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QUADRATURE OF THE CIRCLE.

CONTAINING DEMONSTRATIONS OF

THE ERRORS OF GEOMETERS IN FINDING
THE APPROXIMATIONS IN USE;

With an Appendix,

AND

PRACTICAL QUESTIONS ON THE QUADRATURE,

APPLIED TO THE ASTRONOMICAL CIRCLES.

TO WHICH ARE ADDED

LECTURES ON

POLAR MAGNETISM

AND

NON-EXISTENCE OF PROJECTILE FORCES IN NATURE.

BY JOHN A. PARKER.

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JOHN WILEY & SON.
1874.

their fractional relations to the square, are opposite to one another in ratio of the squares of their diameters.

Sixth, The difference between a line inclosing and a line coinciding with the utmost limit of the area of any circle is the essential difference in the properties of curved lines as they differ from any possible number of straight lines in the polygon.

Any criticism which meets these points understandingly and fairly, showing that the writer has himself understood the subject, will be entitled to an answer; but anything which does not meet these points cannot be considered as being relevant to the points at issue, and therefore cannot be considered as worthy of a reply.

I am not aware of any errors in the book which can mislead any one in regard to the principle or result intended to be shown by the various calculations in figures. It is quite possible, however, that in transcribing for the press, some errors may have occurred; but if any exist, I think they can be only such as the reader will be able to understand and correct for himself.

The work is commended to the examination and candid judgment of all those who may feel interested in the development of truth.

THE AUTHOR.

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knowledge of the quadrature has never ceased to be a cause of some perplexity, requiring explanation, and necessarily, also, a cause of more or less error in these most important sciences.

Unfortunately, but very much like other men in similar circumstances, the professors of the schools, unable to demonstrate the truth, and unwilling to acknowledge their deficiency, have thought it necessary to explain away their error, and in doing so, they have taken the opposite ground, and concluded, that what *they have been unable to attain, is unattainable by human intellect.* Hence the college lectures on this subject, published and unpublished, abound in learned speculations and hypotheses, tending to place the quadrature of the circle without the range of demonstrable mathematics.

All geometrical truth whatsoever, in nature, rests on two simple things—the properties of straight lines, and the properties of curved lines,—difference of angle and difference of curve are but modifications of the same principles. Of the properties of straight lines, geometers have long supposed themselves to be perfect masters, but of the properties of curved lines, and their relative value to straight lines, geometers have yet known nothing whatever, except by approximation. To find the quadrature of the circle, is simply to determine the relative value of straight lines and curved lines,—and in

view of these facts, to an unprejudiced mind, it sounds equally strange and ridiculous to hear the professors of an exact science, condemn as *useless* the solution of a problem, *which is in itself an elementary truth*, and involves, *to say the least, one half of all the geometrical truth in nature.*

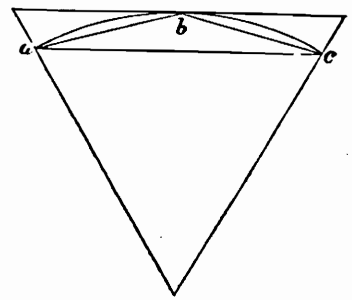
Without any disrespect to the learning and general intelligence of those who have adopted the conclusion that it is impossible to find the exact quadrature, (and it seems to be general among the professors,) I may be allowed honestly to doubt both its truth and its reasonableness. It may be admitted, if it pleases them, that it can never be done by any principles at present known or taught in the schools; but this proves nothing more than a deficiency in knowledge of the principles which govern it. To prove satisfactorily that it can never be demonstrated by *any means*, it is necessary, *first*, to prove that no principles *can be true in nature*, but such as are already known to science. To this, however, no mathematician will for a moment pretend, and for aught *any one can know*, there may be other principles in nature, as yet unknown or untaught, and which are equally true with any that are known, by which, when understood, the demonstration may be made. To reason otherwise, is to assume that we are already acquainted with all the principles which the Creator brought into action in reit both convenient and necessary to adopt such terms and forms of expression, as, in my judgment, would best convey my own meaning and suit my own purpose, without regard to their common scientific application, preferring always the attainment of truth, rather than elegance. Thus, for example, the term "circumference," when it will best suit the idea intended to be conveyed, is often applied to the square, the triangle, to polygons, and other angular shapes, and is synonymous with perimeter. "Diameter" is also applied to triangles and rectangular figures, and diameter, when not otherwise explained, always means twice the least radius (which, in all regular shapes, is always the diameter of an inscribed circle) when a figure is to be measured by circumference and radius; but in the equilateral triangle when it is to be measured in the usual way of measuring angular shapes from half its side by the perpendicular, then the perpendicular from the base is the diameter. This seeming indiscriminate use of terms will, I think, be better understood by the reading, and will not be objected to except by those who reason for elegance rather than truth, and those, who, for lack of reason, confound terms and definitions with principles.

DEMONSTRATION OF ERROR
IN THE
APPROXIMATION OF GEOMETERS TO THE CIRCUMFERENCE OF A CIRCLE.

PROPOSITION I.

The circumference of a circle is greater than any number whatever of mean proportionals from an inscribed straight line.

PLATE I.



We have here a section of circumference with an inscribed and circumscribed perimeter of a polygon. The circumference itself gives ocular demonstration, that it is greater than any number whatever of mean proportionals, as the straight lines *a b*, or *b c*, from the inscribed straight line *a c;* because, as all such mean proportionals must forever continue to be *inscribed* straight lines, they can therefore never equal the circumference; and without this quality the circumference *could not be a perfect curve. The proposition is therefore demonstrated.*

From the above proposition it is evident, that, if we error in the scale of proportions used amounts to one or more, and where they necessarily pass to the inside of the value of circumference, all effort to gain any nearer approach to it, is nothing more than an effort to arrange a vulgar fraction of remainder into a line of decimals, which because it is not of decimal value makes a line of figures without end.

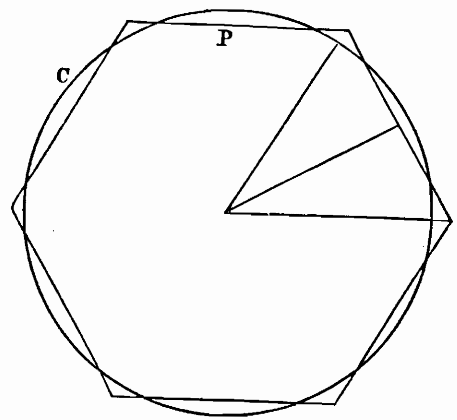
I have stated the case in such general terms as that the error of geometers shall be seen to apply, as it really does, to *every method in use* for finding the approximation by means of straight lines, or by any fluxionary series, the basis of which is the value of straight lines; and in order that we may understand it fully, we will now look further at the mechanical causes which can have this influence. And

First. Between curved lines and straight lines there is an essential difference in principle and properties, as is evident from the fact, that if two shapes be formed, one with straight lines and the other with curved lines, and of equal magnitude or area, their circumferences will be totally different, and again if their circumferences are equal, their areas will be totally different; and this difference of principle or property, is therefore seen to consist chiefly in the different powers of the two descriptions of lines to inclose area; therefore,

PROPOSITION II.

The area of a circle is greater than the area of any shape formed of straight lines and of equal circumference, and hence greater than the area of any polygon of any possible number of sides, and having the same circumference with the circle.

PLATE II.



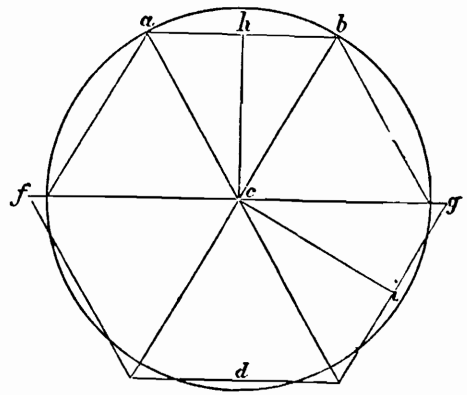
In the figure (Plate II.), let C be the circumference of a circle, and let P be the perimeter of a polygon, and let C and P be equal to one another in length. It is evident that C is a perfectly curved line, and that P is composed of several straight lines. It is known also that all regular shapes formed of straight lines and equal sides, have their areas equal to half the circumference, or half the length of all its sides, multiplied by the *least* radius which the shape contains, than which all other radii contained in the shape are greater; whereas, the circle has its area equal to half the circumference by *the radius*, to which every other radius contained in the circle is equal. (prop. I., chap. ii.). It is evident some additional value after all mean proportionals have ceased; which is, when the inscribed and circumscribed so-called have become thoroughly equal; and whatever the amount of this infusion may be, it is evident, that there must be a point in the expression of numbers, where its value becomes *fixed*, and where it shall amount to *one* or *more;* and because this infusion into the value of circumference, is a fixed and unalterable law of nature, by which the two descriptions of lines are made essentially to differ, therefore, whenever we attempt to find the value of circumference by the properties of straight lines alone, we shall always fall short of the truth, at that point in the expression of numbers, where the value of this essential difference equals one or more. To get rid of this essential difference of property in the two descriptions of lines (for they are not ignorant of its existence) geometers have assumed that it is "Infinity," and therefore call it nothing! but I shall show, that whatever an infinity may be, it is always such, that in material things it is capable of *increase*, and is therefore a value *not to be thrown away.*

Secondly. The perimeter of a circumscribed polygon of any number of sides, is a proportion to the *inscribed* of an equal number of sides, in precisely the same ratio as the radius is to the perpendicular from the center to the chord of the arch. Now, therefore, if the circumference

be greater than *any number whatever* of mean proportionals from the inscribed, as we have ocular and mathematical demonstration in proposition I. that it is, and without which it could not be a perfect curve, then it is evident again, that the circumscribed, is to the inscribed, in *less ratio of value* than the circumference. Here again we see a mechanical necessity of any proportions between the inscribed and the so-called circumscribed lines, other than such as are exact relatives to the circumference, necessarily bringing the two to agree with each other, at a point of value within the value in area of the circumference; therefore

PROPOSITION III.

PLATE III.



The perimeter of any polygon whose circumference or the length of whose sides is equal to the circumference of a circle, is always such that if the polygon be placed or drawn upon the circle, the point or angle formed by the hypothenuse of each right angle which the polygon contains, lies wholly outside *of the circumference of the circle.*

carry the angle of hypothenuse *outside* the circumference, is an approximation *not* of a full circumference, but of a circumference whose *diameter is less than one*, in its ratio of value to the area of the circle: therefore—

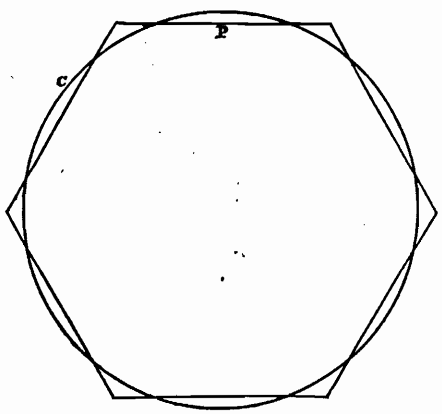
PROPOSITION IV.

If the perimeter of any polygon be brought into the form of a circle, both the diameter and area of the circle are greater than the diameter and area of the polygon.

By diameter of a polygon, I mean always its least diameter, or that which, being multiplied by one-fourth of the perimeter, or its half multiplied by half the perimeter, gives its area, which is always the diameter of an inscribed circle.

By proposition II., it is shown that if P be the perimeter of a polygon, and C be the circumference of a circle, and P and C are equal to one another in length, then P shall always have a greater and a lesser radius, and the radius of C shall always be greater than the least, and less than the greatest radius of P, therefore, C multiplied by

PLATE IV.



radius is greater than P multiplied by least radius. It is evident, therefore, as seen in Plate IV., that if the perimeter of any polygon be brought into the form of a circle, both diameter and area are increased by the transition of shape. *The proposition is therefore demonstrated.*

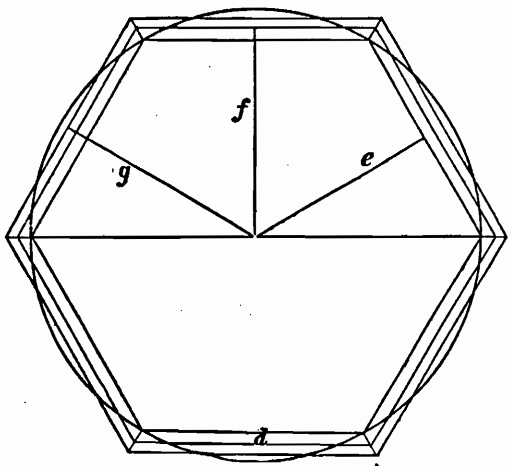
Now, it is known that geometers regard a many-sided polygon as *equal* to a circle, to the extent to which two polygons (the inscribed and circumscribed, so called) agree with each other; and with them, to the same extent, diameter is treated as having a *fixed* and equal *value*, whether the shape may be a many-sided polygon or a circle. Let us then examine the effect of considering the two shapes as equal to one another, and each having the same *fixed diameter*. Let P (Plate IV.), be the perimeter of a polygon, and let C be the circumference of a circle, and let C and P be equal to one another in length. It is already known that the area inclosed by C is much greater than the area inclosed by P. If, therefore, the diameter of each, C and P, be considered as *fixed*, and treated as *one*, then, in order to give such expression to the circumference of the circle as that its half being multiplied by $\frac{5}{10}$, or half the diameter, the result shall express the whole area of the circle, we must add to the length of C an amount equal to *four times* the difference between the area inclosed by

PROPOSITION V.

If the area of any polygon shall equal the area of a circle, the perimeter of such polygon shall lie wholly outside of, and inclose a polygon, whose circumference equals the circumference of the circle.

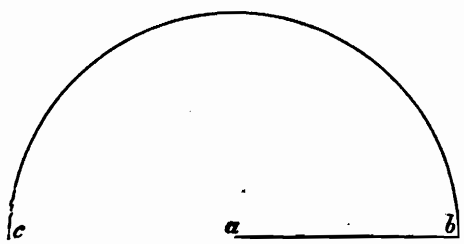
This proposition is already proved by the demonstration of proposition II., and needs not to be repeated, because, if the area of a circle is greater than the area of a polygon having the same circumference as the circle, then by reciprocity, if the *area* of a polygon shall *equal the area* of the circle, then the circumference of such polygon shall be greater than the circumference of the circle, and shall be able to inclose another polygon, whose circumference *equals* the circumference of the circle.

PLATE V.



We have then, in Plate V., three polygons, one whose perimeter rests on the perpendicular *e*, and which differs from an inscribed polygon less than any assignable quantity, and which, therefore, equals the approximation of geometers, at six sides of a circumference. Another, whose perimeter (*d*) rests on the perpendicular, *f*, and which *equals the circumference* of the circle; and still another, whose perimeter rests on the perpendicular, *g*, and which incloses an area *equal to the area* of the circle. Between these three polygons, at six sides of a circumference, there is a very sensible difference of *area* and *diameter*, and to whatever number of sides these polygons may be bisected, they can never be brought to agree with each other, but there will always remain a positive difference of area, and a difference of diameter, which is more than three times equalled in the circumference. It is known, and I have proved (in proposition XII., chapter ii.), that the true ratio of circumference to diameter of all circles, is four times the *area* of one circle inscribed in one square for the ratio of circumference, to the area of the circumscribed square for the ratio of diameter; and since it is proved that the area of a circle is greater than the area of any polygon of any possible number of sides, and having the same circumference with the circle, it is therefore evident, that if we would find the ratio of circumference which shall express the full value of the area of the circle by proportions between the perimeters of an inscribed and circumscribed polygon, we must seek it *mechanically* shown the utter impossibility of ever squaring the circle by the application of straight lines, or by any method now in use, and I am as completely satisfied on that point, as the skepticism of any modern Professor respecting my solution of the problem could desire. But this does not prove that the circle is incapable of being squared, or that no equality exists between the circle and the square, which I shall presently show by direct propositions *does exist*; and unless it can be shown on the other side, that no principles in geometry can be true but such as are already known, I have yet hopes of arriving at the quadrature. That whenever it is reached it will be greater than the approximation of geometers, and greater at no very remote point from unit, than the so-called perimeter of a circumscribed polygon which is made to agree with the inscribed within less than any assignable quantity, I think already established; if not, I think additional facts will not be wanting to place my position beyond a doubt.

PLATE VI.



The value of the area of a circle, is the value of a straight line *a b* (Plate VI.), equal half the diameter, multiplied by the value of the curved line *c b*, equal half the circumference. The

value of the curved line *c b* is, as we have seen, greater than the same length of straight line at any possible number of sides, and by my ratio of circumference to diameter, which I affirm to be the true one, the expression of numbers by which circumference and diameter are made equal, is 20612 parts of circumference to 6561 parts of diameter, or *a b*=3280.5 and *c b*=10306, which I now propose to prove.

the equilateral triangle equals the lines $a\ b\ c \times d\ e$, or half the circumference by the least radius which the triangle contains, which is always the radius of an inscribed circle, and it is evident that every other radius contained in the triangle is greater than the least. In like manner the areas of the square and the hexagon are equal to the lines $a\ b\ c$ (half the circumference) by the line $d\ e$ which is the least radius either shape contains, than which every other radius contained in either shape is greater. But the area of the circle equals the line $a\ b\ c$, half the circumference, by the radius $d\ e$, to which every other radius contained in the circle *is equal. The proposition is therefore demonstrated.*

Here then is a relative property between straight lines and curved lines, showing us conclusively, that since straight lines have been made the basis of area in mathematical science, some compensation must be made to the circumference of the circle for this difference of *relative property*, if we would give to the circle the full expression of its value, by the properties of straight lines, *which is the thing demanded by the quadrature;* and a future demonstration will show what this compensation shall be.

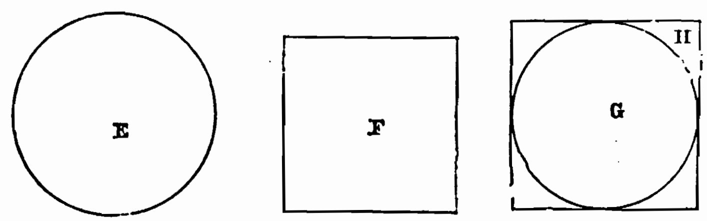
The next relative property of straight lines and curved lines which I shall notice is contained in the following proposition.

PROPOSITION II.

The circumference of any circle being given, if that circumference be brought into the form of a square, the area of that square is equal to the area of another circle, the circumscribed square of which is equal in area to the area of the circle whose circumference is first given.

Explanation.—In the figures E, F, G and H, Plate VIII., let the circumference of the circle E be given,—let it be for example 36 (or any other number), and let the circumference or four sides of the square F be also 36, then one side of F=9, and $9 \times 9 = 81$, which is the area of F. Now let the area of the circle G=81, then by the proposition the area of the square H circumscribing G equals the area of the circle E whose circumference is 36.

PLATE VIII.



My own mode of demonstrating the foregoing proposition is simply to test the principle by more than one ratio of circumference and diameter. If it be true of these, then it is a general principle, and true of every square (H) is such, that its inscribed circle (G) is equal in area to the area of another square (F) whose circumference equals the circumference of another circle (E) of equal area with H. And in this view of the proposition let it be particularly noticed, that nothing is assumed, and the result is subject to no conditions as in the direct proposition, which is conditioned on the equality of G. to F.

This 2d proposition is not only wholly original *with me*, but is, I believe, entirely new in mathematics; and certainly it is very beautiful as showing the principles and the manner in which the areas of circles and squares unfold and display to each other. It was discovered by me more than 30 years ago by the method of demonstration here given, and it has since been frequently demonstrated by algebraic formula; but I forbear to insert any of these demonstrations and give as a reason for this omission,—*first*, that I never make use of algebra in demonstration, and *secondly*, that, although the principles of algebra aided by geometry are amply capable of demonstrating the proposition when *discovered and stated*, it yet to my mind embodies no principles or process of reasoning by which the discovery *can be made*. It is not in the power, I think, of algebra alone, by the same formula, and without the aid of numbers or geometry, to contemplate the transition and alternation of

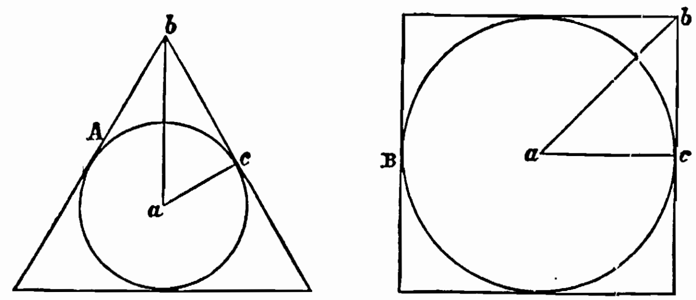
shapes necessary for the discovery of this proposition. And hence I attribute to the almost exclusive use of algebra in the schools, and the little attention paid to the mechanical properties of numbers, that this relative property of the circle and the square, or of straight lines and curved lines, has so long remained unnoticed and undiscovered.

The following proposition is, I think, appropriate here to the course of my reasoning.

PROPOSITION III.

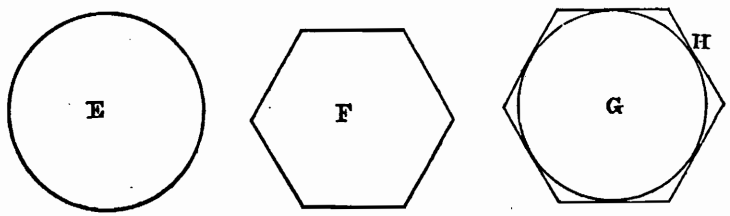
The circle is the natural basis or beginning of all area, and the square being made so in mathematical science, is artificial and arbitrary.

PLATE IX.



By proposition I. it has been shown, that all regular shapes have their areas equal to half the circumference by the radius of an inscribed circle, which is the least radius the shape contains; the circle is therefore the area of F. Now let the area of the circle G=.4330127 +(—F) then the diameter of G (by my ratio)= √.551328+, the perpendicular or diameter of H ($c\ d$) =√1.240489+ and the side of H=√1.653985+ and √1.240489+ × half √1.653985+ gives the area of H= .7183315+ which is also the area of the circle E whose circumference is 3.

PLATE XI.

