primes. The general answer is that the number of digits in the period of $1/p$ is *some* divisor of $p - 1$. Obviously the recurrence will begin when the remainder 1 appears in the division. Putting this with Fermat's Theorem is a sufficient hint for a proof of the general statement.

Here is an easily explained peculiarity of some such periods, namely all those in which the number of digits in the period is even. This is the case for 1/7, where there are 6 digits. Write the first half of the period as one number, and the second half as another. From 1/7 we get 142 and 857. Add the two numbers. The result is always a string of as many 9's as there are in half the period length. In our example $142 + 857 = 999$. I regret that I can find no arithmetical truth which yields this upside down and backwards, namely as 666. Playing with these periods provides endless entertainment. Try multiplying them by 1, 2, 3, 4, ... in turn, and see what happens. Then see *why* it happens.

13. Perfect Numbers

For the definition of these, see chapter VI. Euclid proved that every *even* perfect number is of the form $2^{p-1}(2^p - 1)$, where p is prime *and* $2^p - 1$ *is also prime.* To verify this is a fairly easy exercise for anyone who remembers a little algebra.

This raises the interesting and extremely difficult question of finding those primes p which make $2^p - 1$ prime. Any help here

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would be appreciated by arithmeticians. For $p = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127$, it is known that $2^p - 1$ is prime. Can anything *general* be stated? If so, whisper it to some mathematician and you will probably be heard from now till at least the year 4000 A.D.

14. Regular Polygons

The consequences of Fermat's Theorem support much of modern algebra. It is a foundation stone of many other edifices besides arithmetic, for example the vast modern theory of groups of operations, which in turn appears to be at the bottom of much of the new physics of the atom. This is no place to go into such things. But there is one beautiful and mysterious property of numbers where the cornerstone is indispensable if not quite so plainly evident. For its own sake this is worth looking at. For another reason however it is worthy of perpetual remembrance. It induced one of the greatest mathematicians of all time, Karl Friedrich Gauss, to go into mathematics as his life work instead of into philology as he had intended. He changed his mind at the age of 17 when he not only discovered this beautiful thing but proved it.

At school we learned to draw a triangle with three equal sides *by the use of straight-edge and compass alone.* Then perhaps we constructed a regular pentagon—five equal sides and five equal angles—with the same tools. But we did *not* construct regular polygons of 7, 9, 11, or 13 sides, although we did do a 15. We might have done a 17 or a 65537 with straight-edge and compass, but we could not have done an 18. What is the general fact? The Greeks knew how to do the three and five. No progress was made for over 2000 years, till the boy Gauss proved that *an m-sided regular* (equal angles and equal sides) *polygon can be constructed by means of straight-edge and compass alone if m is a number of the form pqr . . .t, where p, q, r, . . . , t are all different primes, and each of them is of the form* $2^n + 1$, *or if m is a power of 2 times such a number, and ONLY in these cases is the construction possible.*

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The proof was first published in the *Disquisitiones Arithmeticae* of Gauss, (1801), the greatest single work on the theory of numbers ever written, and one of the mathematical masterpieces of all time. It was entered in competition for a prize offered by the French Academy of Sciences. If the work which won the prize has survived, I have never heard of it. So mathematicians as well as numerologists sometimes make grievous blunders. We are all so human—and so damned stupid.

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Sans Tache

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Sans Tache

IN THE "elder days of art" each artist or craftsman enjoyed the privilege of independent creation. He carried through a process of manufacture from beginning to end. The scribe of the days before the printing press was such a craftsman. So was the printer in the days before the machine process. He stood or fell, as a craftsman, by the merit or demerit of his finished product.

Modern machine production has added much to the worker's productivity and to his material welfare; but it has deprived him of the old creative distinctiveness. His work is merged in the work of the team, and lost sight of as something representing him and his personality.

Many hands and minds contribute to the manufacture of a book, in this day of specialization. There are seven distinct major processes in the making of a book: The type must first be set; by the monotype method, there are two processes, the "keyboarding" of the MS and the casting of the type from the perforated paper rolls thus produced. Formulas and other intricate work must be hand-set; then the whole brought together ("composed") in its true order, made into pages and forms. The results must be checked by proof reading at each stage. Then comes the "make-ready" and press-run and finally the binding into volumes.

All these processes, except that of binding into cloth or leather covers, are carried on under our roof.

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The printing craftsman of today is quite as much a craftsman as his predecessor. There is quite as much discrimination between poor work and good. We are of the opinion that the individuality of the worker should not be wholly lost. The members of our staff who have contributed their skill of hand and brain to this volume are:

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