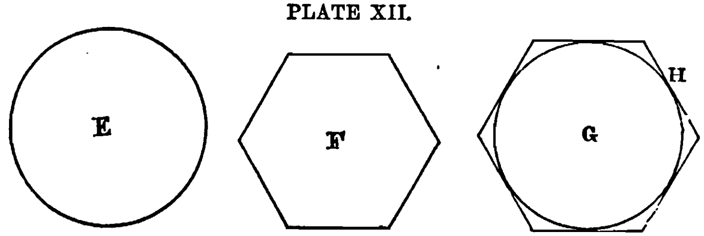


Also if F and H (Plate XI.) be hexagons,—then if the circumference or 6 sides of F shall equal the circumference of E, and the area of G shall equal the area of F, then the area of H will also equal the area of E, and the same is true of an octagon or polygon of any possible number of sides and of *every ratio* of circumference and diameter of a circle which can be used. The examples given will enable any one to prove the truth by figures for themselves. *The proposition is therefore demonstrated.*

It is a remarkable fact, that among all the modern attempts at analysis of the circumference of the circle, there is not one which gives to circumference a fixed and

definite locality, and I have never seen the man among the mathematicians of the schools, who, on a circle being placed before him, could point to the position of circumference and say, it is *there, in that place*, simply because the imaginary line without breadth, having no existence, it can therefore have no fixed and definite locality, either in fact or in imagination;—being in itself a mechanical fallacy, it can never be mechanically applied, and for the present purpose is therefore useless. The laws of geometry are undoubtedly the laws of perfect mechanics, and anything which can have no existence, such as an ideal line *without breadth*, is a *mechanical fallacy*,—and it is a *mechanical truth*, that nothing can have a definite and fixed locality, or occupy a definite and fixed position in space, without magnitude. The definition, therefore, "position without magnitude," whether applied to a line or a point, means nothing else, and cannot understandingly be made to mean anything else, but the *place* of a magnitude without the developed magnitude itself. The existence of any shape signifies *limit*, and it is evident that a circle, in order to *be a circle in nature*, must have *limit*, and a boundary definitely located, and mechanically defined, which boundary is its circumference; otherwise a circle cannot exist in nature, not even in imagination, if imagination be definite. It is evident then that the circumference of a circle having a fixed and definite locality, it is ties, and H is formed of straight lines, and controlled by *their* properties; but it is evident, whatever may be the number of sides, that H is greater than G, and because G equals F in area, therefore H is also greater than F; and because H is greater than F, and F is a polygon of the same number of sides as H, therefore the circumference of H is greater than the circumference of F; and because the circumference of F equals the circumference of E, therefore the circumference of H is also greater

PLATE XII.



than the circumference of E, and if brought into the form of a circle, will wholly circumscribe E. But the circumference of H will constantly approach more nearly to E by any increase of the number of sides of H, yet so long as H shall have the properties of straight lines, though the number of its sides were the greatest possible, its circumference shall always be greater than the circumference of E (proposition II., chapter i.), and if brought into the form of a circle, will wholly circumscribe E, though the difference may be such that it *cannot be less.* And in material things, any difference such

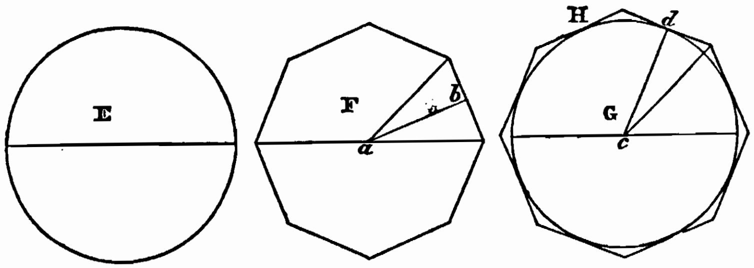
that it cannot be less, is one ultimate particle of whatever material or thing is under consideration, which is also the essential difference between the properties of straight lines and curved lines. *The proposition is therefore demonstrated.*

By this demonstration, it is shown, that in order to give to the circumference of a circle the full expression of its value by the properties of straight lines, it is not to be measured as angular figures are, and after the manner of geometers, by a line *coinciding* with the greatest extent of diameter, but by a line *outside*, wholly inclosing the diameter, and the difference between the two is the compensation due to the circumference of the circle, to answer to the relative difference of property shown in proposition I., this chapter.*

* A very fine illustration from nature, of the difference between the line approximated by geometers and the *true line of circumference*, may be had by placing a glass of water before us. If we suppose the tumbler to be a perfect cylinder, then, the surface of the water it contains will be the area of a circle. Now, the line of circumference approximated by geometers, is a line, lying wholly *inside* of every part of the tumbler, and *coinciding* with the *outer limit of the water.* The line which I say is the true circumference is the interior of the tumbler not coinciding with the water, but lying wholly outside of it, and inclosing the whole area of the water. It is evident that *every* part of the interior of the tumbler is farther from the centre of the circle than any part of the water, consequently the least possible line of the tumbler is greater than the greatest possible line coinciding with the water, because the one wholly *incloses* the other. It is evident, also, that the difference between the two lines is such, that "*it cannot be less;*" because, in the nature of the water, it would by its own gravity adjust its particles to fill the whole circle, until there is not room in the same plane for one particle more in its

mation of geometers as a ratio of circumference, viz., 3.1415926535+, and let this be the circumference of a circle, and of a polygon.

PLATE XIII.

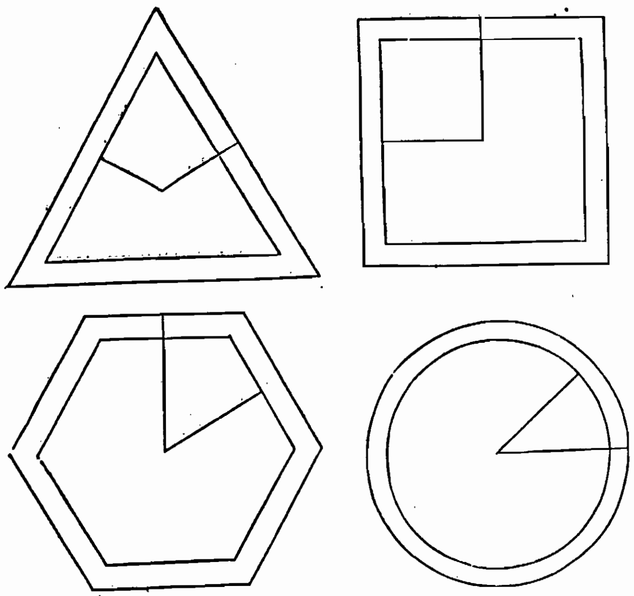


Let the diameter of the circle E=1; then the circumference of E, by the above approximation,=3.1415926535+, and the area of E=.7853981633+. Let F be a polygon of 6144 sides, in which the line *a*, *b* is a perpendicular from the centre to either side of the polygon, and let the circumference or perimeter of F equal the circumference of E; *i. e.*, let it =3.1415926535+. Now, the perpendicular, *a*, *b*, mathematically determined after Playfair and Legendre's method, is found to equal .499999934636, and the area of F (half the circumference by *a*, *b*), =.78539806072+, which is seen to be less in the seventh decimal place, than the area of the circle E, having the same circumference. It has already been proved (propositions II. and IV.), that if G be a circle equal in area to F (Plate XIII.), and H a polygon of 6144 sides, circumscribing G, then the area of H equals the area of E.

Now, therefore, let us see what is the circumference and diameter of H, which is equal in area to E. By the same approximate ratio of geometers (viz., the circumference of one diameter = 3.1415926535+), if the area of G equals the area of F, then the diameter of G = .999999934636+, and the radius of G = 4.99999967318+, and the radius of G is seen to be the perpendicular on either side, or least radius of H (*c*, *d*). If then, *a*, *b* (figure F), give for circumference 3.1415926535+, then *c*, *d*, will give 3.1415928589+, which is the circumference of H, and half 3.1415928589+ × *c*, *d*, = .7853981633+ = the area of H, which is also the area of E. Now, let it be remembered that the circumference of F equals the circumference of E, and the *area* of H equals the *area* of E, but the circumference and diameter of H is greater than the circumference and diameter of F, by .0000002054+, which is in the seventh decimal place of circumference, and by .00000006536+, which is in the eighth decimal place of diameter. And if the diameter of F be considered as *fixed*, and the whole expression of these values be given to circumference, in order to make F equal in area to E, then F should have a circumference = 3.141593+, which is seen to be greater than the approximation of geometers in the sixth decimal place of circumference, and greater also than the *so-called* perimeter of a circumscribed polygon.

that the areas of any two or more equilateral triangles, squares, hexagons, or circles, are, to each other of the *same* shape, in ratio of area, as the squares of their diameters; but no triangle, square, hexagon, or circle, is, to either of the *other shapes*, in ratio of area as the squares of the diameters of each. Therefore, the negative part of the proposition, that the square of diameter

PLATE XIV.



is *not* the natural and legitimate element of area by which different shapes are made equal to one another, is proved. Referring to the figures again (Plate XIV.), it is seen, that any triangle, square, hexagon or circle, has an area equal to half its circumference by *the radius* (the

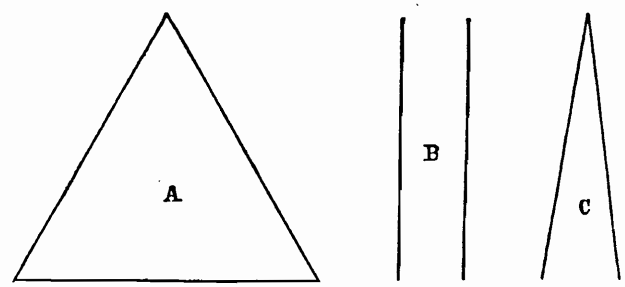
least radius in all those shapes formed of straight lines), and consequently, they are all, to one another, in ratio of area, as *half their circumference by their radius.*

The proposition is therefore demonstrated; and circumference and radius are seen to be the only legitimate elements of area, by which all shapes, and all areas, are in like ratio to one another; and by which they are made equal to one another.

PROPOSITION VIII.

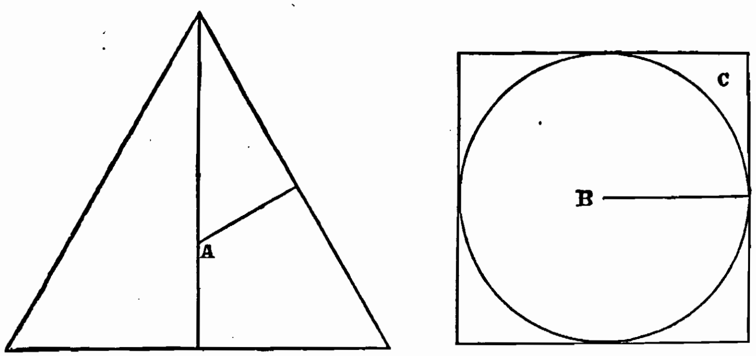
The equilateral triangle is the primary *of all shapes in nature formed of straight lines, and of equal sides and angles, and it has the least radius, the least area, and the greatest circumference of any possible shape of equal sides and angles.*

PLATE XV.



It will be seen that the triangle A (Plate XV.) has three equal sides, formed of three equal lines. Now if we suppose a shape formed of only two lines, as B or C, if such shape shall have breadth or magnitude, then it must have more than two sides, and if more than two has the least number of sides of any possible shape in nature formed of straight lines; and the circle is the ultimatum of nature in the extension of the number of sides. In this particular therefore they are opposite to

PLATE XVI.

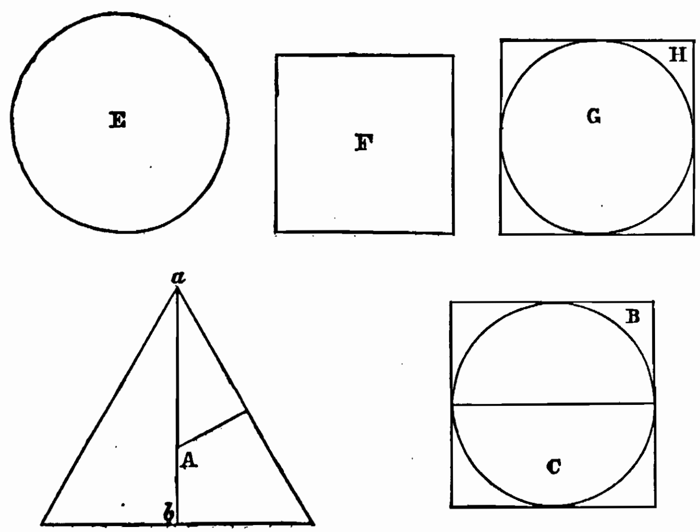


one another in the elements of their construction. By proposition VII. it is shown that circumference and radius are the only natural and legitimate elements of area by which different shapes may be measured alike, and are made equal to one another. By proposition VIII. it is shown, that the triangle has the *least* radius of any shape formed of straight lines of equal sides and of the same circumference, and by prop. II. and IV. chap. i. it is seen, that the circle has the *greatest* radius of any possible shape of the same circumference. By the same propositions the triangle is shown to have the *greatest* circumference and the *least* area of any shape formed of straight lines and equal sides, and the circle is shown to

have the *least* circumference and the *greatest* area of any shape. By a well known law of numbers and geometry by which the greatest product which any number or any line can give, is, to multiply half by half, it will be seen that if we take the aggregate of circumference and radius in each shape, it is most *equally* divided in the circle, and the most *unequally* divided in the triangle, of any possible shape. In *every* case, that which is *greatest* in the triangle is *least* in the circle, and that which is *least* in the triangle is *greatest* in the circle, and in every particular the two shapes are at the extreme and *opposite boundaries of nature*, being the *greatest* and the *least* that *is possible*. They are therefore opposite to one another in all the elements of their construction. Therefore the square being made the artificial basis of area (prop. VII.), if the diameter of the circle B (Plate XVI.) shall equal the diameter of the square C, then, in the fractional relations of B to C such diameter shall be in the opposite duplicate ratio to the diameter of A correspondingly situated. The diameter of A correspondingly situated with the diameter of B to C, it will be seen, is a line drawn across the centre of A perpendicular to either side; therefore the diameter of B in its fractional relation to C is the opposite duplicate ratio to the *perpendicular* or diameter of A. And no other result is possible in the nature of things (see prop. VII.

been proved (prop. II.) that if the circumference of F equals the circumference of E, then F and G are also equal in area. And because one circle which is equal to one square (the area of the square being one) is in 6561 equal fractional parts, therefore *any* circle which is equal to *any square* (the diameter of the circle being a whole number) shall be in some definite and certain number of $\frac{}{6561}$ parts. Hence the areas of the circles C and G (their diameters being each 81) are some definite and

PLATE XVIII.



certain number of $\frac{}{6561}$ parts of B and H. It is proved by the approximations of geometry obtained by the properties of straight lines, that C and G are each greater (much greater) than $\frac{5152}{6561}$ parts of B and H, and less

(much less) than $\frac{5154}{6561}$, therefore (*reductio ad absurdum*) they *shall* be each $\frac{5153}{6561}$ because they *can be nothing else*, there being no other $\frac{}{6561}$ part between 5152 and 5154.

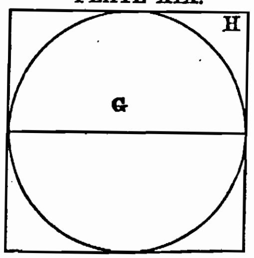
The proposition is therefore demonstrated; and the fractional area of one square which is equal to one circle (the area of each being one) is 6561, and the fractional area of one circle *inscribed* in such square is 5153.

It will now be seen that having determined two parts of each C and G, *i.e.*, having determined the fractional area and diameter of each, if we divide the area by one-fourth the diameter, it will give us the circumference of each. And because the diameter of each C and G=81, and the area of each =5153, therefore 5153÷20.25 ($\frac{1}{4}$ the diameter) =254+ with a remainder forever. It is evident, therefore, as has always been the case with others, that I have not yet reached a circumference which may be expressed in a whole number, and in decimal figures, without a remainder. I therefore add another and *final proposition* as follows.

PROPOSITION XII.

The true ratio of circumference to diameter of all circles, is four times the area *of one circle inscribed in one square for the ratio of circumference, to the area of the circumscribed square for the ratio of diameter. And hence the true and* primary *ratio of circumference to* *diameter of all circles is* 20612 *parts of circumference to* 6561 *parts of diameter.*

PLATE XIX.



It will be known that if the diameter of the circle G inscribed in H (Plate XIX.) = 1, then the area of H also =1. It will be known also, that the area of G equals half the circumference, multiplied by half the diameter, and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$; hence the diameter of G being *one*, then the area of G equals $\frac{1}{4}$ its circumference, and vice versa, the circumference of G equals four times its area. And the diameter of G being one, it therefore equals the area of H, because the area of H = 1. Therefore, the first part of the proposition is demonstrated, and four times the area of any inscribed circle for a ratio of circumference, to the area of the circumscribed square for a ratio of diameter, is seen to be a true ratio of circumference to diameter of *all circles.*

It has been proved (proposition XI.), that by the primary relations existing between straight lines and curved lines, as developed by the opposite ratio of the equilateral triangle and the circle, the fractional area of H = 6561, and the area of G = 5153; *therefore*, the true and *primary* ratio of circumference to diameter of all circles = 4 G for the ratio of circumference to the area of H for the ratio of diameter; and since G = 5153, and H = 6561; *therefore*, the *true* and *primary* ratio of circumference to diameter of all circles = 5153 × 4 = 20612 parts of circumference to 6561 parts of diameter. *The proposition is therefore demonstrated*, AND THE QUADRATURE OF THE CIRCLE IS DEMONSTRATED ! !

NOTE TO CHAPTER II.

THE ratio of Metius, known for more than a century past (113 to 355), is the nearest approximation to the truth ever made in whole numbers; but it does not answer the imperative law contained in our twelfth proposition, and therefore it cannot be true. The circumference cannot be divided by four without a fraction or remainder. By whatever means Metius may have obtained his ratio, its examination shows it to be of the same composition as mine, but improperly divided. For example, if 113 shall be the diameter of a circle, then circumference (355) is $\frac{1}{20612}$ part too little. But if 355 shall be the circumference of a circle, then diameter (113) is $\frac{1}{6561}$ part too big. It thus affords a very perfect evidence that my ratio 20612 to 6561 is the true one, as we have fully proved it to be.

It will be seen by any one who has carefully examined the propositions in chapters i. and ii., that the whole demonstration of the quadrature rests on the last four propositions of chapter ii. All the others are only preliminary, to show the errors of the received mode of demonstration, and to open the way to this. The remark following the eighth proposition, chapter ii., is, I think, a conclusion fully warranted, viz., "that because the equi-"lateral triangle and the circle are the primary shapes "in nature (propositions III. and VIII.), one formed of "curved lines, and the other of straight lines, therefore, "the primary difference between straight lines and "curved lines, and hence their equality one to the other, "is to be found in the relations between the circle and "the equilateral triangle."

This most natural and conclusive inference, drawn from proposition VIII., and those preceding it, leads our reason at once, to examine the relative properties of the equilateral triangle and the circle, and this examination again leads us at once to the ninth proposition, viz., "that the equilateral triangle and the circle are *opposite* "to one another in all the elements of their construction, "which are concerned in plane extension, and hence the "square of diameter being made the artificial basis of "area (proposition III., chapter ii.), therefore the equilat-"eral triangle and the circle are opposite to one another "in ratio of the squares of their diameters."

This, then, is the only question of doubt to settle. If the equilateral triangle and the circle are opposite to one another in all the elements of area which enter into their construction, then are they also opposite to one another in ratio of the squares of their diameters. But if they are not opposite to each other in ratio of the squares of their diameters, then are they not opposite to one another in the elements of their construction. This question, the examination of proposition IX. settles conclusively. And that proposition being proved, all the serial and algebraic formula in the world, or even geometrical demonstration, if it be subject to any error whatever, cannot overthrow the ratio of circumference and diameter which I have established! So long as it remains a truth, that the equilateral triangle and the circle are opposite to one another in the elements of their construction, that ratio of circumference and diameter will stand forever against all argument and all demonstration by the properties of straight lines which can be brought to disprove it, and time will show all the efforts of geometers to disprove this, to be just as idle as all their efforts to prove the value of the circumference of a circle by the properties of straight lines have been. These (the opposite elements, and opposite ratio) are *the particular facts* which govern the circumference of the circle in its relation to the properties of straight lines, according to the idea given out in proposition II., and which make the circumference of the circle to be a *particular fact also*, and *not* a general principle, though true to every thing that is a general principle; because, it is itself the basis of all *general principles* in relation to magnitude or area.

Having therefore arrived at that point where, by the natural conclusion of the course of reasoning adopted, the quadrature of the circle is demonstrated, I may now be allowed some freedom of remark, in respect to the credit due to those opinions, which for the last half century have condemned the quadrature as a useless question, which it was impossible to solve.

In respect to the utility and value of the quadrature, I think it will be sufficient to say, what no one will dispute,—that from the earliest time in the history of geometry, up to the close of the last century, it was considered by every mathematician, geometer, or astronomer, of eminence, to whom the world owes all that is known on these subjects, that the quadrature of the circle *was*, and *is*, an elementary truth, *necessary* to be known for the perfection of mathematical and astronomical science. In the course of the preceding demonstrations, I have shown clearly, I think, that all geometrical truth whatsoever in nature, rests on two simple things, viz., the properties of straight lines, and the properties of

curved lines, and that the relations of these to one another are controlled by the circle and the equilateral triangle as *primary shapes*;—the circle, and consequently curved lines, being *primary of all others*; thus sustaining, and being sustained by, the opinions of all ancient geometers, that the quadrature of the circle lies at the foundation of all geometry, and hence it is an elementary truth, necessary to be known for the perfection of mathematical science. Under these circumstances, therefore, when I hear a learned professor of an exact science, stigmatize the quadrature as "a useless question," and one in which an approximation is "near enough," I am led to conclude, either that he does not understand the nature of the subject, or that his declaration is made to console a wounded pride, in not being able to reach it himself.

Foremost among those who have thrown discredit on the pursuit of the quadrature, has been Legendre, the eminent French geometer; and I confess to an abundant surprise at finding, that the professors of our own day and in our own country particularly, have received what Legendre and a few others have said, as *established facts*, and have adopted their opinions without investigation. A distinguished professor of one of our own distinguished colleges, to whom I sent some of my original papers, once wrote me in reply; and referring to Legendre's note on the subject of the quadrature in his elements of geometry, added, "that after seeing what that great geometer "had said, he presumed I would think no more of the "subject."

In the note in question, Legendre, after having finished a bisection to 8000 sides of a polygon, asserts that he has determined the quadrature as accurately as the root of any imperfect square can be determined to the same number of figures; and after concluding that the perfect quadrature is impossible to be found, he adds, in substance, "that no one having the least pretension to geo-"metrical science will ever make the attempt"!!!

What consideration is due to these declarations of Legendre, may easily be seen. He did not even attempt to measure the exact circumference of a circle; and he did not in *the least* consider the properties of curved lines, notwithstanding that it had long been known that curved lines do possess properties essentially different from straight lines; yet Legendre, by his method, tacitly admitted that he knew nothing about them. He simply bisected the perimeters of two polygons on a given radius, until he brought their sides to agree with each other to a certain extent, and these polygons he then considers as equal to a circle, to the extent to which their sides agree!! But I have demonstrated (and any one may know the fact, almost without a demonstration), that a circle is greater than a polygon having the same cir-

cumference at any possible number of sides (proposition II., chapter i.); especially is it greater than a polygon of only a few thousand sides. The conclusion of Legendre is therefore manifestly erroneous. He measured nothing but polygons, and judging from what he has said, he neither understood nor regarded the effect of transition of shape to a circle. (See remarks following proposition IV., chapter i.)

Again, it will be obvious to all, that by Legendre's method, his errors from quantities neglected or lost in the course of his calculation and reduction of his result to numbers, are errors of *each side* of the polygon, and consequently while in that result, he can have but one error in his diameter, he has 8000 errors in his circumference, and he could not tell whether the sum of these errors was plus or minus!! And again, these errors are not errors in the circumference of a circle, but *in the perimeter of his polygon*, which is of less value than a circle having the same circumference (propositions II. and IV., chapter i.), and hence they stand additional to the errors arising from the essential difference in the properties of straight lines and curved lines, which he has wholly neglected. Yet in the face of these facts, which he must have known if he understood his own work, Legendre has seen fit to declare that he has determined the circumference of a circle "as accurately as the root of any imperfect square can be determined." And who, I ask, will, after such a declaration, feel his pointed rebuke, or trust what he may say on this subject, without first satisfying himself of its truth? For my own part, I cannot but believe that Legendre's reputation as an eminent geometer, great though it deservedly is, would have been more enduring if he had left the note in question out of his work entirely.

Playfair of Edinburgh was nearly contemporary with Legendre, and his and Legendre's Elements of Geometry, or rather their editions of Euclid, have for a long time been standard works in the English language on these subjects. Playfair admits that his and Legendre's method of determining the value of circumference (which was also the method of Archimedes 2000 years ago) is *defective*, but modestly says, that geometers "know no better method." Still, however, Playfair is supposed to have sided with Legendre in thinking that the exact solution of the problem was unattainable.

Legendre and his coadjutors were members of the Academy of Science in Paris, and Playfair and his coadjutors were members of the Royal Society of London.

About the era of the publication of Legendre's work, the Academy of Science in Paris, instigated perhaps by Legendre himself, passed a resolution, that, in order to discourage such futile attempts, the Academy would not

thereafter receive or consider any paper purporting to be on the subject of the quadrature! And within a few years thereafter the Royal Society of London passed a similar resolution!! And under the influence of these tyrannical proceedings it soon became disreputable in learned circles for any one to talk of finding the quadrature. But these resolutions of Academies and Royal Societies needed some support, and Montucla generously comes to their aid in his *History of the Quadrature*, which, so far as I can judge from such portions of it as have been brought to my notice, appears to have been written for the purpose of pandering to the prejudice of the French school, rather than to do justice to the many ingenious but ineffectual attempts which have been made to solve the problem. He remarks with an air of complacency, which he seems to think is a conclusive argument, "that he never knew a man who thought that "he had discovered the quadrature who would ever be "convinced of his error, however clear the argument "might be against him." In this remark he seems to have forgotten the possibility that he and his associates were afflicted with the same human infirmity,—an unwillingness to be found in error; but if we examine into the history of the progress of science we shall find, that the great stronghold of this mental disorder has always been found within the walls of the academy. If this remark needed any evidence to sustain it, I might instance the almost uniform and steady resistance with which the schoolmen have received the first introduction of every great discovery in modern times.

Here then is a brief history, and the whole merit, of that popular prejudice which for the last fifty years has condemned the quadrature of the circle as a useless question which it was impossible to solve, and the only very remarkable thing about it is, that the really intelligent portion of the mathematical world should so long have bowed in meek submission to the tyranny.

The only demonstration ever made or ever pretended to be made, by any body, of the impossibility of finding the quadrature, is only a demonstration that the thing is impossible to be done by means of straight lines alone, and geometers have never used any thing else but straight lines. I have never disputed nor even doubted the truth of this fact,—far otherwise, and so plain and simple is it, that it hardly needs a demonstration, but may be easily understood by almost any one of only moderate mathematical perception. And any one who thinks that there is necessarily any disagreement between what I profess to have proved and what has been proved by the properties of straight lines, or any thing contained in Euclid, will find himself, upon a full examination, to be greatly in error.

I also have demonstrated the same thing, and not only so, but my demonstrations show clearly, I think, that no straight line, nor any number of straight lines *in a circumference*, can in any possible shape or position, be equal in relative value to the same length of curved line. Hence, instead of pursuing the quadrature as all others have done, by reasoning wholly on the value of straight lines, I have first turned my attention to discover by means of shapes, some of the relative properties of curved lines to straight lines, and then I have made use of these properties to determine their relative value,—a mode of proceeding, I believe, never before adopted in any attempt to solve the problem; and few, I think, will disagree with me in believing that this is the only practicable mode. Some may, and no doubt will, find fault with it. I expect this, from the influence of that prejudice, of which I have just given the brief history, for it is hard, for scientific men above all others, to yield a fostered prejudice. But there are many men, even among the most learned of mathematicians, whose power of reasoning is limited to the rules which they have studied, and any thing which comes not within *these rules* is, to their minds, not mathematical. Such men are often heard to say, that "the science of mathematics is "perfect, and that there can be no change or improvement "in it; because," say they, "mathematics are true and "the truth cannot change." But such men, I think, confound the *written science* with the truth,—they substitute a mere method by which truth is determined, for the truth itself, when in fact there is only an affinity between them.

Nature never follows the rules of mathematicians, though she is in fact a much better mathematician than any of them; she arrives at the same results however by methods of her own, and the nearer our methods approach to hers, the more simple and perfect they are.

The whole written science of mathematics is, to my mind, nothing more than a process of inductive reasoning, wherein by laying hold of certain primary and self-evident truths, we thence deduce other truths, and from these again yet others, constantly widening the basis of our reasoning and always ascending from the lower to the higher. In geometry the primary truths adopted, are, simply, a straight line and a given angle, which form the starting points, and from these, all subsequent deductions are made. The whole course of geometrical reasoning therefore which is known to geometers, is based wholly on the properties of straight lines, and all circular or spherical measures as yet known or used, are, only approximations of straight lines, which in circular or spherical magnitudes are both erroneous in principle and subject to error in reducing their value to numbers.

Other systems of geometry might have been formed which would have been equally accurate, yet based on other shapes, and other truths, but whatever might have been adopted as a basis of reasoning, numbers in some one of their infinite capacities of notation and combination, and not straight lines alone, would lie at the bottom of all truth. It is not necessary, therefore, in order that our reasoning should be strictly mathematical, that we should follow the rules laid down in the written science for reasoning on the properties of straight lines. It is only necessary that we should abide by the *same principles* which are laid down for that purpose, viz. *to seek first from nature* some primary and self-evident truth from which to deduce other truths; and then it is only necessary, that the truths deduced, should be *necessary results* of the primary truth which forms the basis of our reasoning, and if we abide by these principles then our reasoning is *strictly mathematical.*

Hence whether we reason from the properties of numbers or magnitudes, shapes or areas, lines or angles, or from all of them together, if our deductions are necessary results of premises founded on the truth of nature, then such deductions are *mathematically true.* Hence I am justified in speaking of a line and an area as equal to one another (prop. XII.). Hence also I am justified in introducing shapes as an element of mathematical reasoning independently of lines, because shapes are *primary things* to which lines are only secondary, and of which they constitute the divisions, dimensions, and boundaries, as stated in the remarks preceding prop. VII.; but without shapes lines could have no existence and would become wholly inoperative.

Every step which I have taken and every conclusion at which I have arrived in the course of these demonstrations, will be seen to be founded in, and a necessary result of, some primary and self-evident or demonstrable truth, and therefore the result *cannot fail to be true.*

In mathematics there is no room for opinion; what a man demonstrates by unerring principles he either *knows* to be true, or he does not understand his own work; and believing that I have understood mine, in submitting these demonstrations to public examination and criticism, I do not ask whether the result is true or not, but I assert that it *is true*, and I hold myself ready to sustain it. Fortunately, however, for my time and industry, the truth, if such it be, being once known, will be able to sustain *itself* without *my* aid.

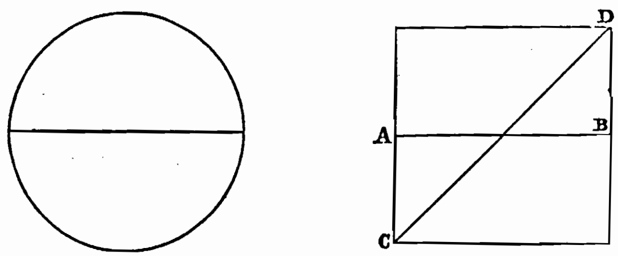
It is proper in connection with this note to remark, that the method of demonstration which I have here used, is not the only method by which I have arrived at the same result, nor is it the method by which the ratio of 20612 to 6561 was first discovered. That ratio was

first discovered by reasoning wholly on the properties of numbers, independently of lines or shapes, and assuming as a primary truth, *first*, that in the material creation, numbers and things (magnitudes or shapes) are identical and inseparable. *Secondly*, (as has here been demonstrated) that the circle is the beginning of all magnitudes or areas (prop. III. chap. ii.), and hence the *beginning of numbers also;* and *thirdly*, that consequent upon these primary truths, numbers themselves are constructed upon the circle, and hence by reciprocity, the *truth of the circle is the basis of numbers.* Reasoning from these premises, I arrived at the conclusion that the circle in its relation to the square (the area of each being one) is composed of 6561 parts, and taking advantage of approximation I inferred, "reductio ad absurdum," that the ratio of circumference to diameter is 20612 to 6561.

In conformity with the premises above assumed it will be seen, that the fraction 6561 divided into its own root (81) produces a repetition of the digits to infinity, the number *eight* being always missing in consequence of our use of decimal numbers in dividing. Also if $\sqrt{6561}$ (81) be divided into *one*, the product is the same—a repetition of the digits to infinity, thus conclusively showing that this fraction is a basis of the digits used in the construction of decimal numbers.

The second method by which I arrived at the same result was by dissecting a circle and a square into the *original whole* and *separate* parts necessary to form a circle and a square in *shape* and *area*, as follows:

PLATE XX.



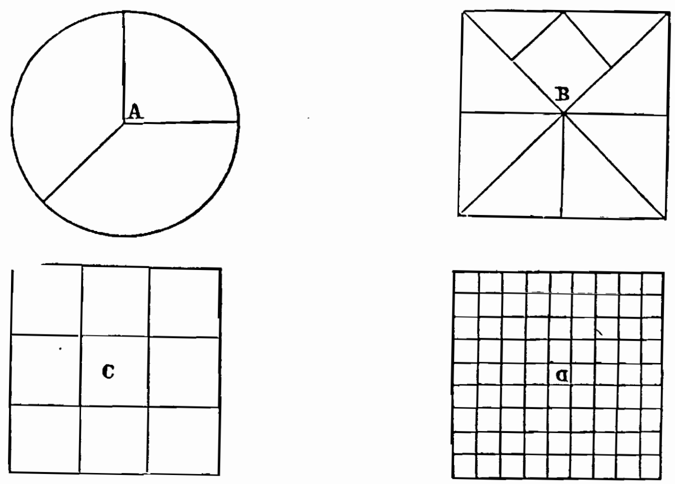
Let us suppose that the circle and the square (Plate XX.) are equal to one another in area, that is each =1. It will be seen that the parts necessary to form a circle in shape and area are *three*, viz., it has one continued line for circumference, and therefore its circumference is one,—it has but one diameter (all other diameters in it being equal) and therefore its diameter is *one* (of a circle)—the square of its diameter is also *one* (of a circle) making in all *three original whole* and *separate* parts; and these are seen to be *all* the parts that are necessary to form a circle in *shape* and *area*, and without these parts a circle cannot be formed.

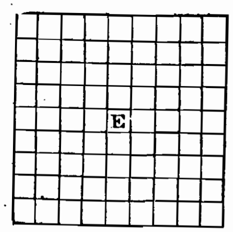
The square is seen to have four distinct and separate lines for its circumference, and therefore its circumference is *four*. It has two diameters, viz., A B and C D (the greatest and the least) and therefore its diameters

are *two*. The squares of its diameters are three (of a square) viz. A $B^2=1$ and C $D^2=2$, making in all *nine original whole* and *separate* parts, and these are seen to be *all* the parts necessary to form a square in *shape* and *area*, and without these parts and in these exact proportions a square cannot be formed.

In the two shapes, however, these parts are, in their relation to one another, wholly incongruous in quantity. Their equality to one another is therefore to be found by the following method.

PLATE XXI.





Let us again suppose that the circle A, and the square B (Plate XXI.), each equal one in area. It has been seen that the circle is composed of three original, whole and separate parts, and the square of nine parts. Now, let us suppose that there are three magnitudes contained in A, of any shape, and of incongruous quantities, and that there are nine other magnitudes contained in B, of any other shape, but individually and relatively to one another, and to the magnitudes contained in A, wholly incongruous in quantity, but the sum of the three magnitudes, and the sum of the nine magnitudes are each equal to one another; and for illustration, let these magnitudes be represented by the areas contained in the lines drawn on A and B. It is now required to find the perfect equality of these incongruous magnitudes in the parts of the square. Therefore, because A = 3; therefore $A^2 = 9$; and we have the figure C equal to the circle A, equal to the square B, and equal to the nine magnitudes contained in B, but the individual magnitudes contained in C and B are unequal to one another; therefore, because B = 9, then $B^2 = 81$; and we have the figure D equal to the circle A, equal to the square B, and equal to the whole of C, but the separate magnitudes contained in C and D are unequal to one another; therefore, because C = 9, then $C^2 = 81$; and we have the figure E, equal to the circle A, equal to the square B, and equal to D, and all the separate magnitudes contained in D and E are equal to one another *in the parts of the square.*

Now, therefore, because 81 is the smallest number

by which the unequal magnitudes of incongruous quantity contained in A and B can be made equal to one another *in the parts of the square;* therefore, 81 is a diameter by which the fractional area of B equals the area of A, and hence the fractional area of A, in its relation to $B = E^2$, and the fractional area of B, in its relation to A, $= D^2$, and because E and D each = 81, therefore the fractional area of A and B each = 81 × 81 = 6561. Therefore it is demonstrated that the fractional area by which one circle is equal to one square (the whole area of each being *one*), is in 6561 equal fractional parts.

Now, therefore, since B = 6561, by proposition XI., chapter ii., it is proved that the area of a circle inscribed in B = 5153, and by proposition XII., it is proved that the true ratio of circumference to diameter of all circles is four times the area of the circle inscribed in B for a ratio of circumference to the area of B for a ratio of diameter; therefore, the true ratio of circumference to diameter of all circles is 20612 parts of circumference to 6561 parts of diameter. Q. E. D.

In proposition III. (chapter ii.), I have demonstrated that the circle being the primary shape in nature, is therefore the natural basis or beginning of all area, and the square is the artificial basis created by science.

From this conjunction of facts, it will be seen, that because one shape (the circle) forms the basis of nature, and the other shape (the square) forms the basis of art, therefore, in the elements of their construction, or in the component parts necessary to form a circle and a square in shape and area (the area of each being one), the original whole and separate parts of the two shapes *shall also be equal*, or the parts of one shape shall be the root of the parts of the other shape. Therefore, it is seen in the foregoing demonstration, that because the circle is *primary*, therefore the parts necessary to form a circle in shape and area are the root of the parts necessary to form a square in shape and area ($\sqrt{9} = 3$), and although these parts, in their relation to one another, are originally and individually wholly incongruous in quantity, yet because one is the root of the other, their perfect equality is readily found, as in the demonstration.

It will be perceived by any one who has read with any care, that this method of showing the relations between the circle and the square to consist of 6561 fractional parts, has no similarity to, or dependence on the opposite duplicate ratio; on the contrary, it is entirely dissimilar, and wholly independent of it in its principles and operation. I consider it perfect in itself, yet I do not advance it here as any part of my argument, but rest the decision of the truth entirely on the opposite duplicate ratio of the equilateral triangle, and the circle as explained and demonstrated in the twelve propositions of chapter ii.

I have introduced these two last methods of finding the quadrature only into this note, and have forborne to treat them at length on their merits, lest I should make my work too large. I would remark, however, that I hold a mass of papers and correspondence in respect to this last method (which was the second in the course of discovery), and which I may make use of at some other time. I consider *one method sufficient*, and I have preferred the method of demonstrating by the opposite duplicate ratio of the equilateral and the circle, which was the third in the course of discovery, as being to my present views the most full and complete. But it is due to the merits of the second method, by dissecting of the parts, to say, that it was the examination of the subject by this method which revealed to me many of the properties of curved lines, and led to the discovery of the third from the opposite duplicate ratio.

The methods which I have used may doubtless be somewhat improved upon, and there are various other methods which may be used for the purpose with equal effect, but if we have found the truth, it is useless at present to descant upon these.

APPENDIX TO THE QUADRATURE.

Many persons, I think, imagine, that the Quadrature of the Circle is only a kind of mathematical puzzle, which if ever solved, some one should at length work out by a single proposition; and few, perhaps, will be prepared to believe that a work so large as this has already become, is really necessary in order to demonstrate satisfactorily any single truth. But such persons, I think, can have very little idea of the numerous ramifications into which mathematical science has extended itself, and how intimately it is associated, not only with *every other* practical science, but with every material truth in the known world.

If the ratio of circumference to diameter had been among the early discoveries made, and the whole superstructure of mathematical science been built upon the knowledge of its truth, it would then have been easy enough to satisfy inquiry by the demonstration of a single proposition; but unfortunately such is not the fact. The foundations of the science were laid without this knowledge; and under the guidance of multitudes

of the most acute minds the world has produced in every age, it has extended itself seemingly in every possible direction, and embraced almost every possible subject, until it must be admitted, that it is at this day, the *most* perfect of *all* the sciences. Yet it is *not perfect* in any of its branches. In geometry especially, the most beautiful and useful of all, there is something yet lacking, and that something lies at the foundation of all truth,—it is the Quadrature of the Circle, or a knowledge of the exact relations between straight lines and curved lines, which has never yet entered into the structure of the science. The science is, I think, rightly esteemed the most noble, most useful and most beautiful structure in existence, the production of human intellect searching after truth, but even this most perfect production of intellectual labor is not yet perfect. It was begun with a knowledge of only a part of the truth,—without understanding *all* the principles which in its upward progress to its present magnificent proportions would be brought into practice,—and as in all such cases a want of a knowledge of *all* the principles which were to be carried out, has necessarily led to some error;—some of its materials are heterogeneous, and they have become mixed and confused;—some of its proportions are unjust, because not exactly true,—some of its parts will not match, and the workmen have tried to make them match by correcting their differential properties. And we all know well enough that in architecture when we attempt to correct a mistake in this way, instead of pulling it down and building up again, we must go on correcting mistakes forever,—if but one stone is out of place, it will go on displacing others until the whole building is marred,—it is a mathematical result that it should be so, and no power of man can correct it without correcting the first error. Such, then, is the condition of the structure of mathematical science at the present day, and to carry out the figure, the building can never be complete. It wants another and a chief corner-stone to rest upon, before the cap-stone can be laid and the whole present a finish which the Deity himself may look upon without pity on the intelligence of his creatures. And to accomplish this, we must first remove all that part of the superstructure which is out of place, and this is in fact the thing proposed when we attempt the solution of the Quadrature.

To supply this chief corner-stone we must go back to the first error, dislodge it from its foundation, and establish the truth in its place, by determining without condition or qualification the exact relations between straight lines and curved lines; and we must then follow up the first error, through all gradations of the received science, and wherever it has established itself as a prinicple, we

must prove such principle to be false by unmistakable evidence, dislodge it, and take it away entirely; or otherwise it will become the foundation of other false principles through every gradation of an infinite series.

Such is the constitution of the human mind, and such the force of education, that the minds of mathematical professors are, with exceedingly rare exceptions, formed *upon the rules of the written science*, and they are unwilling, and often *unable* to comprehend any other. One highly distinguished among them lately remarked, respecting himself, that these ideas (meaning the received theories of mathematical science) "had become a part "of the furniture of his mind, and were too strongly "fixed to allow him to *consider* any other." From this cause I have found the Professors as a body, though learned in the received theories, to be among the *least competent* to decide on any newly discovered principle. Their interest, education, pride, prejudice, self-love and vanity, all rise in resistance to anything which conflicts with their tenets, or which outruns the limits of their own reasoning. So little do they look beyond the principles inculcated by education, and so tenaciously do they hold on to these, that when driven from one principle they fall back upon another, and when beaten from all, they return again to the first, and maintain themselves by dogged assertion, or by charging their assailants with ignorance and a lack of science; such at least I have found to be the character of the professors, in every approach I have made to them; and this being the case, if I would have my work acknowledged, there must no foothold be left for them to rest upon. I think I shall be justified by the candid judgment of well-informed men when I say, that, in consequence of this character of professors, the practical men of the age are at least a century in advance of the schools, in all useful scientific knowledge. I have made these remarks as a reason and in explanation of the necessity of following out in further minutiæ the errors to which various problems in geometry are subject in consequence of the error in the Quadrature.

In the preceding chapters I have made occasional reference to facts and principles not previously demonstrated, and which, in a work strictly mathematical, or which was designed for practical instruction, should have stood first, as elementary truths, on which subsequent demonstrations were to be based. But to have made my preliminary demonstrations too diffuse, would, I think, have diverted attention from the main object; and I have therefore thought fit, under the head of an Appendix, to demonstrate such propositions as will answer to the above references and sustain the argument.

One of the facts stated as above in the course of

this work, but not previously demonstrated, is as follows:

That "the so-called perimeter of the circumscribed "polygon of geometers is *not a circumscribed* perimeter, "but that the center of each side of the perimeter *coin-* "*cides* with a part of the area of the circle, and at an "infinite number of sides is brought *wholly within the* "*area of the circle.*"

The first general proposition on which geometers proceed, in approximating to the circumference of a circle, is as follows,—that "the circumference of a circle is "greater than the perimeter of an inscribed polygon, and "less than the perimeter of a circumscribed polygon, "whatever may be the number of the sides."

Nothing can be more true than this general proposition,—provided, however, that the *conditions* of the proposition be fully adhered to in the demonstration. In the fifth proposition of the first book of Playfair's Supplement to the Elements of Geometry, he demonstrates that "the area of any circle is equal to the rectangle con- "tained by the semi-diameter and a straight line equal to "half the circumference." This proposition is also true, and Playfair demonstrates it by an *inscribed* and *circumscribed* polygon; but the *conditions* of the demonstration are, that the perimeter of the *circumscribed* polygon lies *outside* of the circle "touching it," and on *this condition*, and on *no other*, is the first above named general proposition true (see prop. v., chap. ii.). It will be seen that if the perimeter of the circumscribed polygon lies *outside* of the circle "touching it," then no part of the perimeter of such polygon can coincide with any part of the area of the circle. My object is now to show that the line approximated by geometers as the circumference of a circle, is a line coinciding with the greatest limit of the area of the circle, and exactly equal to the circle, but not *inclosing* or *containing* it according to the true definition and meaning of circumference (prop. v., chap. ii.), —that this result is produced by bringing the so-called circumscribed perimeter *wholly within* the area of the circle, and that consequently geometers by their method of bisection *do not adhere* to the conditions of the first general proposition, and hence their result is not true in its application to the circumference of the circle. Therefore,

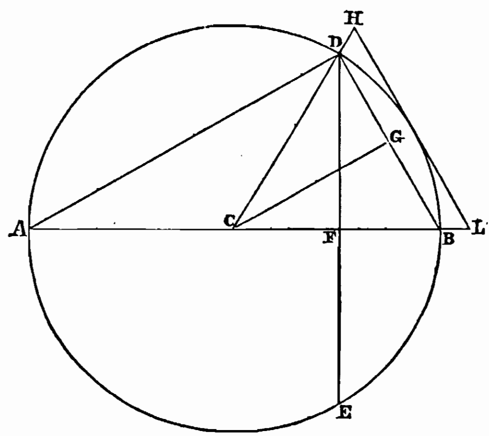
PROPOSITION I.

The line approximated by geometers as the circumference of a circle is a line coinciding *with the greatest limit of the area of the circle, but not* inclosing *or* containing *it.*

I now take the eighth proposition of Playfair's Supplement to the Elements of Geometry, book i. It reads as follows: "The perpendicular drawn from the center of a

"circle on the chord of any arch, is a mean proportional "between half of the radius, and the line made up of the "radius and the perpendicular drawn from the center on "the chord of double that arch. And the chord of the "arch is a mean proportional between the diameter and "a line which is the difference between the radius and "the aforesaid perpendicular from the center." This proposition is also true in every particular in respect to an *inscribed polygon*, which forever remains inscribed within the circumference of the circle, and if it could be carried out in bisection without any quantities being lost in the calculation (which it cannot be), it would constantly approach to a line coinciding with the greatest limit of the area of the circle, but could never equal it, much less inclose it (prop. I., chap. i.).

PLATE XXIX.



In Plate XXIX., we have the same diagram which Playfair uses in his illustration, with the exception that I have added the circumscribed line H L. To reduce the proposition to its value in numbers the proceeding runs thus.

The diameter (A B) being considered as 2, the line D E is the chord of one-third of the circumference; it bisects the radius C B, and since C B=1, therefore C F=.5 and C F is the perpendicular from the center C on the chord D E, and the proposition is, that "the chord of the "arch is a mean proportional between the diameter and a "line which is the difference between the radius and the "aforesaid perpendicular from the center." Therefore C B − C F=F B, and F B×A B=D B², and $\sqrt{}$D B²= D B, which is the chord of the arch of one-sixth of the circumference, or double the number of sides of D E. And in like manner he proceeds to a greater number of sides.

Now, the circumscribed line H L, according to Playfair's method, is a proportion to D B, as C B is to C. G. The chord D B is supposed by geometers to be a line *wholly without* breadth; consequently, it is a line, the center of which exactly coincides with the extreme point of the perpendicular, C G, neither one particle short of it, nor one particle beyond it, the point of the perpendicular itself being, in fact, part of the chord; consequently, the circumscribed line H L, being a proportion to the inscribed line or chord, D B, as C B is to C G, its center (H L) exactly coincides with the extreme point of C G, when C G is produced equal to C B, neither one particle short of it, nor one particle beyond it, so that if

the perpendicular C G shall have breadth given to it, then the extreme point of C G, when produced equal to C B, will form part of the line H L. It is evident, therefore, that the perimeter of Playfair and Legendre's so-called circumscribed polygon does *not lie* OUTSIDE of the circle "touching it," according to the conditions of the fifth proposition, book first, of Playfair's Supplement, in which he demonstrates the area of a circle, as before referred to, in this proposition; on the contrary, the perimeter, at the center of each side of his so-called circumscribed polygon, coincides with a part of the area of the circle, and at an infinite number of sides is brought *wholly within* the area of the circle, and, therefore, does not inclose or contain it. It is evident, also, that the condition of the first general proposition of Playfair and Legendre, that "the circumference of a circle is greater than an inscribed polygon, "and less than the circumscribed," is *not adhered to* in the demonstration, and, therefore, their result is not true by their own showing, but is less than the truth; because the perimeter of their so-called circumscribed polygon does not lie *outside* of the circle "touching it," according to the required conditions; and because, as has been demonstrated (proposition v., chapter ii.), the true circumference of a circle is a line wholly *outside* of the circle, thoroughly *inclosing* its whole diameter, and containing the whole area of the circle *within* it; therefore, the true circumference of a circle is greater than the *so-called* circumscribed perimeter of Playfair and Legendre, at an infinite number of sides. *The proposition is therefore demonstrated.*

A teacher of mathematics, in one of our institutions, in answer to my second proposition, chapter i., that "the "area of a circle is greater than the area of any polygon "having the same circumference of the circle, whatever "may be the number of the sides of the polygon," writes me as follows:

"You endeavor," says he, "to prove that the polygon "can never equal the circle" (each having the same circumference, and being measured in the same way). "Your reasoning on this appears to be correct; but by "comparing this approximation with some others that "are analogous, I am inclined to believe that it is not "correct. Take, for instance, the series $4 + 2 + 1 + \frac{1}{2}$ "$+ \frac{1}{4} + \frac{1}{8}$, &c. Now, this series will approach to 8, but "can never equal 8; but embraced in an algebraic for-"mula, it can be proved, that it does exactly equal 8, "when the number of the terms are infinite. Let the "series $= x$.

" $x = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$, &c., to infinity:
"Then $x - 4 = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$, " $+ 2$:
"Then $2x - 8 = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$, "
"The last series is identical with the first, and things

"which are equal to the same things, are equal to one "another; therefore, $x = 2x - 8$; or $0 = 2x - x -$ "8; or $0 = x - 8$; or $x = 8$."

Now, this "algebraic formula" is, by the learned teacher, called a *demonstration*, showing that my second proposition, chapter i., cannot be true! It looks, on the face of it, almost too ridiculous to be entitled to an answer,—but the learning of the schools must have consideration; and besides, if this proposition be true, then my second proposition is *not true.*

I now desire the reader, therefore, to turn to the second proposition, chapter i., and examine the demonstration which follows; he will see that my demonstration is purely geometrical (not algebraical); the result is a necessity of the immutable laws of numbers,—the reason of that result is palpable to the senses,—the demonstration is therefore accepted as a self-evident truth. Now, what is the character of the *learned teacher's* demonstration? It is an algebraic formula, adopted to prove a thing contrary to the evidence of our senses, and contrary to the operations of numbers; for it is admitted, that in numbers (and numbers are in themselves infinite), the series can never equal 8. It will be seen, that in the treatment of this series by algebraic formula, the conclusion arrived at, or rather assumed, is, that an infinity $= 0$; but I have already promised, in another part of this work, to show that an infinity, whatever it may be, is always such, that in material things it is *capable of increase*, which I shall presently do by this same series. This algebraic formula, then, is called a demonstration; but, in point of fact, it is no demonstration at all. By a demonstration, I understand the making known and certain, something which was before unknown and uncertain. But the whole of the foregoing so-called demonstration by algebraic formula, depends entirely on the assumption or hypothesis—*first*, that infinity = 0; and *secondly*, that the series does actually equal 8. But if the assumption or hypothesis be *not true*, then the demonstration is *not true;* and I say, that in this case, the assumption is *not true;* and unless it be first proved by numbers, the algebraic formula proves nothing but what the contrary may be proved by the same formula.

The absurdity of calling this a demonstration, is, I trust, manifest; yet it is the same which the learned teacher of mathematics, in one of our public institutions, has furnished to disprove my second proposition, chapter i., which is purely a geometrical proposition, geometrically demonstrated. And this is not the only instance which can be found, of the absurd use in the schools, of algebraic formula for demonstrating geometrical propositions,—there are many things thought to be demon-

strated, which will not bear criticism. One rule, however, will apply to all such; if the assumption or hypothesis be true, the demonstration is true; but if these be not true, then the demonstration is *not true;* and in this case, I say, it is not true, that the series $4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ &c., = 8, or that an infinity = 0, because numbers and things are identical and inseparable, and neither in numbers or things, is there any infinity of division = 0.*

The term "infinite," or "infinity," is one often used in mathematics, but no explicit or satisfactory definition has ever been given to it. An infinity, in its fullest sense,

* The same teacher, who so learnedly attempts to refute my second proposition, as above, writes me also in respect to my ratio of circumference, that "it is proved by trigonometry that the length of an arch of 45° to radius = 1;" is equal to a certain series, by which they obtain for circumference 3.1415926+, and hence, he thinks that my ratio cannot be true. And this method, he says, "*is entirely independent of the method of Euclid.*" He would thus argue, it seems, that trigonometry and geometry are two things; and hence the result by what he pleases to call trigonometry, is independent of the result by the geometrical method!! a most potent argument, to be sure. But I trust it will require no argument to prove that the principles made use of in trigonometry to determine the series, being based only on the properties of straight lines, are precisely the same, and involve the same error as Euclid's method, and therefore, come to the same result. If the method, by a fluxionary series deduced from trigonometry, is right, then Euclid's method is right also, because they come to the same result. But Euclid's method has been proved to be wrong, and to be less than the truth; therefore, the series proved by trigonometry is also wrong. In fact, all such series are nothing but approximations; there is not a single absolute truth in the whole range of them; the very name of an infinite series signifies something which never can be equalled.

whether of magnitude or minuteness, is an incomprehensible term,—no mathematician ever did, or ever can understand it. To suit their own purposes of reasoning, however, mathematicians have assumed that an infinity, or a thing infinitely diminished, equals 0, and therefore, throw it away, as having no appreciable value. Of the error of this course, I have already given an example, in the remarks following proposition IV., chapter i. No schoolboy's mind was probably ever satisfied with this *throwing away* of *infinity*, until, by instruction and habit, he has at length reached the full grown prejudice of his teachers; for he sees, that they are sometimes obliged to reverse the case, and then they endeavor to prove that *nothing* may equal *something!!* If an infinity be really *nothing*, the term cannot be applied to material things, nor can we reason on the two (finity and infinity), or from one to the other, with the slightest ground of truth for a basis. Therefore—

PROPOSITION II.

An infinity in minuteness is always such, that it is capable of increase; therefore, in material things, an infinity equals one ultimate particle of matter, such, that in the nature of the material or thing under consideration, it cannot be less.

I propose to demonstrate this second proposition by

the series $4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ &c., and I say that this series, infinitely extended, equals 8, *minus one infinity*, or minus one ultimate particle of matter, such that, in the nature of the matter or thing considered, it cannot be less. The demonstration is by numbers, and it proceeds upon the supposition, that the so-called infinity of mathematics is a point of division *beyond* the power of numbers; therefore, $4 + 2 + 1 + 0.5 + 0.25 + 0.125 + 0.0625 = 7.9375$. I have here carried the division to one-sixteenth only, and it is seen that the sum of the whole is deficient of 8, one part of the last division, and in order to make the sum of the whole equal 8, the last addition must be $\frac{2}{16}$ instead of $\frac{1}{16}$; and the same is seen to be the case, to whatever point in numbers the division may be carried,—two parts of the last division must be added, to make the sum of the whole equal 8; therefore, let the number of divisions be the greatest possible; then, because the number of the terms or divisions is the greatest possible, and two parts of the last division are necessary to be added, in order to make the series equal 8, and by the addition of one part only, an equal part is left; therefore, the part left is infinity, which numbers cannot divide; hence, it is evident that numbers themselves are infinite,—hence the divisions of numbers equal infinity,—hence an infinity equals the greatest possible divisions of numbers,—hence an infinity equals one part of the greatest possible division of any magnitude—hence also, by reciprocity, in the above series, an infinity is such, that by constant doubling, it shall amount to 4; therefore, an infinity is such, that it is *capable of increase. The proposition is therefore demonstrated.*

It will be seen from the above demonstration, that because *numbers themselves* are thus proved to be *infinite*, as, indeed, our own perceptions tell us they are, and because the series 4 + 2 + 1 &c., can never be made to equal 8 by *numbers*, therefore, the assumption that the series does actually equal 8, or that an infinity = 0, is absurd, and any demonstration by algebraic formula, to the effect that they do, is a pure assumption, contradictory of evidence, and is therefore absolute nonsense.

The general idea which we get of magnitude in the abstract is, that it is something which entirely fills a definite portion of space. Now if we suppose any definite portion of space to be divided first into two equal parts, then four, then eight, and so on to *infinity*, it is evident that the sum of all the parts into which the space is divided is equal to the whole, and one part equals infinity, but if we suppose that infinity = 0, then each part = 0, and the whole space is annihilated, which is absurd; because although we cannot say with certainty that matter cannot be annihilated, we do say and know with certainty that blank and abstract space cannot be annihilated. It is evident, therefore, that *both numbers* and *space* are infinitely divisible, and no minuteness can annihilate either. But material things, or magnitudes developed to the senses, are governed by laws which the existence of *space* or magnitude in the *abstract* does not involve. Such developed magnitudes, as, for example, those composed of metals, of wood, water, earth, &c., do not always, and I think never, fill all the space within their boundaries. Their parts may be united by cohesion, but the lines which separate their parts, though infinitely diminished, are not annihilated. And the bodies are filled with porosities which allow of the existence of other material elements of nature within them, as air, moisture, light, heat, electricity, &c. We know, also, that all material or developed magnitudes are formed from *original elements* which had a separate atomic existence, and which have become united by their affinity, and hence they may be so divided that they *cannot be divided any further* without resolving themselves into their original elements, or when so divided they may perhaps, by forming new affinities, take some new form of existence, by which they may appear to be annihilated; and this we know is the process of nature, by which all material things are subjected to decay and renewal; but there is no such law governing space or magnitude in the abstract, which is subject to no decay or renewal. Therefore it is evident that in *material things* magnitudes are *not infinitely divisible* in the fullest sense of the term Infinity. I would, therefore, define the term infinity in its application to material things (of which alone we are cognizant) in a limited sense, and say that an infinity is one ultimate particle of whatever material or thing we are considering, such that it cannot be divided again without resolving itself into its original elements, and therefore such that it *cannot be less.*

This, I think, is the only comprehensible meaning which can be given to the term "infinity" in its relation to matter, and if we reason from things which are *incomprehensible*, we reason of things which we know nothing about, and must fall into error. If we apply the term infinity in its fullest sense to material things, it will result, that a drop of water may be divided just as many times as a square foot or an ocean of water, and we shall have one infinity greater than another infinity of the same thing, which is absurd. But if we apply the term as I have defined it, then no infinity *can exist* which is greater than another infinity of the same thing, and a most important truth is brought within comprehensible limits.

I have made these remarks concerning infinity, as being applicable to the propositions which follow, and to

the *difference* between a line coinciding with the greatest limit of the area of any circle, and a line *inclosing* the same circle, which is infinity, in the sense in which I have defined it.

I shall here lay down as axioms certain truths which have been proved.

First. The circumference of a circle is a line outside of the circle thoroughly inclosing it, and of itself forms no part of the area of the circle. (Prop. V., chap. ii.)

Second. The line approximated by geometers, if it could be correctly determined, is a line coinciding with the greatest limit of the area of the circle, but not inclosing it. (Prop. I., Appendix.)

Third. The line approximated by geometers is consequently the circumference of a circle whose diameter is less than one in its relative value to the area of a circle. (Prop. I., III., and IV., chap. i., and prop. I., Appendix.)

Fourth. The difference between a line coinciding with the greatest limit of the area of any circle and a line inclosing the same circle, is an infinity, such that it cannot be less. (Prop. II., Appendix.)

Fifth. In material things an infinity equals one ultimate particle of whatever material or thing is under consideration, such that it cannot be less. (Prop. II., Appendix.)

Sixth. An infinity is a value, such that it is always capable of increase. (Prop. II., Appendix.)

I now propose to show, that by the method of geometers, the omission of the difference between the radius of a line *coinciding* with the greatest limit of the area of any circle and the radius of a line *inclosing* the same circle, being an infinity, the value of such infinity is increased in the process of bisection, so that it shall always equal one or more in the sixth decimal place at some great number of sides of a polygon; and may be increased, so that it shall equal circumference itself.

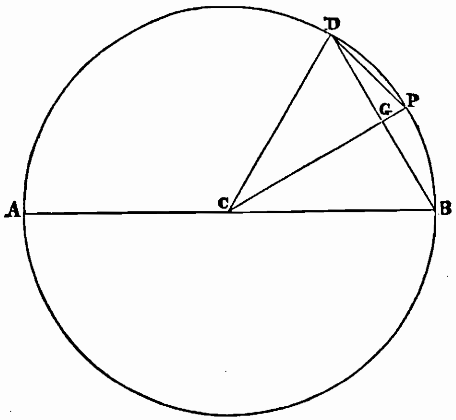
It will be seen that if the radius of the inscribed line, or line coinciding with the greatest limit of the area of the circle, shall equal *finity*, then because the difference between the inscribed and circumscribed lines equals *infinity*, therefore the radius of the circumscribed line equals infinity added to finity. Therefore let finity=F, and let infinity added to finity=I F.*

PROPOSITION III.

The value of that infinity which is the difference between the inscribed and circumscribed lines (axiom 4th), and which is omitted by geometers, is increased in the process of bisection of a circumference, so that at some great number of sides of a polygon it will always equal one or more in the sixth decimal place, and may be increased, until it shall equal circumference itself.

I now take Playfair's eighth proposition, first book, of the Supplement to the Elements of Geometry, in which he bisects a circumference to 6144 sides of a polygon.

PLATE XXX.



Let the radius C B (Plate XXX.) equal *one* of thirteen decimal places, that is, C B = 1.0000000000000, then let C G equal the perpendicular on the chord of the arch of 6144 sides, then C G = .9 9 9 9 998692723+ then $C\,B - C\,G = G\,P$, and $G\,P \times A\,B = D\,P^2$ and $\sqrt{D\,P^2} = D\,P$, which is the chord of the arch of 12288 sides. Now let F (finity) equal radius=C B=1.0000000000000, then $F - C\,G = G\,P$, and G P is seen to equal 1307277— as follows:

* In examining this proposition, we cannot do better than to place a glass of water before us, and supposing the tumbler to be a perfect cylinder, let us then suppose the radius of the greatest possible line coinciding with the water=F, and the radius of the least possible line coinciding with the interior of the tumbler=I F, which last is the circumference of the circle.

$$F = 1.0000000000000$$
$$C\,G = .9999998692723+$$
$$G\,P = \quad 1307277-$$

Now let an infinity, such as I have defined it, equal one in the thirteenth decimal place, than I F (infinity added to finity) =1.0000000000001.—then I F—C G=G P and G P is seen to equal 1307278—as follows:

$$I\,F = 1.0000000000001$$
$$C\,G = .9999998692723+$$
$$G\,P = \quad 1307278-$$

It is seen that while radius (F and I F) are lines of *fourteen figures*, or a left hand unit and *thirteen* decimal places, G P is a line of only *seven figures*, or a left hand unit and *six* decimal places, and G P=I F—C G is seen to be greater by one in the *sixth* decimal place than G P =F—C G, and because G P×A B=D P² and √D P²= D P, and D P×12288 (the number of sides of the polygon) gives the circumference, therefore G P=I F—C G will give a circumference greater in the *sixth decimal place* than G P=F—C G, viz., G P=F—C G×A B at 12288 sides of a polygon will give a circumference=3.1415925+, which is less than geometer's ratio, and G P =I F—G G×A B will give 3.1415948+which is *greater* than *my* ratio. It is evident, therefore, that if that infinity which is equal to the *essential difference* in the properties of straight lines and curved lines, and which is

consequently equal to the *difference* between a line coinciding with the greatest limit of the area of any circle, and a line *inclosing* the same circle, shall equal one in the thirteenth decimal place of any line of figures, the omission of the value of that infinity, will, in the process of bisection, to 12288 sides of a polygon, be an error in the sixth decimal place of circumference.

Again, let F (finity) equal one of 16 decimal places or 1.0000000000000000, and let infinity equal one at the sixteenth decimal place, then I F (infinity added to finity)=1.0000000000000001. Now let the circumference be bisected fifteen times, this will give an inscribed polygon of 196608 sides, then let C G equal the perpendicular on the chord of the arch of 196608 sides of an inscribed polygon, then C G=.9999999998723363+and F —C G is again seen to be a line of only seven figures, or a unit with six decimal places, and I F—C G is also again seen to be greater by one in the sixth decimal place than F—C G, and at 393216 sides of a polygon, an infinity which had a value of only one in the sixteenth decimal place, is increased in value in the process of bisection, so that it becomes one or more in the sixth decimal place of circumference. The error of geometers is here *palpable* —it does not even require the calculation to be made in order to demonstrate it,—the number of figures left when C G is deducted from F is alone sufficient to show at what point the perimeter of the polygon found, is less than the true circumference of the circle; and it is perfectly obvious, that this error arises from mechanical causes perceptible to our senses; and from an inherent property of numbers which cannot be obviated by any method of geometers. It is not a defect or discrepancy of numbers, but it is the perfection of their power, and is easily understood to have its origin in the essential difference in the properties of straight lines and curved lines. It is perfectly obvious also, from the examples given, that if we could go on with the process of bisection until the perpendicular on the chord of the arch should equal radius, the error arising from an infinity omitted at the first, would then equal circumference itself. It is equally obvious, that at whatever point in a line of figures we may place the value of infinity by hypothesis, whether at the sixteenth, twenty-fifth, fiftieth or even at 1000 decimal places from unit, by the process of bisection the value of such infinity will be increased, so that at some great number of sides it will equal one in the sixth decimal place, and finally equal circumference itself. I do not say that one in the sixteenth, twenty-fifth or any other particular decimal place *is an infinity;* but if a little estimate be made of its value, we may form some conception, whether it may, or may not, be an infinity, according to my definition of the term.

A difference which equals one in the sixteenth decimal place, is such, that if the magnitudes be miles, it is less than one hair's breadth in the distance from our earth to the sun! and it is less than the *four-thousandth* part of one hair's breadth in the circumference of our earth! Whether such a difference is equal to an infinity in the surface of a glass of water (to which I have requested reference in a note to this proposition; also in a note to proposition v., chapter ii.), or, in other words, equal to one particle of water in its least possible natural division, and hence equal to the difference between the *greatest* possible line coinciding with the water, and the *least* possible line coinciding with the interior of the tumbler inclosing the water, I leave to the decision of those whose perceptions and judgments are more acute than mine.

The principle shown in the examples given, is a clear one, and proves conclusively, that if the ocean were spread out in a circle, and the difference between a line *coinciding* with its utmost limit, and a line *inclosing* it, should be of the value of one particle of water in its least possible natural division, the omission of that value would, in the process of bisection by geometers' method, at some great number of sides of a polygon, become of the value of one or more in the sixth decimal place of circumference, and finally equal circumference itself.

The proposition is therefore demonstrated.

Having completely demonstrated in the six propositions of chapter i., that there is an essential difference in the properties of straight lines and curved lines, which has been entirely overlooked by geometers,—having proved, also, in the fifth proposition, chapter ii., that the circumference of a circle is a line *outside* of the circle, thoroughly inclosing it,—in the first proposition of this Appendix, that the line approximated by geometers is a line *coinciding* with the greatest limit of the area of the circle, but *not inclosing* it,—and by the last proposition, that the method of geometers is such, that by the difference in value between these two lines, they are liable to an error in the sixth decimal place (the point at which their ratio differs from mine): It would seem as if all these demonstrations were sufficiently conclusive, and that no professor of mathematics would, hereafter, object to my ratio of circumference and diameter, on the ground that "it differs from their approximation, in the sixth "decimal place," unless he can first disprove all these demonstrations, by some other means than by assuming to be true, just what has here been proved to be false, a very common way of repelling truth, and then dignifying it with the name of argument. But I have no idea that professors will so easily surrender the point. It is a part of human nature, that men who are joined to their idols will never let them go,—neither will *they*.

As I have said in the introduction to this Appendix, "if "driven from one principle, they will fall back upon "another; and if beaten from all, they will return again "to the first;" and all we can do, is, to reduce them to this necessity, and there leave them.

In the last demonstration, I have introduced, in a note, for illustration, the natural lines which are seen in a glass of water, to show the difference between a line *coinciding* with, and a line *inclosing* the circle. It will be asserted, no doubt, as an objection to the truth shown in the last demonstration, that these two lines (the line of the water, and the line of the tumbler) meet at a line without breadth, which is the common boundary of both, and which they hence assume to be the common measure of both; in other words, that because the water and the tumbler are said to touch each other, that they are therefore equal. We must, therefore, anticipate their resort, and take away the subterfuge, before they turn to it. If they would make this objection, and abide by the principles it involves, it would be quite sufficient for my purpose, for thereby, they would admit that the circumference of the circle is a line *outside* of it, and it is then easily shown, that the difference between this line, and that which they measure, is that infinity for which I contend, and the omission of its value lays them liable to an error in the sixth decimal place. But they will not abide in argument, even by the principles of their own objections, because it is not their purpose to find the truth, but only to object, lest, in finding the truth, *they* should be found in error. I shall, therefore, treat the subject on its true merits, and show that no curved line, or line of circumference, can be, at the same time, the common boundary, and the common measure, of its two sides.

Before proceeding to demonstrate this principle, I shall here again lay down as axioms, certain truths, which have been proved, or are self-evident.

First. Space is infinitely divisible. (Proposition II., Appendix.)

Second. Any imaginary line (not a material line), which shall have breadth, is equal to the same portion of space.

Third. Any such imaginary line is, therefore, infinitely divisible.

Fourth. Any such imaginary line may, therefore, be divided, until each part or division is less than any magnitude which is, or can be, developed to our senses.*

* Theoretic men have a science, which they call the "science of vanishing quantities," and which, I think, has some affinity to this idea of the existence of magnitude in the abstract, beyond the means of development by any magnifying power. I know nothing about it,—I only know that they have such a science, by which they assume to prove that a thing may be annihilated, and yet continue to exist. I can well nnderstand how abstract space or magnitude may be diminished, until it is entirely beyond our perceptions by any aid which we can control. And I can well understand how a developed magnitude may, by some process of nature, become so divided, as to return to its original elements; or, by forming some new affinities, pass to some new form of existence, and be seemingly destroyed. But I cannot understand how a thing can exist, and yet not exist at the same time. The thought appears to me to have its origin in the same class of abstract absurdities, which calls an infinity "nothing," and then, as occasion may require, seeks to prove that nothing equals something. If asked my opinion, what is the limit of developed magnitudes, I would answer, that, so far as I am able to reason, without much reflection on the subject, the atmosphere we breathe, seems to be the boundary line. Being the medium of light which discloses magnitude, it cannot, itself, be developed to the senses. We can feel its mass, by its own motion, or its resistance to motion; but we can neither see, nor feel, its separate particles, though convinced of its existence in that form, by the evidence of its motion. I should infer, from these facts, that particles less than those of the atmosphere, cannot be made perceptible to our senses; and that particles greater than those of the atmosphere, are within the scope of possible development.

Fifth. At whatever point the division of such a line may be arrested, because the sum of all the parts is equal to the whole; therefore, each part must have breadth, though the breadth of each part may be such, that no conceivable number of them would form a developed magnitude.

Sixth. One line cannot occupy two places at the same time; neither can two lines be in one and the same place, at the same time.

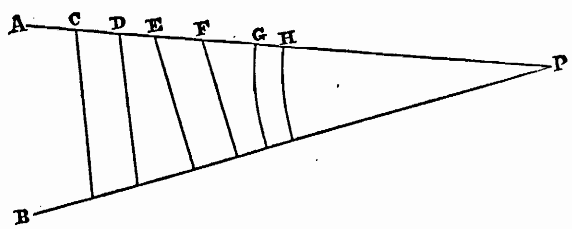
Seventh. Two lines without breadth, cannot exist with no breadth between them.

Eighth. The existence of shape signifies limit; hence, no shape can exist without a boundary line definitely located, which forms no part of the shape itself, which boundary is its circumference.

PROPOSITION IV.

No two lines lying in the same plane, parallel to each other, and between two other straight lines, which are at an angle to each other, can possibly coincide, and be equal, except they shall become one and the same line.

PLATE XXXI.



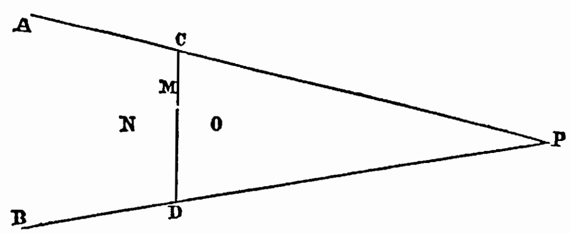
Now, let A and B (Plate XXXI.) be two straight lines, at an angle to each other. They are seen to meet at the point P, therefore, P may be supposed to be the center of a circle. Now, let C and D be two other lines parallel to each other, lying in the same plane, between A and B, and at any angle to A or B. It is seen that C and D are not equal; that which is farthest from the center, P (C), being greatest, and that which is nearest the center, P (D), being least; and because one is greater than the other, therefore, if brought together, they cannot *coincide*, but *in part;* and if either C or D be divided

through the center, lengthwise, in halves, the halves cannot coincide; and if they be divided thus, to infinity, no one part of either C or D can coincide with any other part, because no one part of such division is equal to any other part,—that which is farthest from the center P being greatest, and that which is nearest the center P being least. So also with E and F, or G and H, and if either be divided to infinity, none of the parts so divided can coincide with any other part, because no one part of such division is equal to any other part, but each part coincides with, and is equal to itself *only*. *The proposition is therefore demonstrated.*

PROPOSITION V.

All lines which have a fixed and definite locality must have breadth, whether they be lines of circumference, or lines of division.

PLATE XXXII.



Let A and B (Plate XXXII.) be two straight lines at an angle to each other as in the fourth proposition, and meeting at the point P, which is supposed to be the center of a circle. Let A and B form two sides of a developed magnitude equal to M, and let N and O be two divisions of that magnitude. Now let an imaginary line supposed to be without breadth fall on M dividing it into two parts as at C D. It is known that every part of M which is nearer to the center P than such imaginary line, will fall on the inside (O), and every part of M which is farther from the center P than such imaginary line will fall on the outside (N), therefore (proposition IV.), every part of N parallel to C D is greater than any part of O parallel to C D. And because every part of N is farther from the center P than any part of O, therefore the line C D being a line of division has breadth, however we may have imagined it, though such breadth may be a less portion of space than any separate existing portion of the material or thing divided. *The proposition is therefore demonstrated.*

From the demonstration of the last two propositions and their preceding axioms these necessary deductions follow:

First. In the illustration of the glass of water, the water and the tumbler being two materials which cannot mingle, they therefore occupy two places, and (axiom 6th) "one line cannot occupy two places at the same "time," therefore the lines coinciding with each (the water and the tumbler) are not one and the same line. And (axiom 7th) "two lines without breadth cannot

"exist with no breadth between them," therefore the line between the water and the tumbler *has breadth*, though such breadth is evidently less then one particle of water in its least possible natural division (see Note to prop. v., chap. ii.).

Second. *Every* part of the tumbler is farther from the center than *any* part of the water, hence (prop. iv.) the line of the tumbler is greater than the line of the water, therefore a line between them being greater than the inside and less than the outside, *cannot* be the *common measure* of both its sides,—it must be assumed as the measure of one or the other of its sides, but not of both, and in this particular it differs essentially from a straight line. It is the common boundary of both its sides only, as it limits *extension from* the center outward, and diminution *towards the center* inward, which are opposite qualities. From the mere fact of its being the common boundary, it cannot therefore be the *common measure of extension* on both its sides.

Third. The line between the water and the tumbler being a line of division which separates all parts of the water from all parts of the tumbler, leaving one wholly inside and the other wholly outside, therefore (proposition v.) that line has breadth, and being defined and considered as having breadth, therefore when we say that the circumference of the water is the least possible line of the tumbler, we mean, of course, that line which limits the *diminution* of the interior of the tumbler *toward* the center, and which limits the *extension* of the surface of the water *from* the center, and is the common *boundary* of both, but not the common measure of *extension* in both. But if the line between the water and the tumbler be defined and considered as having *no breadth*, then the line which would limit the extension of the water would be a line *coinciding* with the tumbler and not interior to it, and the line which would limit the diminution of the tumbler toward the center would be a line coinciding with the water and not outside of it. And because the circumference of a circle is a line outside of the circle thoroughly inclosing it (prop. v., chap. ii.), therefore if the definition of a line shall be that it has *no breadth*, then the circumference of the water is a line coinciding with a part of the tumbler. But if the definition of a line shall be such that all lines which are lines of division and have a fixed and defined locality *have breadth*, though such breadth may be infinitely diminished (axioms 3d, 4th, and 5th), then the line between the water and the tumbler, having breadth, and being wholly outside of the water, is the circumference of the circle, and it is evident (axiom 8th) that no circle can exist in nature which has not a fixed boundary or line of circumference which separates it from all surrounding

things. These differences which grow out of the definition of a line require to be carefully considered.

Fourth. It is self-evident that *contact* is not *union*, and it is a mathematical error to consider them as *one*. Therefore if two magnitudes be separated by any given line or space between them, and if they are then brought together until they are in contact or seem to touch each other, the line or space between them is only divided or diminished—it is not annihilated. And because space is infinitely divisible (axiom 1st and prop. II.), therefore it may be infinitely diminished, but it can never be annihilated unless the two sides shall become one, which is impossible. They may approach so closely as to exclude all other matter and cohesion may take place by affinity of the parts, but the lines of contact or cohesion are still in existence, terminating the boundaries of the two sides, and hence there is space between them, and if not, then space is *not infinitely* divisible. Hence the water and the tumbler being wholly distinct and unmingled there is space or breadth of line between them, though such breadth is evidently diminished until it is less than one particle of water in its least possible natural division.

The foregoing demonstrations and the deductions from them, naturally lead to some few remarks respecting the properties of numbers, and of magnitudes, with the mathematical definitions of a line, and a point, and their application in geometry.

I will not undertake to say, that because it is proved that all lines having a fixed and definite locality have breadth also, that under the circumstances, it is absolutely necessary in *all cases* to change the mathematical definition of a line or a point; but I do say, that to regard these definitions, in their abstract, arbitrary, and limited signification as essential to truth, is *wholly unnecessary.*

Nature will work in her own way in spite of our definitions, and if in geometry *we* work according to nature's truth, the result will be correct, whether we define lines as having breadth or no breadth.

The science of mathematics in the most universal sense in which the term can be applied, is, I think, the science of numbers, which have infinite capacity, both as to notation and enumeration.

Algebra means *nothing*, or it may mean either one thing or another, as suits the fancy of him who works in it, until its results are reduced to numbers; and then there is no value in the universe, either simple, compound, or relative, whi.h some notation, enumeration, or relative fraction of numbers is not capable of expressing. A little examination into the character of numbers, will, I think, be sufficient to convince us of this truth. And

what are numbers? Have we any definite ideas of their nature, capacities, origin, or end? Are they a creation of God, existing by his power independent of other things? or are they simply the result of a previous necessity which the order of creation only fulfills, and which hence follow as a mere consequence of the perceptions with which the Creator has endowed our minds? I am compelled to think the latter, and hence whether in the material or immaterial world, to our perceptions, numbers and existences are identical and inseparable. Before creation began, then, numbers had no existence, except in the infinite eternal ONE. But when the first particle of original matter was brought into being it constituted the *finite one*, and the second particle *two*, and finite and progressive numbers then had their beginning, and have ever since, with every succeeding production, been moving forward towards the INFINITE. But our notation of decimal numbers, is only one of the forms which nature employs for herself; and it is because they *are* one of the forms which nature employs for herself, which gives them their power, accuracy and clearness; hence decimal numbers will be found to have an important part in the order of created things. But nature also employs other forms or notations of numbers besides decimals, and whenever she does so, decimal numbers are not accurate, or rather they have not the power to tell us the exact truth without the loss of some fraction or remainder.

We know very well that all notations of numbers must have their beginning in *one*, which is less, and their end in *one*, which is greater; thus when we say "one "thousand," we simply mean a thousand ones, or if we consider it in its unity, we mean *one*, a thousand times greater than the original one. Hence it is plainly seen that when the first original particles of matter were collected together and moulded into shape,—when our earth assumed her opaque body, and when the first star sprang into existence, it was nothing else but a following out of the progression of numbers from an original creation of *one* less, to the accumulation of *one* greater. At least such is the only understanding which our perceptions can give us of the order of production in created things: and it appears to me, that in material things, *truth* is nothing more than the perfect agreement between nature and our perceptions, and error is their disagreement. Hence if we had been differently endowed, natural truth might have been to us quite another thing, and to our minds numbers might not have been what they now are.

If then this agreement between the order of production in nature and our natural perceptions *be truth*, then it is self-evident, that there exists not in the wide

universe, a single particle of matter, or combination of particles into shapes or things, which numbers have not once told out with unerring exactness; and what numbers have done once, they can do again. It is evident, therefore, that in the material world, numbers and things, that is, shapes and magnitudes, are identical and inseparable, and we cannot comprehend one without the other, therefore, *numbers* are the legitimate medium of determining mathematical truth, and no solution can be complete until it is reduced to numbers. It is an error, therefore, of mathematicians, to give to numbers the inferior place in mathematical science.

The science of geometry, as a part of mathematics, is generally defined as the science of measure, or of quantity; but I think this definition is too limited, and that it may, with greater justice, be termed the science of perfect mechanics, by which all forms and proportions are *produced*, as well as measured, and their relative properties and values determined. If, for example, we would make any shape or form out of brass or other material, in order that its proportions may be as nearly accurate as possible, we must first *produce it* geometrically, and then make one from brass, as near like the geometrical form produced, as we can. The first is, then, in principle, a *perfect* form, according to nature's working; but the second is only an imitation, and is imperfect, by reason of our inability to measure or detect so small a quantity of matter as one original particle, which is the only perfect standard of measure; and if the form of brass shall differ from the true geometrical form but one particle, it is imperfect. The first is an operation of geometry, and is nature's work; the second is a mechanical operation, and is the work of the artisan. The first is the development of the pure principles which govern the form; the second is only the labor of the workman with his tools. If, therefore, we separate geometry from mechanics, we leave the latter without a science, and degrade the mechanic arts to the character of a blind imitation, without rule or principle. The really scientific mechanic is one who constructs his work according to geometrical principles, and the only difference between his work and his principles is, that his principles are perfect, being according to nature's laws, but his work is imperfect, from lack of skill to make it more perfect. The power of mechanics is altogether constructive, it is not creative; she can fashion things, but she cannot make them,—we must first furnish her with materials, and she will then mould them; and being furnished with the necessary parts of things, she can put them together. So, also, with geometry; she can create nothing,—all her powers are constructive, only; she finds all her proportions in magnitudes and forms, and we have seen that

numbers and magnitudes, or forms, are inseparable. These, then (numbers and magnitudes), are the materials of geometry, and until she is furnished with these, she can do nothing. Let it be required of geometry to produce a hexagon, and she requires you to furnish her with six equilateral triangles, and placing them together, shows you a hexagon; and in producing this hexagon, geometry has done no more than mechanically to construct a form out of other forms, and built up a magnitude out of lesser magnitudes; and this is the limit of her power, because she can create nothing. If you now require to know the value of this hexagon, in proportion to other shapes, decimal numbers will tell you that each side of the hexagon being one, its value in proportion to the equilateral triangle of an equal side is *six*, and in proportion to the square, it is the square root of 6.75. If, in this last, you wish for any more definite expression than square root, decimal numbers will tell you to go and acquaint yourself with some other notation, besides decimals, which can give the needed fraction, or otherwise to sit down in ignorance. Now, let it be required of geometry to produce a form without the use of magnitudes, and she tells you that she can do no such thing, that form and magnitude are inseparable ideas,—that all shapes and forms have both extension and limit, and are, therefore, finite,—that she deals in finite magnitudes, and nothing else,—that abstract space is *infinite*, and if she ever considers space, she considers it only in finite portions, and in reference to some developed magnitude having the same boundaries; hence, if geometry be required to measure any portion of space, she demands to be furnished with some magnitude as a standard of measure, or otherwise she cannot do it.

Let it be required of geometry to give a fixed and definite locality to a line without breadth, or to a point in space, without magnitude, and she will tell you that locality, place, and position, are all of the same import, and all of them mean a *portion of space*,—that there can be no division of space equal nothing (see page 95); and, therefore, there can be no locality, place or position, without magnitude; and hence, your line without breadth, and position without magnitude, are fallacies, both in nature and mechanics, and, therefore, beyond her power.

Ask geometry to *measure a form* by a line without breadth, and, if you please, let the form be a hexagon of uncertain size. Geometry at once answers you, that a line without breadth has no existence, and if you furnish her with no other materials she cannot do it,—that magnitude is only magnitude by comparison with some standard of measure, and hence things can only be measured by comparison with other things, and, therefore, it

is out of her power to measure something, by comparing it with nothing. Even a standard of measure (an artificial standard) is without meaning, only, as it refers to some other standard; as, for example, a carpenter's foot-rule is without expression as to quantity, only as we mentally refer the rule to the foot. When we measure a thing, therefore, we do nothing but simply determine its quantity, in its relation to some other quantity already determined; how, then, can relative quantity be determined, by comparing any quantity with itself, or by comparing it with no quantity? But furnish geometry with a positive magnitude, which shall be her standard of measure, and which she can apply to any part of the hexagon, and then knowing what the form is, numbers will perform the rest and you may imagine your line to be just what you please—as having breadth, or no breadth, it is all the same to geometry. Having determined the extension of the form in one direction, by a positive magnitude, or line with breadth, and knowing what the form is, geometry can now determine its extension in all directions, from any point, and the result will be correct, and geometry is satisfied; but without the help of this positive magnitude, you could never have known anything of the value of the hexagon.

And now if there be anything in the world to personate common sense, I would ask, has geometry measured this hexagon by a line with breadth or by one without breadth? Certainly by one *with breadth*, will be the answer, and by no other means did geometry ever yet measure anything! It seems therefore that geometry can do nothing, with a line *without* breadth independently and alone,—you must first furnish her with a line *with breadth* with which to measure the imaginary one *without breadth*, before she can proceed one step in her work towards determining the quantity of any figure. In proposition v. (this Appendix) I have shown that in dividing any material or developed magnitude, geometry does it by a line *with breadth*, and it has been shown, also, that such a line may exist, and yet be less than any portion of the material magnitude divided. Let us now see what sort of a line geometers do actually use in dividing a magnitude. Let the magnitude to be divided be a plate of gold one inch square, and to be divided equally. We know that gold is constituted of original particles, because it can be dissolved or dispersed and its particles collected together again and deposited in a new place, the particles cohering as before; therefore let the plate to be divided be of the imaginary thickness of one original particle of gold. The principles of geometry fix the line *where* it is to be divided with perfect accuracy, so that just as many particles shall lie on one side of the line as on the other. But the geometer not

being able to locate, to perceive, or to understand the exact place of his imaginary line *without breadth*, in order to aid this deficiency of his perceptions, he draws a line *with breadth* across the face of the plate of gold at the points indicated by geometry; and knowing that this line by its breadth covers a portion of the gold, he proceeds mentally to diminish the line towards the center, until its breadth is *less* than any portion of the gold, or, in other words, until its breadth is equal to the lines of cohesion which unite the particles of gold. And the magnitude or plate of gold is thus mentally divided, without the loss of any portion of its quantity; and because the line of contact or cohesion of the particles of gold has two sides, leaving all the particles of one half on one side of it, and all the particles of the other half on the other side, it therefore has breadth, though such breadth is diminished, until it is less than one particle of gold, and less than any of our perceptions of quantity. The particles of gold do not, by their cohesion, unite, to become one and the same,—they simply approach each other within a distance, such as to exclude all grosser matter, while each particle of gold remains as before distinct from the other.

And now I think that no geometer who is capable of examining the nature and extent of his own perceptions, will tell me, that in dividing any magnitude geometrically, he has ever made use of a line mentally or physically, in any other way than just as I have described above; and if so, then no geometer has ever yet (except in name) used a line without breadth in dividing any magnitude. As an evidence of the truth of this conclusion, we need only make the attempt, mentally, to fix the position of a point without magnitude, or the locality of a line without breadth; as we diminish toward the center, our very thought expires with the expiring magnitude, and we have neither recollection nor comprehension of its exact place.

I have, in another place, explained natural truth to be nothing more than the agreement between nature and our perceptions, and error as their disagreement. Now it appears to me that in the application of lines and magnitudes to geometry, the explanation which I have given of their use, forms a perfect agreement between the operations of nature, and the perceptions with which nature has endowed us; and if so, then it is true; and the ideas of lines without breadth, and position without magnitude, are misnomers,—mere illusions of the imagination,—altogether unnecessary,—wholly without use, and in opposition to all natural truth and evidence.

The mathematical definition of a line,—"that which "has length but not breadth," at first strikes the mind as an absurdity, because it implies quantity of one kind, and

yet it has no existence. But on examining it with the application made of it by geometers, it is found to mean *mere distance* from one point to another, and where nothing but mere distance is intended, it is wholly immaterial to the result whether the line shall have breadth or not. We may consider it an inch wide, or a foot wide, or as broad as it is long, and no difference will follow in the result. The definition is therefore exceedingly imperfect, even if true. But it is *not true* in nature, and therefore *not true* in geometry, because nature is the perfection of geometry. Lines in their practical application in nature, and consequently in geometry, have a more enlarged use and meaning than mere distance. They in reality constitute the divisions of all magnitudes, and the boundaries of all shapes,—offices which mere distance is incapable of performing. We have seen that shape or form (which is identical with magnitude) is essential to the first principles of geometry. We have seen also that no shape can have any positive existence without *limit*, and a boundary clearly defined, definitely located, and separating it from all surrounding things. And the proposition is self-evident that lines which perform these offices *must have a positive existence*, and therefore *must have breadth*,—therefore the circumference of a circle *must have breadth.*

A definition does not necessarily form any part of mathematical truth. It is only a part of the method by which truth is determined, and is not always essential even to this; as we have already seen that in certain circumstances, it is entirely immaterial to the result what the definition may be. The chief object of a definition is to enable the mind to comprehend a truth when it is determined, rather than to constitute any part of the truth itself. A definition must be conformable to truth in the circumstances in which it is used. Hence the definition of a line without breadth, being conformable to truth in such circumstances, is a good definition in all those cases where distance only is intended, in which it is entirely immaterial what may be the breadth of a line or whether it has any at all. But in all those cases in which a line *with* breadth differs in value from one without breadth (which it always does in a continuous line of circumference), the mathematical definition is *not conformable* to the truth of nature, and therefore leads to error.

I do not say, therefore, that it is absolutely necessary, *in all cases*, to change the mathematical definition of a line, but I do say that it is wholly unnecessary to consider the definition as a mathematical truth, which is fixed and unalterable, and that *it is* absolutely necessary to consider the circumstances in which a line is used, and to modify or change the definition in all those cases in

which a line with breadth differs in value from one without breadth, as in the circumference of a circle. In such cases, I would define a line as that which has length and the least possible breadth with locality, and I would define a point as that which has position and the least possible magnitude with locality.

It will be seen, however, by all who examine the whole course of my reasoning attentively, that lines having breadth, is *not a condition* on which the Quadrature of the Circle is based, but only a *conclusion* to which the developments as they proceed inevitably lead us. The twelve propositions of chapter ii., which embrace the whole demonstration of the Quadrature, rest entirely on the relative properties of shapes, and of straight lines to curved lines, and *are wholly independent of the fact* whether lines have breadth or not. If, therefore, any one should choose for any reason to reject the idea of lines having breadth and points having magnitude, it cannot in any wise affect the truth of these demonstrations or the main object of this publication.

We have seen, in the course of the foregoing reasoning, that *shape* or *form* is essential, both to the *principles* of geometry, and to the *practice* of geometry, and that form and magnitude are identical and inseparable ideas. We have seen, also, that *numbers* and *things* (which, in the material world, are the same as magnitudes) are also inseparable. We have seen, also, that geometry, being the science of *quantity*, can only compare indefinite quantity with other quantities which are known, and definite. We have seen, also, that geometry—being, as she is, the basis of pure mechanics—when she is required to *produce* a *form*, she does it by using, or putting together, *other forms*, just as we build a ship, or a house; with this difference, only, that all her forms are perfect, but ours, from lack of skill, are imperfect. And when geometry is required to produce a magnitude, she does it with the use of lesser magnitudes, which are her standards of measure, and without the use of both form and magnitude, it is not in the power of geometry, either to produce anything, or to measure anything; she positively refuses to proceed one step, either in the development, or measurement, of any form, until she is furnished with a positive magnitude, as a standard of measure, which she can consider, and treat, as *one;* and no abstract solution of any geometrical problem has any meaning in it, until the result is compared with some known quantity of definite form, as a standard of measure. We have seen, also, that the power of geometry is limited to finite things, or things having *limit* or *boundary*,—that she considers space, only by comparing it with finite and definite magnitudes; that *infinite space*, or *infinite magnitude*, is entirely beyond her reach or

comprehension; and that all abstractions are necessarily infinities.

These conclusions form a perfect agreement between the operations or developments of nature, such as we can understand, and the perceptions and powers of comprehension, which nature has given *us*, and *they* are, *therefore, true*. What then becomes of the idea of the existence of geometry as an abstract and metaphysical science? In my opinion, the notion of abstractions in geometry sinks at once to a level in value with the chimeras of a sick man's brain; they are mere illusions, floating in the mind of the operator, without identifying themselves with the practical truths of which they are only the mental images, and, I believe, were never thought of by geometers, until the introduction of algebra, and its abstract mode of reasoning, which is without use or meaning, until it is brought down to the standard of some definite form and magnitude. I cannot, therefore, do otherwise than conclude, that geometry has to do with *nothing but the relations of physical or material things*, and is, therefore, purely *a physical science*.

And since all that is known in geometry includes but a very small portion of either the general principles, or individual truths, which govern the relations of things to one another, it is self-evident, that the science, as practiced, is open to improvement, and capable of progress, just as much as the mechanic arts, the study of chemistry, or anything else; and notwithstanding its many conveniences, I regard the use of algebra in geometrical demonstration, as deserving no higher character than that of an ingenious invention to supply the lack of a knowledge of numbers; and I regard the consideration of geometry as an *abstract* and *metaphysical* science, as a sort of *ignis fatuus* light, which, by blinding the eyes of the beholder, renders more obscure everything else around him, and which, from its first introduction, until now, has been the chief bar to a rapid progress in the science, which may be realized the moment this idea of abstractions is dismissed from our reason.

The opposite duplicate ratio of the equilateral triangle and the circle.

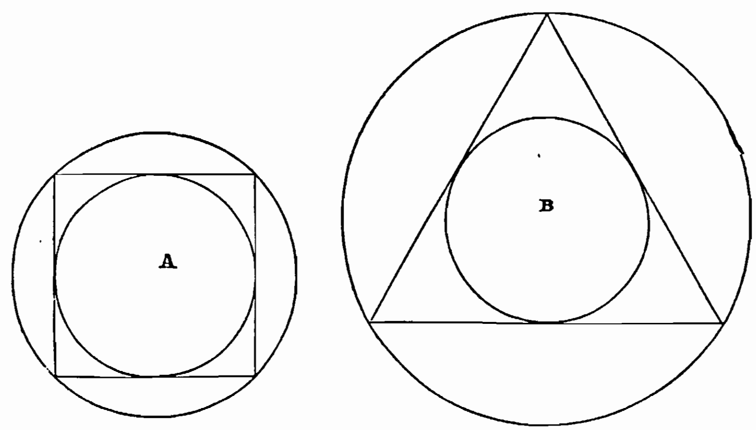
Duplicate ratio is a universal property of area, hence, the square being the standard of value, therefore, the square of diameter is duplicate ratio, and all superficial magnitudes, of any shape, are to each other of the same shape, in ratio of area, as the squares of their diameters, or in *duplicate* ratio. Explained in its simplest and most practical form, duplicate ratio of area means only, that

the increase of area in any shape is in duplicate ratio to the increase of circumference and diameter; that is to say, that area quadruples as often as circumference and diameter double; hence, because the equilateral triangle and the circle are, in their relations to the square, in the *opposite* duplicate ratio to one another (propositions VIII. and IX., chapter ii.); therefore—

PROPOSITION VI.

The circle inscribed and circumscribed about an equilateral triangle, is in duplicate ratio to the circle inscribed and circumscribed about a square.

PLATE XXXIII.



Let the areas of the inscribed circles A and B equal one another; then the diameters of A and B are also equal. Now, let the side of the square circumscribing A equal one, then the diameter of A = 1, and because the diameter of the circumscribed circle equals the diagonal of the square, therefore, the diameter of the circumscribed circle = $\sqrt{2}$, and because the areas of all shapes are to others of the same shape, in duplicate ratio, or as the squares of their diameters, therefore, the area of the circle circumscribed about the square, twice equals the area of the circle *inscribed* in the same square. Now, the diameters of A and B are equal to one another; hence, the diameter of B = 1. It is proved by geometry, that the perpendicular of the triangle circumscribing B equals the diameter of B, plus the radius of B; therefore, the radius of the circle circumscribing the triangle, twice equals the radius of B, and $2 \times 2 = 4$; therefore, the area of the circle circumscribed about the triangle, four times equals the area of the circle *inscribed* in the same triangle. *The proposition is therefore demonstrated.*

It is evident from the foregoing proposition and its demonstration, that the equilateral triangle and the circle are, in their relations to the square, in some form or other, in duplicate ratio to one another. In what form this duplicate ratio exists, remains to be proved, if not already sufficiently proved in prop. VIII. and IX., chap. ii. In proceeding to prove this, I will first state certain truths, which are self-evident, or have been definitely proved already.

First. Circumference and radius (and not the square of diameter) are the only natural and legitimate elements of area by which all regular shapes may be measered alike and made equal to one another. (Prop. VII., chap. ii.)

Second. The equilateral triangle and the circle are exactly opposite to one another in the elements of their construction, which are circumference and radius. (Prop. VIII. and IX., chap. ii.)

Third. The equilateral triangle is the primary of all shapes in nature formed of straight lines and of equal sides and angles (prop. VIII., chap. ii.), and has the least number of sides of any shape in nature formed of straight lines, and the circle is the ultimatum of nature in the extension of the number of sides. Therefore,

PROPOSITION VII.

In all *the elements of their construction which serve to increase or diminish area, the equilateral triangle and the circle are exactly* opposite *to one another in respect to the* greatest *and the* least *of any shapes in nature, and hence they are opposite to one another in ratio of the squares of their diameters, or in duplicate ratio.*

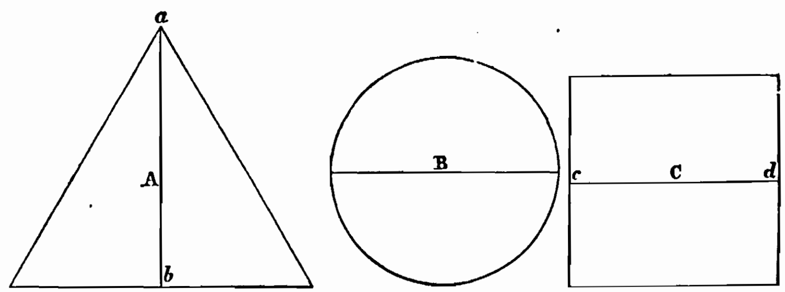
Now it is one of the plainest principles of geometry and arithmetic, that if G be greatest and L be least, then $G \times L$ and $L \times G$ are equal, because they are reciprocals.

Hence it would appear, on general principles, that the circle and the equilateral triangle should be equal, because one has the *greatest* possible radius and the *least* possible circumference of any regular shape in nature, and the other has the *least* possible radius and the greatest possible circumference of any regular shape, and half the circumference multiplied by radius are the only legitimate elements of area in each, by which they may be measured alike. But although this is a general principle in regard to *reciprocals in numbers*, yet it is only true in geometry when G and L are of corresponding or equal proportionate values. But in the relations of circumference and radius in respect to their relative values in area, *radius* is greatest, and *circumference* is least (radius=1, circumference =4), and because in the reversal of the order of *greatest* and *least* in the circle in its relation to the triangle, the greatest in relative value (radius) is also made greatest in relative magnitude, therefore the circle and the triangle are *not equal* as reciprocals of corresponding value are equal, but *opposites* in ratio, and the circle is to the triangle in its relation to the square as $G \times G$, and the triangle is to the circle in its relation to the square as $L \div L$, because $G \times G$ and $L \div L$ are in opposite ratio to $G \times L$ or $L \times G$, and hence the square, or square of diameter, being made the artificial basis of area, they are opposite to one another in ratio

of the squares of their diameters, or in the proportion of square and square root.

The term opposite signifies an intermediate, or a point, relative to which, the things spoken of are opposite to one another, and in this case the thing necessary to be known is, the point in numbers relative to which the circle and the triangle are opposite to one another in the proportion of the square and square root. Therefore let A be an equilateral triangle, B a circle, and C a square, each equal *one* in area.

PLATE XXXIV.



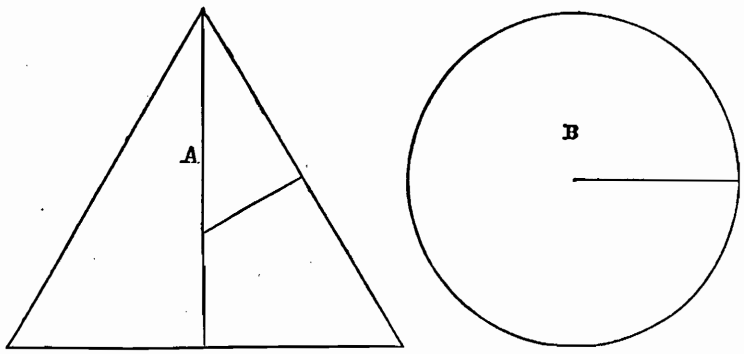
Now because A is a shape formed of straight lines and angles, therefore the value of $a\ b$ may be known, but because B is formed of curved lines, therefore (by hypothesis) none of the parts of B are known, hence the point in numbers relatively to which the two shapes are opposite to one another must be determined from A alone. It has been demonstrated (prop. IX., chap. ii.) that if C and B are equal in area, then the diameter of C ($c\ d$) in its fractional relation to B is in the opposite duplicate ratio to $a\ b$, and the area of C ($c\ d^2$) in its fractional relation to B, is in like opposite duplicate ratio to $a\ b^2$.

Now when the area of $A=1$, then $a\ b$ is found to equal 1.316074+and $a\ b$ expressed in decimal figures is seen to be an infinite fraction, therefore, $a\ b$ is an imperfect number, and all our ideas of fractions or magnitudes in order to be definite must be formed from whole and perfect numbers or units; for we cannot conceive of a fraction unless it shall have reference to some unit: therefore square $a\ b$, and $a\ b^2$ is seen to equal 1.7320508+ which is still an imperfect number, then square it again, and $a\ b^2 \times a\ b^2=3$, which is a whole and perfect number; therefore 3 is the first point in numbers from which the opposite ratio to $a\ b$ may be deduced in *whole and perfect numbers.* Now because $a\ b=\sqrt{3}$, therefore $c\ d$, being in the opposite duplicate ratio,$=3^2 \times 3^2$ viz., $3 \times 3= 9 \times 9=81$, and 81 is seen to be the smallest number which can be found which is in the opposite duplicate ratio to $a\ b$, when $a\ b$ is brought to a whole and perfect number; hence $81 \times 81=6561$, is the smallest whole and perfect number by which the fractional area of the circle and the square are equal to one another when the whole area equals one. (Prop. x. and xi., chap. ii.) The number 3 is therefore the point in numbers relatively to which the circle and the triangle, in their fractional rela-

tions to the square, are opposite to one another in duplicate ratio, or in the proportion of the square and square root.

The *opposite* ratio is simply a *necessity* resulting from a universal law of nature. The planets could not move forward in their courses without the action of opposite forces (centrifugal, centripetal). The north pole could not exist without an opposite or south pole. A shape or figure of one side cannot exist without another and opposite side, and no point can be fixed within the boundaries of nature which is not capable of opposite extension to the utmost limits of nature. In fine, no intermediate can possibly exist without its opposites. Hence the square being an intermediate shape and being made the standard of value, its opposites are the two extremes of nature in the production of shapes. And if we examine the whole subject in accordance with this universal law, it will be seen that in the production of shapes, the opposite duplicate ratio is a pre-existing necessity. It is self-evident that shapes, which are thus opposite to one another in ratio, must, in their relative construction to the square, form the two extremes of nature in respect to all their elements which serve to create, increase, or diminish area, and hence, that not more than two shapes *can exist* at the same time which are thus opposite one another. The whole course of development in this work shows conclusively that the equilateral triangle and the circle are the *only two shapes* in *existence* possessing the qualities necessary to render them opposite to one another in duplicate ratio, or as the squares of their diameters. The truth of their relation to one another in their relative value to the square by opposite duplicate ratio is therefore self-evident. The square being made *one*, it is an intermediate shape relatively to which the two extremes in nature are necessarily opposite to one another. Any other conclusion would, I believe, in the nature of things be an absurdity.

PLATE XXXV.



The reciprocals of numbers by which the parts of the circle and the triangle may be made numerically to equal one, are to be found by reversing the order of the parts of each; therefore, if we multiply the greatest in relative value, by the greatest in relative magnitude, and the least in relative value, by the least in relative magnitude, then the two products, multiplied together, equal one.

Therefore, if the areas of A and B each equal one, then, if the radius of B, multiplied by half the circumference of A, shall equal C, and the radius of A, multiplied by half the circumference B, shall equal D, then C $\times$ D $= 1$. Also, if the radius of A be divided by half the circumference of B, and the radius of B be divided by half the circumference of A, the two products will be equal to one another; but if the area of A $= 1$, and the diameter of B equal 1, then C $\times$ D equals the area of B.

This reciprocity of parts, and hence of numbers also, exists between the circle and all regular shapes whatsoever, which are formed of straight lines, when the area of each equals one. But all other shapes, *except the triangle*, lack the condition of being opposite to the circle, *in the elements of their construction*, in the particulars of the greatest and the least which are possible in nature; hence, *no other* shape, except the circle and the equilateral triangle, *can be* in the opposite duplicate ratio to one another, but *all other* regular shapes, formed of straight lines, and of any greater number of sides than the square, *approach* to the opposite ratio, "*ad infinitum*," in proportion to the extension of the number of sides.

It has been suggested by some friends, that I should add some examples of my method of deducing arithmetical truths from the problem of the circle, but this would extend a work already larger than I intended, and until my principles of reasoning, and the truth of my ratio of circumference and diameter shall be acknowledged, would be productive of no other benefit than the gratification of curiosity.

CHAPTER III.

PRACTICAL QUESTIONS ON PROGRESSIVE ASTRONOMY.

> To ask or search I blame thee not, for Heav'n
> Is as the book of God before thee set,
> Wherein to read his wondrous works, and learn
> His seasons, hours, or days, or months, or years:
> This to attain, whether Heaven move, or earth,
> Imports not, if thou reckon right; the rest
> From man or angel the great Architect
> Did wisely to conceal, and not divulge
> His secrets to be scann'd by them who ought
> Rather admire; or if they list to try
> Conjecture, He his fabric of the Heavens
> Hath left to their disputes, perhaps to move
> His laughter at their quaint opinions wide
> Hereafter, when they come to model Heav'n
> And calculate the stars, how they will wield
> The mighty frame; how build, unbuild, contrive
> To save appearances; how gird the sphere
> With centric and eccentric scribbled o'er,
> Cycle and epicycle, orb in orb.

It will be understood, from what has been said in the preceding chapters and note, that I claim for the ratio of circumference and diameter which I have established, that it is *the primary* ratio—*that* which is *one in nature*, and on which all other *true ratios* depend. I therefore denominate it, *the primary circumference*, and a circle having its component parts, as a *primary circle*.

All circles are of themselves and in their own nature, primary circles, because all circles, without regard to magnitude, have the same relative and constituent parts,

—that is to say, a small circle contains within itself the value of just as many angular spaces, and has the same ratio of circumference to diameter, which a larger circle has, the only difference being in the magnitude of those parts. But when we set aside any limited and definite quantity, calling it *one*, and set that up as a standard of measure by which to determine other quantities, then all other circles, either greater or less than such standard, cease to be primary in their relations to that standard.

In introducing practical questions here, let it be understood, that it is not my object to write an elementary book of instruction, but only to test the applicability of *the primary ratio* of circumference and diameter to practical purposes, and at the same time to test the *truth* of that ratio by applying it to other *primary truths* in nature, the sole object and value of the discovery being its applicability to practical purposes.

In adapting the ratio of circumference and diameter of a circle to mechanism of any sort, it is evident from the almost inappreciable difference between my ratio and that of geometers, being scarcely equal to one hair's-breadth in any arch ever constructed by men, that the only application of it which can be made to any natural truth as a test of its accuracy is, to the great astronomical circles, in whose vast magnitude alone, the value of

the difference becomes important or even perceptible. I therefore proceed to make the mechanical application of my ratio of circumference to these great circles, upon the principles of mechanical motion, and in a manner which I think is peculiarly my own, but which I believe to be undeniably correct and a demonstration in itself. But in order to be understood, and to fix the attention of the examiner on the points necessary to be considered, I must first state a few preliminaries as follows.

First, Time (or perhaps I should be better understood by saying the standard by which time is measured) is nothing else but the relations existing between light and motion. Therefore time is *altogether relative*, and the motion of the revolving bodies by which time is measured, being measured by time, is *also relative*.

Secondly, Time and space are, for all purposes of calculation with respect to motion, *one and the same thing*, because the measure of time is the circumference of a circle, and its length or duration is the revolution of a circle. Therefore the circumference or area of *one* circle may be reduced to time, and the length of a day or a year may be considered and treated as circumference or area.

The area of one primary circle (5153) being reduced to time (corresponding with the revolution of one solar day) until it is without remainder, is 23h. 51′ 23″ 20‴.

One sidereal day, or the revolution of the earth on her axis from opposite a fixed star to opposite that fixed star again, is 23h. 56′ 4″ 6‴.

The length of one solar day is exactly 24 hours, and for convenience I will call these a *circular day*, a *sidereal day*, and a *solar day*, the solar day being greatest, and the *circular day* least of all the three.

Reduced to their lowest denominations, these three days stand as follows:

The length of 1 "Circular day" is 5153000‴.

The length of 1 Sidereal day is 5169846‴.

The length of 1 Solar day is 5184000‴.

The difference between 1 Circular and 1 Solar day is 8′ 36″ 40‴.

The difference between 1 Circular and 1 Sidereal day is 4′ 40″ 46‴.

The difference between 1 Sidereal and 1 Solar day is 3′ 55″ 54‴.

The *excess* of difference between 1 Circular and 1 Sidereal, and 1 Solar and 1 Sidereal day, is 0′ 44″ 52‴.

Let it now be understood, that in computing the motion of a circle by time, we are to bring it at last to *solar time.*

Thirdly. All natural periods of time are, I think (in accordance with the above table), greater than one primary circle, because all the heavenly bodies by whose

motions time is measured, or whose motions are measured by time, are themselves also in motion. For example, the earth is *more than* 24 hours in revolving on her axis from the moon to the moon again, so as to bring the same meridian exactly opposite. But the earth revolves on her axis from the sun to the sun again in 24 hours exactly, and a lunar day is greater than a solar day, therefore the moon is in motion, and the earth performs on her axis more than a complete revolution in space in turning from opposite the moon's center to opposite that center again. But the earth revolves on her axis from opposite a fixed star to opposite that fixed star again in 23h. 56′ 4″ 6‴, and a solar day is greater than a sidereal day, therefore either the earth or the sun is in motion in an orbit, and in either case the earth performs in space more than a complete revolution on her axis in turning from opposite the sun's center to opposite that center again. But the difference between a circular and a sidereal day is greater than the difference between a sidereal and a solar day, therefore *both* the *earth and the sun* are in motion in an orbit.*

The above are mechanical truths, easily proved, and if they be true, as I affirm, then the relative motion of

* The evidence of this fact does not depend on the truth of my ratio of circumference, and if the geometer's ratio *were true*, the difference would be greater still.

the sun and of the fixed stars, so-called, may at length be found and demonstrated by the ratio of *one primary circumference!!*

Before proceeding to apply the ratio of circumference to the astronomical circles, it is necessary, first, to solve the problem of three gravitating bodies. I therefore submit the following proposition.

PROPOSITION I.

The respective and relative motion of three gravitating bodies revolving together and about each other, is as four to three, or one and one-third of one primary circumference.

I have always considered this proposition as self-evident on the face of it, and that no mathematician would deny it and hazard his reputation on sustaining the denial with proof. But as I shall perhaps be called on for proof, I add here, at some length, the solution of the problem, after my own method, as follows:

The problem of three gravitating bodies revolving together and about each other, is one which, like the Quadrature, has hitherto baffled all attempts of mathematicians to solve. But since this, like others of the kind, is of itself a problem, which is daily performed and consequently solved by the mechanical operations of nature, the failure of mathematicians to reach the solution

proves nothing but the imperfection of the reasoning applied to it.

It is a principle I think clearly demonstrable, that whatever can be constructed by mechanics out of given magnitudes, can be *exactly* determined by numbers, and that which cannot be constructed by mechanics out of any given magnitudes, cannot be exactly determined by numbers, having the same relation as the magnitudes one to another. It is for this reason, and for this reason only, that we cannot *out of* the *same magnitudes* construct a square which is just twice as big as any other perfect square, neither can we find the perfect root of such a square by decimal numbers. If this reasoning be true, then, because the problem of three gravitating bodies is a mechanical operation daily performed in nature, it is hence a thing capable of being proved by numbers. The great difficulty of this problem has arisen, I think, from the impossibility of its full display by diagram, and the difficulty of embracing in any formula, all the conditions contained in its elements. The plan of exacting a display by diagram of all geometrical propositions is *safe*, and perhaps it is the only plan by which the yet untaught mind can be initiated into the truths of geometry, but is it always necessary in every original demonstration? Are there not other means equally true and *equally safe* in the hands of one accustomed to examination, and acquainted with the properties of numbers and of shapes? I think there are, and without taking the least unwarrantable latitude, or departing from the clearest perceptions of reason, I think this problem may be easily and accurately solved.

The thing required of every demonstration is, that it shall give a *sufficient reason* for the truth which it asserts. But in order that a reason may be *sufficient*, and the conclusion drawn from it *safe*, it is necessary, not only that the relations of cause and effect shall be made clear to our perceptions, but also that the conclusion, *when drawn*, shall abide the test of practical application. Any demonstration which does less than this cannot be relied on, and no demonstration ever made has ever done more than this.

We know very well that things are possible or impossible to be done, only in proportion as the means applied are adèquate or inadequate to the purpose. We know also, that because different principles exist in the various forms of matter, therefore it is impossible to demonstrate every thing by the *same means* or *same principles*. It is a narrow-minded prejudice, therefore, which exacts that every demonstration shall be made by the prescribed rules of science, as if science already embraced every principle which exists in nature. Yet none are more frequently guilty of this narrow-mindedness than mathematicians, who often require that things shall be done by

the means which the written science affords, well knowing at the same time that such means are *inadequate.* Such has always been the case in respect to the quadrature of the circle. Mathematicians have demanded that it should be demonstrated by the properties of straight lines, knowing, at the same time, that straight lines are *inadequate;* therefore (*and therefore only*), the thing has been found impossible, and all other demonstrations are rejected, because they cannot be shown by straight lines. I do not consent to such unreasonableness of decision, but in every proposition where the *sufficient reason* is manifest, I hold the proposition to be demonstrated until it can be disproved.

In entering upon the solution of the problem of three gravitating bodies, we must first examine and see of what elements the problem is composed.

The elements which I shall consider, in this case, will not be such as a mathematician of the schools would think it necessary to consider. They will be far more simple, more conclusive (for such as the schools can furnish, have yet decided nothing), and, I think, more comprehensible, yet equally true to nature (for I consult nature's laws only, and not the method or opinions of any other man), and equally accurate and precise with any which can be given by any other method.

And *first*, each revolving body is impressed by nature with certain laws making it susceptible of the operation of force, which, being applied, impells motion. These laws may all be expressed under the general term *forces*, which, though various in their nature, possess an equalizing power, controlling each other in such a way, that neither can predominate beyond a certain limit; and, consequently, these bodies can never approach nearer to each other than a certain point, nor recede from each other beyond another certain point. Hence these forces are, at *some mean point*, made *perfectly equal*, and therefore they may be considered as but *one force* and hence but *one element* in the problem.

Secondly, these revolving bodies have magnitude, shape, density, &c., which affect the operation of force in producing motion. These properties of revolving bodies have all the same inherent power of equalization as forces. For example, if density be greater in one than another, then magnitude will be relatively less, force will be less (the direct force), and the momentum from velocity greater, but the whole shall be equal. On the other hand, if magnitude be greater, and density less, then force will be greater, and velocity less, but the whole shall be equal. The second element of this problem may therefore be comprehended under the term *magnitude*, which shall include shape, density, and every other quality or condition which affects the operation of force

in producing motion, and the whole constitute but one element in the problem, which I term *magnitude*, as referring to the bodies themselves rather than to any of their qualities, as density, gravity, or otherwise.

The *third* element in this problem is *distance*, by which I would be understood to mean the *chosen distances* from one anothor, at which these bodies perform their revolutions in space. It is well understood, that from the nature of the case, these revolving bodies must take up their mean distances from one another in exact proportion to their respective magnitudes and forces, and in proportion as these are greater or less, the distance from each other will be greater or less. Hence it is seen that the same inherent power of equalization exists in respect to distances as in respect to the forces and magnitudes, and whether their distances from each other be greater or less, equal or unequal, they still constitute but one element in the problem.

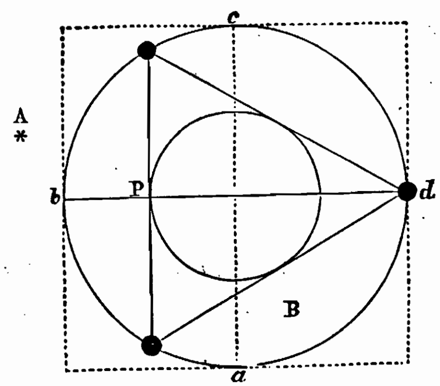
The *fourth* and *last element* in this problem is *motion*, or *velocity*, by which distances are to be performed or overcome by revolution. And here again it will be seen, that because the distances to be thus performed by revolution depend entirely on the *chosen distances* from one another, and these again depend on magnitude and force, therefore the same equalizing power exists in regard to *motion* or *velocity*, as exists in regard to all the other elements, and therefore this also constitutes but one element in the problem, which I will term velocity, as including momentum, and every other quality, condition, or effect of motion.

These, *four in number*, are all the elements necessary for the *mechanical* performance of the problem, and consequently all that are necessary for its *determination* by *numbers*, and it has been seen, that such is the nature of the problem itself, and the power of these elements over one another, that every other quality or condition affecting either, is equalized by, and held in subservience to these, and these again are equalized by, and held in subservience to *one another*, and all controlled by magnitude, so that the whole constitute but *one problem* or *mechanical* operation in which *four elements* are concerned.

The difficulty of reducing impalpable things to a palpable standard of measure is generally conceded, but in this case, I think the difficulty does not exist, and that these elements may all be as truly represented by numbers and magnitudes, as if they were palpable things in themselves, having the qualities of length, breadth, and thickness. For example, let a stone be a magnitude having shape, bulk, density, &c. Now, a *force* which can raise this stone one foot from the ground, and hold it suspended there, is, in its relation to the magnitude or stone, exactly equal to *one foot* of measure, and because the stone is held suspended, and does not descend again, nor

rise higher, it is evident that the force and magnitude have *become equal at that point of elevation*, and therefore, *vice versâ*, the magnitude or stone is, in its relation to the force, exactly equal to *one foot* of measure, and consequently distance and motion are each also seen to be equal to one foot; and the same principles of applicability to measure exist in *three* bodies suspended in space, and made to revolve about each other, by forces inherent in themselves. It matters not that other and disturbing forces exist outside or inside the space in which these bodies revolve, because, if another and disturbing force be considered, then it ceases to be a problem of *three* gravitating bodies; and also, because such disturbing forces, if they exist, operate proportionally on all *three* of the revolving bodies, and in the course of a revolution, and consequent change of *relative position*, these disturbances *must find* their perfect equality.

PLATE XXII.



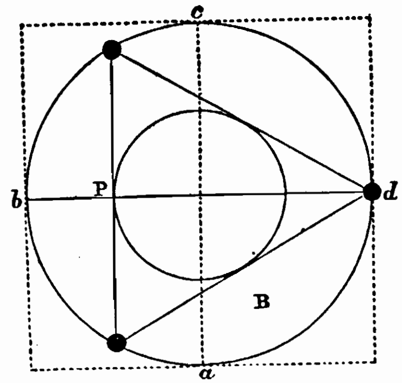
Now, let us suppose that we have here, three bodies revolving together in space by their own gravitating power, and let the magnitudes of these bodies be exactly equal to one another;—then their forces shall be equal, their distances equal, and their velocities equal, and it will be seen that they *cannot* revolve *about each other*, but must *follow each other* round a common center, and their relative motion in respect to any point in space (as the point or star A), must be on the value of the circumference of the circle B, which passes through the center of each body, as in Plate XXII.

Now, let us suppose that each of the elements contained in the problem of *three* gravitating bodies, is an equal portion of the *area* of the circle which these bodies describe in a revolution; then the circle will be divided from the center into four equal parts, as at the points *a*, *b*, *c*, *d*, and let each part equal *one*. It will be seen, that in each relative change of position, each revolving body passes over an area equal to *one and one-third*. In other words, their relative motion is as *four to three*. So also, if each element shall be an equal portion of the circumference of the circle B, or an equal portion of the square of the diameter of B, the same result is manifest, and the relative motion of each revolving body is, as *four to three* of such magnitude as is made the standard of measure. Again:

Secondly. Let the area of the circle *inscribed* in the equilateral triangle, whose sides make the distance between these revolving bodies, be *one*, as in the marginal Plate XXIII. It is seen that the circle B, whose circum-

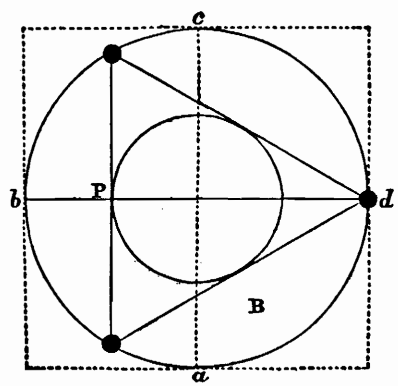
ference these bodies describe by their revolution, is four times greater than such *inscribed* circle. (See illustration, Plate XXXI., Appendix.) Hence, again, their relative change of position is seen to be as *four to three*, or one and one-third of the primary magnitude which is made the standard of measure, and [proposition I., chapter ii.], it is seen, that the circle *inscribed* in the triangle as above, forms the basis of the area of that triangle, when it shall be measured by circumference and radius, which are the only legitimate elements of area in all shapes alike.

PLATE XXIII.



Again: *Thirdly*. It is seen that the equilateral triangle (Plate XXIV.), whose sides make the distance between these revolving bodies, is an angular shape, and being measured in the usual way of measuring angular shapes, its area equals the perpendicular, P, *d*, by half the side. Now, let the perpendicular, P, *d*, equal *one*. Then it is

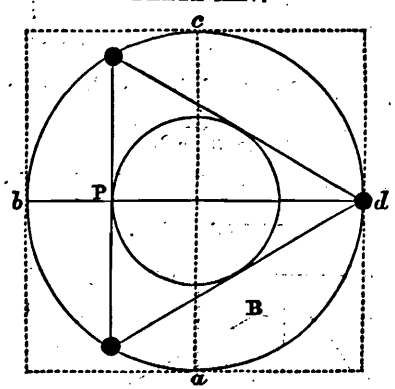
PLATE XXIV.



seen that the diameter of the circle B, which these bodies describe in a revolution, is one-third greater than the perpendicular. Hence, in performing a complete revolution, these bodies describe a circumference equal to *one and one-third* the circumference of *one diameter*. In other words, their relative motion is again seen to be as *four* to *three* of *one primary* circumference.

Fourthly. These bodies, which are revolving together, are known (by hypothesis) to be equal to one another in magnitude, and consequently, equal to one another in all the elements concerned in their revolution.

PLATE XXV.

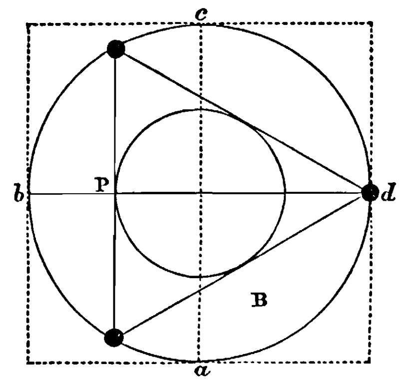


Now, let us suppose that their distance from each other equals *one*. That distance is seen to be the *side* of an equilateral triangle inscribed in the circle B, whose circumference they describe in one complete revolution. (Plate XXV.)

Now, the side of an equilateral triangle inscribed in a circle equals the perpendicular from the base of an equilateral triangle, *whose side* equals the diameter of the aforesaid circle; and therefore, because the *square* of the side of *any* equilateral triangle, equals one-third added to the *square* of its perpendicular, and because the *square*

of the side of the equilateral triangle inscribed in B equals *one*, therefore the *square* of the diameter of B equals *one and one-third*. Hence the area of B equals one and one-third the area of a circle whose diameter is *one*. Hence, in describing the circumference of B, the relative motion of these three revolving bodies shall be as *four to three*, or *one and one-third* the *area* of a circle whose diameter is *one*.

PLATE XXVI.



By prop. XII., chap. ii., it is shown that the true and primary ratio of circumference to diameter of all circles, which can be expressed in whole numbers, is *four times* the *area* of one circle *inscribed* in *one square* for the ratio of circumference, to the *area* of the *circumscribed square*, for a ratio of diameter. Therefore it is evident, that if the circumference of B shall be resolved into such primary parts, as shall express the circumference of *one* diameter in whole numbers, and in its exact relation to area and diameter, without a remainder in either, then the circumference of B, shall equal *one* and *one-third* of one primary circumference, such as may be expressed in whole numbers; because the area of the square *circumscribing*

B, equals *one and one-third*, when the side of the equilateral triangle *inscribed in* B equals *one.*

Fifth and lastly. These revolving bodies must be supposed to revolve upon a value, in which diameter and area form exact and equal portions, and the only circle in nature whose diameter and area are equal to one another, and identical in numbers, is a circle whose circumference is *four;* hence the relative motion of three bodies of equal magnitude, revolving together, cannot be otherwise than one and one-third of *such parts.*

It is evident from all the foregoing demonstrations, that, if we suppose the elements of which this problem is composed, to be *magnitudes*, and take *them* as a standard of measure, whether such magnitudes shall be equal portions of the *area* of a circle, or of its *circumference*, or of the *square* of its *diameter*, or whether we take as our standard of measure, the *distance between* these revolving bodies, which makes the *side* of a triangle, or the *perpendicular* of such triangle, or its *inscribed circle*, in all cases, and in every case, the relative motion of these three revolving bodies *must be* as *four to three*, or *one and one-third* of *such magnitude as is made the standard of measure*, and there is no other standard of measure which can be mathematically assumed in the premises, which I have not here considered.

The proposition is therefore demonstrated, that three

gravitating bodies of *equal magnitude*, revolving together, their relative motion *shall be* as *four to three*, or *one and one-third* of *one primary* circumference!!

It will be obvious to any one, that in the foregoing demonstrations, I have assumed, that the *magnitudes* of the revolving bodies are all *equal* to *one another*, and hence their forces, distances, and velocities, are all *equal to one another;* consequently, they all revolve on the same circumference, as shown in the several plates, from XXII. to XXVI.; therefore, they cannot revolve *about* each other, but must *follow each other* round a common center. But in the problem of the revolution of the moon about the earth, and the earth and the moon together about the sun, the magnitudes are *all unequal*, and hence their distances from each other, their forces and velocities, are all *unequal*, and they are known *not to follow each other*, as in the foregoing demonstration, but to revolve about each other in the order above stated.

It may, perhaps, therefore be inferred that the foregoing demonstration is not applicable to *such gravitating bodies.* But it must be observed, also, that the EQUALIZING POWER of all the elements of the problem, *are in full force and operation here*, as well as in the problem just solved, and that the chosen distances, forces and velocities, are in exact proportion to the relative magnitudes of the bodies revolving; and hence their *relative* motion shall be *still the same*, with this difference only, that because the moon revolves about the earth, and the earth and the moon together revolve about the sun, *therefore* their relative motions being expressed by time (which is also relative), the following proportions ensue:

First proportion. As *one primary circumference of a circle is to the moon's time about the earth, so is the moon's time about the earth to the earth's time about the sun!* (See the practical application in propositions II. and III., which follow in this chapter.) It must be borne in mind, however, that in the above proportion, reference is had to the revolution of the earth and the moon over the value of a *complete circle*, and *not* to the full sidereal lunation or mean year, each of which are *greater than one circle.* (See introduction to this chapter.) Also, the time here meant is *circular time*, or one revolution of the earth in space. (See table of time in the introduction to this chapter.) It must also be borne in mind, that in the above proportion, reference is had only to the *relations of decimal numbers*, and no reference is made to any geometrical standard of measure in the revolutions of either body. But because *magnitude* is the controlling element in the problem, with the power of equalizing all the rest, therefore, the first proportion as above given being true, a *second proportion follows*, which is strictly

geometrical in its character, and which makes the whole definite. It is as follows: *The square of the diameter of the moon is to the square of the diameter of the earth, as the moon's time round the earth is to the earth's time round the sun,—the time here meant being circular time, as before.*

The calculations showing the method and the result of these proportions, will be found in prop. IV. and V. which follow in this chapter.

The true and *primary* ratio of circumference to diameter of all circles, as shown by the twelve propositions of chapter ii., is 20612 parts of circumference to 6561 parts of diameter; and by the solution of the foregoing problem it is shown, that the relative motion of three gravitating bodies, as of the moon, the earth and the sun, is as *four to three*, or one and one-third of one *primary circumference.*

With the solution of these two problems, and keeping in mind the preliminary remarks made at the commencement of this chapter, we are prepared on simple, original, and *mechanical* principles, to reduce to numbers, the great circles performed by the revolution of these gravitating bodies.

The *first*, and to us relatively, the *primary* orbit of nature, which is fulfilled by these revolving bodies, is a sidereal lunation, or the passage of the moon round the earth from opposite a fixed star to opposite that fixed star again,—first because the motion is directly around our earth, and secondly, because the fixed star, so-called, has the least apparent or to us relative motion, of all the heavenly bodies; therefore,

PROPOSITION II.

The moon's orbit (or moon's time) round the earth in a sidereal lunation, over the value of a complete circle, is one and one-third of one primary circumference.

The ratio of circumference and diameter being 20612 to 6561, the moon's orbit $=20612\times1\frac{1}{3}=27.48266666+$, which pointing off 2 figures to the left for days (because the first figure or left hand unit of diameter being 6 (6,561), therefore circumference has two left hand places of units) I say is the exact time of the passage of the moon round the earth, over the value of a complete circle, the time being in circular days of 23h. 51′ 23″ 20‴ each, and therefore 27.48266666+ ×5153000‴ (the value of 1 circular day) =141618181.3333+ ÷5184000‴ (the value of 1 solar day) equals 27.3183220164+ which reduced to the proper divisions of solar time =27d. 7h. 38′ 23″ 1‴ 20⁗, which I say as before is the exact time of the passage of the moon round the earth over the value of a complete circle.

But because, as has been shown, all natural periods of

time are *greater* than one circle, and because a sidereal day, or the revolution of the earth on her axis from opposite a fixed star to opposite that fixed star again, is greater than a *circular day* or one revolution of the earth on her axis *in space*, the difference being 4′ 40″ 46‴, and because by the moon's passage round the earth she gains or performs one motion of the earth, *therefore* the difference between one *circular day* and one sidereal day is to be added to the moon's motion to complete the lunation, and therefore 27d. 7h. 38′ 23″ 1‴ 20⁗ +4′ 40″ 46‴, =27d. 7h. 43′ 3″ 47‴ 20⁗, which I say is the exact period of a sidereal lunation, the only error being the astronomical error in the length of one sidereal day (in adding the difference 4′ 40″ 46‴), which by long observation is known to be less (much less) than one-tenth of one second.* The error therefore is less (much less) than one-tenth of one second of time in a lunar month!!

When I say that the above is the exact period of a sidereal lunation, I must be understood that it is the *mean period* which the moon observes through all time. Whether the moon's motion is or is not sometimes accelerated for a long period and again diminished as much, does not touch this demonstration, but is a question standing by itself. It should be observed, however, that

* It is less than one hundredth part of one second.

the period of a sidereal lunation, as given by me above, is nearly *one-fifth of a second* in a lunar month, less than the period given in astronomical time; and the difference applied to the time of eclipses which happened before Christ, is found to agree exactly with the amount of acceleration which the moon is supposed to have received in the last two thousand years; and hence my period is known to be the true period of the moon's motion round the earth.

The second orbit fulfilled by these three revolving bodies, is that of the earth and the moon together revolving about the sun. This revolution, in consequence of all natural periods of time being greater than one circle, becomes a little more complex than a lunation. According to the main proposition of three gravitating bodies, however, it proceeds upon the same principles and is equally mechanical, but with this difference, that instead of the sun passing round our own earth as the moon actually does, the earth as the center of the moon's orbit moves round the sun, carrying the moon with her, while the sun *appears* to move round the earth and the moon together in the same period of time.

In the solution of the problem of three gravitating bodies I have shown the existence of the following proportion between the motion of the earth and the moon,

viz., "That as one primary circumference of a circle is to "the moon's time round the earth over the value of a "complete circle in space, so is the moon's time round the "earth to the earth's time round the sun over the value "of a complete circle in space." Hence, in calculating the earth's orbit round the sun by the relative motion of three gravitating bodies, we must take the moon's orbit (or moon's time) as our primary circumference, or standard of measure; therefore,

PROPOSITION III.

The earth's time round the sun over the value of a complete circle in space is as four to three, *or* one and one-third *the moon's time round the earth over the value of a complete circle in space.*

Therefore the moon's time round the earth, being 27.48266666+, therefore the earth's time round the sun =2748266666+, $\times 1\frac{1}{3}$=366.4355555+, which pointing of *three* figures to the left for units, for the same reason that two figures are pointed off in a lunation, viz., because diameter has become such that circumference has three places of units, *I say*, is the exact time of the earth's motion round the sun over the value of a complete circle in space, the time being in circular days of 23h. 51′ 23″ 20‴ each, therefore 366.4355555+ ×5153000‴

(the value of one circular day) =188824211.77777+ ÷ 5184000‴ (the value of 1 solar day)=364.244293552+ which being reduced to the proper divisions of solar time =364d. 5h. 51′ 46″ 57‴ 46⁗. But because a sidereal day is greater than the mean between a *circular* day and a solar day, by an excess of 44″ 52‴ (see table of time, page 148), which difference necessarily belongs to the sun's motion in his relative position to the earth and a fixed star so called, therefore we are to add the above period 44″ 52‴ ×1⅓=59″ 49‴ 20⁗ and 364d. 5h. 51′ 46″ 57‴ 46⁗+59″ 49‴ 20⁗=364d. 5h. 52′ 46″ 47‴ 6⁗. Now also, because the earth, in passing round the sun from opposite a fixed star to opposite that star again, gains one revolution on her axis from opposite that star to opposite that star again, or one sidereal day, *therefore*, we are to add to the above period one sidereal day to complete the mean year. Therefore 364d. 5h. 52′ 46″ 47‴ 6⁗+23h. 56′ 4″ 6‴=365d. 5h. 48′ 50″ 53‴ 6⁗, which I say is exactly the period of the mean year, the only errors being 2⅓ the amount of error in the astronomical time of the length of one sidereal day, viz., once in adding the sidereal day gained by the earth's motion round the sun, and once and one-third in adding the difference, 44″ 52‴, ×1⅓. And as such error is known to be less (much less) than one-tenth of one second of time, therefore the sum of the errors in the above period

are less (much less) than two-tenths of one second in the mean year!!

If from the above period of the mean year, we deduct the excess of difference between one circular and one sidereal day, and one sidereal and one solar day, viz. 44″ 52‴; or, if instead of adding one sidereal day for the motion which the earth gains in passing round the sun, we add *one circular* day and the difference between one sidereal and one solar day, we shall then have the period of the solstitial or solar year, viz. 365d. 5h. 48′ 6″ 1‴ 6⁗, which is the known truth within the smallest appreciable division of time. This also is a mechanical necessity of the whole principles advanced, because as it will be seen that the earth gains exactly one motion on her axis while performing the value of a complete circle in space while passing round the sun, therefore the amount of the sun's precession of a fixed star shall exactly equal the excess of difference between a circular and sidereal and a sidereal and solar day.

I have several methods of determining from the above periods the amount of a solar lunation, or the moon's synodical period, and by every method I find the exact period to be 29d. 12h. 44′ 2″ 50‴ 31⁗, which also agrees with the most exact time of her conjunctions as observed by astronomers for any number of centuries past.

I have thus, without the use or help of observations of any kind, but with the aid only of the solution of the Quadrature and the problem of three gravitating bodies, and operating only with the simple properties of numbers and with perfectly mechanical principles, determined what the periods of *four* great astronomical circles SHALL BE, viz., a sidereal and a solar lunation, the mean year and the solar year. And these periods, though differing very minutely from astronomical time as given, are found, nevertheless, upon examination, to agree exactly with the conjunctions of nature without correction or allowance. If the same calculations be made by geometers' approximate ratio of circumference to diameter, the result is more than one second of time in a lunar month and near fifteen seconds in the mean year *less than the known truth*, and will agree with no natural period of time whatever. If therefore the solution of the problem of three gravitating bodies *be true*, then it is certain that the geometers' ratio of circumference to diameter is *not true*, but that it is less than the truth, as I have demonstrated both by inverse and direct propositions, chap. i. and ii. And if the solution of the problem of three gravitating bodies *is true*, then it is equally certain that my ratio of circumference and diameter *is true also*, because it agrees with the truth of nature, in the revolution of these great circles. It alters nothing, and matters not in the least, that these revolving bodies do not move exactly in circular orbits, because by the well-known law, that they "describe equal areas in equal times," their orbit motion is made *exactly equal to a circle*. If, therefore, geometers admit the solution of the problem of gravitating bodies, they must admit my ratio of circumference and diameter *also*, or they must deny the truth of these astronomical periods, which are established by long and the minutest possible observations of time as shown by the conjunctions of nature. And if they reject my ratio, they must disprove both the quadrature and the problem of gravitating bodies, neither of which can they do, by any show of reason, which cannot be proved to be unsound and false.

It must be observed, that astronomers have arrived at their great accuracy in time, not by a simple mathematical problem, as I have done, nor by the geometrical accuracy of their ratio of circumference and diameter; but at a great expense of time, money and labor, through a long period of centuries, by taking a great number of careful observations at remote times and distances from each other, then by comparing the whole together, and taking the mean of all the differences for the truth; and by thus taking the mean of all their errors for the truth, they arrive at an accuracy in the computation of time, which their ratio of circumference and diameter cannot give them; not, however, without liability to some amount of remaining error. But my system requires no such outlay of time, money or labor, nor does it claim the indulgence of a correction of error. It proceeds only upon the simple properties of numbers, and principles of mechanics, and points out to us not merely what the truth *is*, but what the truth necessarily *shall be*, and gives a reason for it, why it shall be so, and cannot be otherwise.

It will *not* be understood, however, that the foregoing is the limit of the application which may be made of the quadrature to the astronomical circles. I have already applied it *extensively, and with success,* to others of the most important problems in astronomical science; and my present judgment is satisfied, that it is capable of being applied, *ad infinitum*, to new discoveries of the laws and combinations which enter into the system of the universe, even (if the mind of man could embrace so mnch) to determining the time, distance, magnitude and motion of every revolving body within the range of telescopic observation. It will contradict *no known law of nature.* It will confirm the truth of Kepler's law, and the law of gravitation, as discovered and principally explained by Newton; but it will *not* confirm all else that Newton has said on kindred subjects. And if my present judgment is not mistaken, these problems, when understood and received, will, by the simplest possible *mechanical evidence,* put to silence, and to rest forever, some of

the stupendous theories which have occupied the reasoning of men's minds for centuries, and which have been *received, only,* because they emanated from great minds, and *believed, only,* because no one knew to the contrary. It is not my intention, however, at present to make a show of mathematical curiosities or astronomical wonders; I desire only to prove the soundness of the basis on which I reason, by the well-known truths of nature, in which there can be no mistake, leaving the further discovery of new truths, and the correction of old errors, to future development.

The periods of time as given by me in the foregoing problems, and astronomical time as given by the best authorities, will stand as follows:

A Sidereal Lunation.

Astronomical time	My period.
27d. 7h. 43′ 4″.	27d. 7h. 43′ 3″ 47‴ 20⁗

Solar Lunation.

Astronomical time as commonly given. 29d. 12h. 44′ 3″. The synodical period as given by McKay, the English navigator. 29d. 12h. 44′ $2\frac{8}{10}$; or, 29d. 12h. 44′ 2″ 48‴	My period. 29d. 12h. 44′ 2″ $\frac{84+}{100}$ or 29d. 12h. 44′ 2″ 50‴ 31⁗

MEAN TIME.

Astronomical time as given by the best authorities thirty years since. 365d. 5h. 48′ 49″. As given by the latest and now esteemed the most accurate authorities, taken from a work of Dr. Dick's. 365d. 5h. 48′ 51″.	My period. 365d. 5h. 48′ 50″ 53‴ 6⁗

Solar Year.

Astronomical time 365d. 5h. 48′ 6″.	My period. 365d. 5h. 48′ 6″ 1‴ 6⁗.

From the above comparison it will be seen that my period differs from McKay's but the *thirtieth* part of a second in a mean solar lunation, or lunar month; from Dr. Dick's but the *tenth* part of a second in the mean year, and in the solar year from astronomical time, but the *fiftieth* part of one second of time; and it is known, as I have already said, that astronomers, notwithstanding their great accuracy, are still liable to some very small remaining error, and it is admitted, that because time is the infinite division of motion, therefore no two relative periods can be stated *exactly;* yet the differences in all the above cases are portions of time so incomprehensibly small that no one can estimate them.

The periods of time esteemed to be correct thirty years ago, are those first given in the foregoing table, viz.:

A sidereal lunation, 27d. 7h. 43′ 4″.

A solar lunation, 29d. 12h. 44′ 3″.

A mean year, 365d. 5h. 48′ 49″.

And on these, I am induced to believe, most of the astronomical tables in use were calculated. It will be seen that my periods differ from these a trifle more than from those established by the more recent surveys, being *less* than either lunation by nearly *one-fifth* of a second, and in the same proportion *greater* in the mean year, or nearly two seconds in the year. Within the last thirty years, however, new sets of observations have been made in Europe, and a new deduction made of the mean year, making it, as before quoted from Dr. Dick's work, 365d. 5h. 48′ 51″, thus agreeing with *my period* within the *tenth part* of a second in the year;* but I am not aware whether any correction has been made in the time formerly given for the lunations. It can, however, be shown, I think, that in the assembled motion of any number of revolving bodies, as of the planetary system, if the period of any one of them be fixed *too small*, observation will show the period of any other one to be *too*

* It is worthy of remark that I was not aware of this correction of the mean year which makes it agree with my period, until within the past year, and sveral years after I had made all my calculations.

great to meet the conjunctions which nature makes, and to make the conjunction by calculation, that body whose period is given too small must be retarded or its period made greater, and the other must be accelerated or its period made less, it being a *mean* of *both their motions* which brings them into line. It is self-evident, therefore, that in changing the astronomical period of the mean year as before stated, making it *greater* by nearly two seconds than formerly, astronomers should have changed the periods of the lunations also, making them proportionately *less* than formerly, or otherwise the two (the earth and the moon) will not perfectly conjoin according to the calculations made on their respective periods; because, as before remarked, it is a *mean* of *both their motions* which brings them into line. And if *this be done*, then the periods of the lunations last above given will be made to agree with *my periods*, to the sixtieth part of a second of time, and they will also agree within the thirtieth part of a second with the synodical period or mean solar lunation as quoted from McKay in the foregoing table, which has long been supposed to be the nearest possible approach to the mean time of the conjunction of the earth, sun, and moon.

Let it here be noticed that the mean of the difference, which my period of the mean year is *greater*, and the lunations are *less* than the periods formerly in use, is

exactly equal to the supposed acceleration which astronomers say the moon has received in the last two thousand years or more; and this difference being applied by time to the relative motion of the earth and the moon, will place the two in conjunction with the sun, just at the time and place where an eclipse is known to have happened more than four hundred years before Christ, whereas the astronomical tables will be fully an hour and a half out of the way. It is, therefore, certain that my periods are, to the smallest appreciable divisions of time, the *true periods* of the mean *relative* motion of the earth, sun and moon, and the inference from all these truths is a perfect and practical demonstration of the truth of my ratio of circumference and diameter. If these be facts, they seem to contradict the existence of the acceleration of the moon, such as is supposed by astronomers to exist, and to imply instead a perfectly equable mean motion of that body.—It is not my intention at present to argue the points of acceleration, or *no acceleration*. I do not say at present that acceleration is impossible; because nothing is impossible with the infinite wisdom and purpose which governs the universe; *but I do not credit it*, notwithstanding the high authority on which it is asserted, because, *first*, it is a notion which grew up only on the discovery of error in the calculation of eclipses through long periods of time, and in efforts to account for that error. *Secondly*, all the appearances which indicate it may be easily accounted for, without the necessity of its existence, and nothing exists in nature which is *unnecessary*. *Thirdly*, it indicates defection in nature's laws, and therefore it cannot be true.

In the solution of the problem of gravitating bodies I have established certain *proportions*, which, in order not to be misunderstood, it is proper to reduce to calculations; therefore,

PROPOSITION IV.

First proportion. As one primary circumference of a circle is *to the moon's time about the earth over the value of a complete circle in space*, so is the moon's time round the earth *to the earth's time round the sun over the value of a complete circle in space.*

It will be evident that any two parts of the above proportion being known, the third may be found. By proposition III. (this chapter) it is shown that the moon's time round the earth is, to one primary circumference, as 27.4826666+ to 20,612; therefore,

: 20612 :: 274826666+ : 274826666+ = 366.435555+

The time, in the above periods, is *circular time*, and pointing off two figures to the left in the first period, and three in the second for units, on reference to propositions

II. and III. it will be seen how these periods are brought to the *solar time* contained in a sidereal lunation and the mean year.

The second proportion, in the problem of gravitating bodies, is a geometrical proportion, as follows:

PROPOSITION V.

Second proportion. The *square of the diameter of the moon* is to the *square of the diameter of the earth*, as the *moon's time round the earth* over the value of a complete circle in space, is to the *earth's time round the sun* over the value of a complete circle in space.

Three parts of the above proportion being known, the fourth may be found; therefore let the moon's diameter be the part unknown. We have already seen that the moon's time =27.4826666+ and the earth's time =366.435555+ (circular time), and the diameter of the earth has been ascertained by actual measure to be 7,912 miles, which it no doubt is, very nearly. Admitting, then, that the earth's diameter is 7,912 miles, then the square of her diameter =62599744, therefore : 366.435555+ :: 62599744 : 27.4826666+ =4694980.8+ and : $\sqrt{4694980.8+} = 2166\frac{7}{10}+$ which, I say, is the true diameter of the moon, and neither one mile, or tenth of a mile, more or less, the only condition being, that the diameter of the earth is 7,912 miles.

Dr. Bowditch gives the moon's diameter as 2161 miles, and others have given it at 2180. It will be seen that the diameter proved by me, is nearly 6 miles greater than that given by Dr. Bowditch, and 13 miles less than is given by some others. But the fact that astronomers differ at all, proves their method to be imperfect, and consequently liable to error, sometimes greater and sometimes less, while the close approximation on each side is a very strong argument in favor of the truth of my proportion, even if it were not here seen to be accurately deduced from mathematical principles.

In my introduction to the Quadrature (chapter ii.), I there signified that my course of reasoning would be strictly original, and wholly independent of any arbitrary rule,—*perfectly conformable to nature*, yet not confined to the rules of art; and I recall attention to this remark, because it is necessary to be borne in mind by those who may undertake an examination of the subjects treated of in the present chapter.

In respect to the astronomical circles, it must be observed, that the manner in which I have treated them embraces no other facts or principles, than the simple relations between numbers, shapes and motion, and no reference is made, or intended to be made, except in the fifth proposition, to either magnitudes or distances. In treating of the astronomical circles, therefore, I have

simply treated them each as *one circle*, made up of the parts which compose the *primary relations* between the circle and the square, but without any reference to any standard of measure in art.

It is to be regretted, I think, for the sake of science, that so little examination has been made into the recesses of nature, to supply us with standards of measure. So far as I know, science has given us no *natural standards*. The French standard, derived from the measurement of an arc of the meridian, is but an imperfect attempt. We are told, in English, that "three barleycorns make one inch," and the length of three barleycorns, which grew in the time of one of the English kings, seems to be the only contribution which nature has been called upon to make, to supply us with standards of measure. It is not at all wonderful, therefore, that such standards have no applicability to time, as created by the motion of revolving bodies, or to any of the fixed laws of nature whatever.

It has been objected by some caviling minds, that calculations like these astronomical circles, which are based only on the properties of numbers, or of shapes, but which have no standard of value, cannot be of any practical use. But this is a very short-sighted objection, and to make them of practical application and use, we have nothing to do, but, just as is necessary in any other case, to erect a standard of value in *our own minds.* But in order to make any standard available to us here for any intelligible purpose, it is necessary that it be selected *from nature*, and it must also be a *fixed fact* in nature, and not an accidental truth, like the childish conception of the length of "three barleycorns." To answer the objections made, and to illustrate and prove this position, I am induced to add *another practical question*, which was not intended in the commencement of this chapter.

It is well known that the United States has lately expended a large sum of money for the erection of an observatory in southern latitude, for the purpose of co-operating with others at the north, in determining the sun's distance from the earth; and my purpose is now to show, that this truth may be determined with much greater precision by my principles of reasoning, than by any other method, and *without the help of observations of any kind.* As there is an uncertainty with astronomers, at least to the extent of several millions of miles, what the sun's mean distance really is, it may not be uninteresting to compare the results of my principles of reasoning with the actual observations, when they shall be completed. Therefore—

PROPOSITION VI.

The mean distance of the sun's center from the center of the earth, or that at which the earth would revolve, if the area or plane of her elliptical orbit were made the area of a circle, is *eleven thousand six hundred and sixty-four* diameters of the earth, neither more nor less; admitting, therefore, that the earth's diameter is 7,912 English miles (which it no doubt is pretty nearly), then the sun's center is distant from the earth's center as above 92,285,568 English miles, and neither one mile more or less.

In following out the above proposition to demonstration, in order to make the connection of my principles of reasoning clear and manifest to the perceptions of others, it is necessary here to lay down as axioms certain truths which have been proved.

First. The circle is the basis or beginning of all magnitude or area. (Proposition III., chapter ii.)

Secondly. Any expression of numbers in relation to material things is also an expression of magnitude. (Proposition VI., Appendix.)

Third. A point is therefore a magnitude when considered as *one.* (Proposition VI., Appendix.)

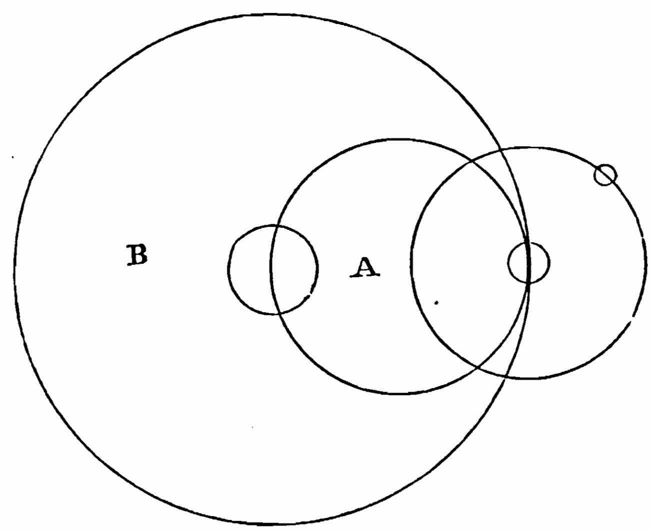
Fourth. A point in reference to space or extension on all sides of it, is therefore a molecule or globe, and in reference to a plane, it is a circle. (Proposition VI. Appendix.)

A point, when considered and treated as *one*, is therefore the least possible existence of magnitude,—the original principle of matter; a *fixed fact* in nature, unchangeable, imperishable, and such, that the union of many points according to their chemical affinities, becomes matter developed to the senses, and because we can have no other comprehension of the development of matter, therefore, relatively to us, this is an *absolute truth.* A point is therefore *such*, that it has an *exact relation* to *every development of matter* in *our world* and its *atmosphere;* therefore, if the magnitude of a point were a thing within our comprehension and grasp, *it would form a perfect standard of measure*, and by enumerating points beginning with one, and counting upward, numbers would at length express the magnitude of our world; and in the process of counting, we shall have enumerated the exact *relative* magnitude, one to another, of *everything contained in it.* A point is therefore a *perfect standard of measure*, and *any number of points* is a perfect standard of measure for any *greater number* of points. Hence our earth being a magnitude made up of points, and a *fixed fact in nature*, is therefore a perfect standard of measure for all greater magnitudes that surround it.

When we attempt to comprehend or to estimate the

distance from our earth to the sun, we enter on a higher order of creation, and mentally pass from the contemplation of things in the world, to things in the universe, where *worlds are points*, bearing exactly the same relation to the *infinite whole*, which the incomprehensible and undeveloped point bears to our world, because each runs to *infinity*, and because a point is *one*, and therefore emphatically *the one* to which all other and greater magnitudes are *exactly related;* therefore, let the earth be *one*, and let that be the standard by which to measure the sun's distance.

PLATE XXVII.



By proposition I., this chapter, I have shown that the relative motion of three gravitating bodies, as of the earth, the sun, and the moon, is as *four to three* of *one primary circumference* of a circle; but this, as has been seen, is without reference to any definite standard of measure. By relative motion, is meant, of course, the *relative change of position* of one body to another. By that proposition, therefore, the measure of a year is the measure of a circle in which the earth and the sun change their relative position, and return to that position again. And by the same proposition, and proposition III., this chapter, it has been shown, that the time in which the earth performs the value of a complete circle in space, being reduced to circumference, it has a diameter of 11664 parts of that which I say is *one primary circumference* in nature, viz., $6561 \times 1\frac{1}{3} = 8748, \times 1\frac{1}{3} = 11664$. It will be evident, on reference to the illustration on the last page (Plate XXVII.), that a circle which is the measure of the *relative change of position* of two of these gravitating bodies, and around which they move *relatively to one another*, is the circle A, whose circumference passes through the center of each body (the earth and the sun), hence the circle A is *the circle* whose circumference I have measured in determining the mean year in proposition III., this chapter, and whose diameter is 11664; in which proposition, I have also made the earth (by her revolution on her axis) to be the *unit* or *standard* of *measure.* It will be seen, also, that the diameter

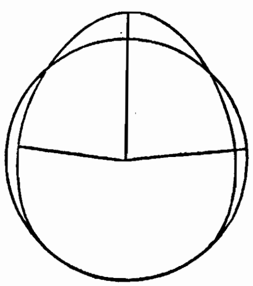
of the circle A is the radius of the circle B, which the *earth shall describe in passing round the sun*, and therefore the diameter of the circle A *is the earth's distance from the sun.*

Now, therefore, because the earth is a *primary* magnitude, a *fixed fact* in nature, and a *point in the universe whose value is one*, bearing the same relative value, in the order of creation, to things in the universe, which the undeveloped point bears to things in the earth, and is therefore a *perfect standard of measure*,—and because she is *herself* the unit or standard of measure which by her revolution determines the value of the circle A in measuring the mean year, and is also, by hypothesis, here made the *unit for determining her distance*, and because the diameter of the circle A is 11664 parts of the diameter of one primary circumference, of which the earth is but *one part*, therefore, the earth's mean distance from the sun, from center to center, is 11664 diameters of the earth, *neither more nor less;* and therefore, admitting that the earth's diameter is 7,912 English miles (which it is pretty nearly), then $11664 \times 7912 = 92285568$ miles, which is the earth's mean distance from the sun, and *not one mile more or less.*

The proposition is therefore demonstrated!!

In order not to be misunderstood in respect to the result in the foregoing demonstration, it is proper for me here to add some explanation of the difference between that which I have called the *sun's mean distance*, and that which is commonly understood by astronomers as such. It will be self-evident to all, I think, that admitting my demonstration to be true, the distance shown is *that* at which the earth *would* revolve in a perfect circle, if the sun *were fixed in the center;* and if this be the fact, then it is equally evident, I think, that the distance shown is the radius of a circle whose area is exactly equal to the plane of the earth's supposed elliptical orbit; because it is self-evident, that if the earth shall move through an elliptical orbit by an *unequal* motion, *passing over equal areas in equal times*, it is precisely the same thing as passing over the circumference of a perfect circle having the same area as the ellipse by an equal motion in exactly the same period of time. It will be seen from

PLATE XXVIII.



the illustration (Plate XXVIII.), that the ellipse and the circle having the same area, the radius of the circle is *greater* than the *least*, and less than the greatest radius of the ellipse; and this will be true, whatever elongation the ellipse may receive, and whatever center may be taken as *the center.*

It will be known, also, from the laws which govern

these shapes, that the difference between the radius of the circle and the least radius of the ellipse, is *less than half the difference* between the least and greatest radius of the ellipse; therefore, if the sun's mean distance be taken to be half-way between the least and the greatest radius of the ellipse, it will be greater than the distance which my demonstration shows; and if the sun's mean distance be taken to be the mean of the *squares* of the two radii of the ellipse, then the distance will be greater still; the latter I believe to be the mean which is mostly adopted by astronomers; but in either case, it will be seen, that *any distance* shown by them, even if measured with perfect accuracy, will be greater than mine. The angle of parallax, as deduced from the last transit of Venus, is given in Vose's Astronomy, as from the best authorities, as 8″.52 at the sun's greatest distance, and 8″.65 at the sun's *mean distance*,—this latter would give a radius of about 94,300,000 + miles as the mean distance. La Place, who has been esteemed the most accurate authority in these things, thought that the deductions made from this transit were within *one eighty-seventh* of the truth, more or less, he could not tell which; thus leaving an uncertainty of considerably more than *two millions of miles;*—deduct this uncertainty from the distance given above, and with a very moderate allowance for the difference of mean intended, the sum will very closely approximate to that which I say is the *exact distance* at which the earth would revolve in a perfect circle, if the sun were fixed in the center, and the area of that circle is *exactly equal* to the plane of the earth's elliptical orbit, *as she moves at present.* The only qualification to this is, that the earth has a diameter of 7,912 miles, neither more nor less. But putting aside *all qualification*, to make the thing *perfectly accurate*, I say, that the sun's distance at the mean, as given by me, is 92,819,114 + of those parts, of which the circumference of the earth is exactly 25,000, and its diameter 7,957 +. And I say, moreover, that these are the *true parts* at which the circumference and diameter of the earth *should be considered*, according to the French standard of measure, which takes the circumference of the earth as *one*, and proceeds by decimation to fix the value of smaller measures.

It is known, that in consequence of the elliptical form of the earth's orbit, she must move faster in one part of it than in the opposite part. It is known, also, that all observations of the sun, or any of the heavenly bodies, taken from a position on the earth's surface, are liable to more or less error, from the fact, that the earth is *at all times* in rapid motion through her orbit, and on her axis. Hence, if two sets of observations be taken; one, when she is in the largest part of her orbit, and the other,

when she is in the smallest, the errors in calculation arising from the earth's motion, will be greater in one case than in the other, because the earth moves faster in one part of her orbit than in the other. But if the mean distance be taken, at which the earth would revolve in a perfect circle, having the same area as the ellipse, and with a perfectly equal motion, this liability to greater error at one time than another, will be corrected. I do not hesitate, therefore, in declaring, that the mean distance, as shown by me, is the most *accurate*, as well as the most convenient, for all astronomical calculations made from observations, even if any other distance *could be* accurately determined, which it *cannot be* by any method adopted by astronomers, without an uncertainty of considerably more than two millions of miles.

Having thus determined with accuracy, the mean distance from the sun, at which the earth would revolve in a circle having the same area as the ellipse, by Kepler's law, that "the squares of the times are as the cubes of the distances," we have a *correct basis* on which to determine the mean distance from the sun, *of every planet and satellite in the solar system*, a thing never before attained. And the only question for astronomers to decide, is, is my demonstration true to the operations of nature, according to the principles set forth in it?

I affirm that it is, to the smallest fraction, and challenge them to the disproof by any means in their power, which is not liable to error equal to the disagreement which they may find ! !

QUADRATURE OF THE CIRCLE,

A LECTURE READ BEFORE THE

GEOGRAPHICAL SOCIETY.

HITHERTO UNPUBLISHED, CONTAINING CURIOUS FACTS CONCERNING ITS PROBABLE ANTIQUITY.

QUADRATURE OF THE CIRCLE.

I AM here to-night for the purpose of placing before you some new views regarding the old and long-since exploded question of the quadrature of the circle. The importance to astronomy and navigation of a correct knowledge of the circle, is my reason for stepping out of my proper sphere of business, temporarily, to become a public lecturer.

I am perhaps as well satisfied as any man present of the utter impossibility of ever finding the exact quadrature by the geometrical application of straight lines to its measurement.

I have, indeed, very often gone farther than any one else in this particular, and said (perhaps not very charitably) that any one who did not know at sight, and without any demonstration, that the measurement of the circle by the application of straight lines or by plane Trigonometry is impossible, did not know anything at all of the subject.

In every natural truth which we may attempt to develop, there will of necessity exist certain principles which we may lay down as axioms to guide our reason in following out the subject to a careful development.

Some such principles exist in the circle in its relation to other shapes, and to the square especially, which is made the standard of superficial quantity. I will therefore state a few of these principles, such as I think no one will venture to dispute, and we may receive them as axioms in the further pursuit of our reasoning; and,

1st. Between straight lines and curved lines there is an essential difference in properties, and before we can determine with accuracy what is true of the circle and what not, we must first understand what that difference of property is.

2d. The circumference of a circle *is a line lying wholly outside of the circle thoroughly enclosing it.*

3d. The circumference of a circle, in its power of enclosing area, is greater than any possible number of straight lines of the same aggregate length in any shape; hence there is a point in a line of figures representing circumference, where such excess shall amount to one or more.

4th. That which geometers call a circumscribed polygon is not a circumscribed polygon, because the centre of each side coincides with the area of the circle, and the true circumference lies wholly outside, enclosing the whole area; at a great number of sides, therefore, the so-called circumscribed lines are brought *wholly* within the true circumference.

These are all demonstrable truths, and are, I believe, entirely new as *elements* to be considered in demonstrating the Quadrature.

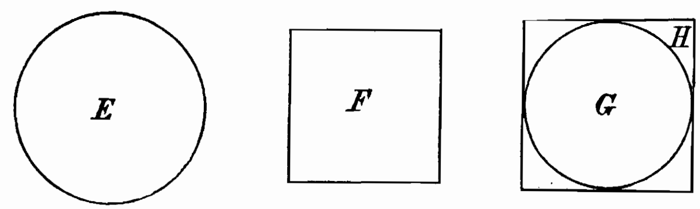
Accepting them as axioms, we will proceed to give some

illustrations of the relative properties of the circle and the square.

The great and fatal error of geometers is seen in their declaration that the circle and the square are "incommensurable," that there is no co-relation between circumference and diameter, and our first proposition shall be to dissipate this fatal error; therefore,

Proposition First.

The circumference of any circle being given, if that circumference be brought into the form of a square, the area of that square is equal to the area of another circle, the circumscribed square of which is equal in area to the area of the circle whose circumference is first given.



Let the circumference of the circle E be given; let it be 36 or any other number; now let the four sides of the square F also$=36$; then each side equals 9, and $9\times 9=81$, which is the area of F; now let the area of the circle $G=81$, then the area of the square H circumscribing G, equals the area of the circle E, whose circumference is 36. The reverse of this proposition will read as follows:—Any square H is such that its inscribed circle G is equal in area to another square, F, whose circumference, or four sides, equals the circumference of another circle E of equal area with H.

The foregoing proposition is entirely new in Mathematics, and was never known until published in my Quadrature in 1851. It does not prove what the ratio of circumference is; it is a general principle, true of all ratios alike, but it sets aside forever the chimera that the circle and the square are incommensurable, or that there is no co-relation between circumference and diameter.

The truth and beauty of this proposition and its general principle are seen by applying it to other shapes. For example, let the circumference of E be given, let it be 36; then let F be a hexagon whose six sides shall equal 36; then the area of F shall equal the area of the circle G, whose circumscribed hexagon is equal in area to the area of the circle E, whose circumference is given. The same is also true of the triangle, the pentagon, the octagon, or any other regular shape, whatever may be the number of its sides.

If, then, we shall let F be a polygon of 6,144 or 8,000 sides, and G a circle equal in area to F, then H, being a polygon of 6,144 or 8,000 sides circumscribing G, is equal in area to the area of the circle E, when circumference is given. And what does all this prove? Why it proves the truth of every axiom we have just laid down, and the reader is referred to each of them for application of the rule. The truth of this proposition is first proved by

numbers. It may be proved algebraically, also, when discovered, but I think algebra contains no formula that will regard the transition and alternation of shapes to enable you to make the discovery.

It is not possible in a single lecture on an intricate subject, and especially one that has been so much studied, criticised, and abused as the Quadrature of the Circle has been, to present and illustrate all the gradations of reason which enable us to master the first elements of a great natural truth, and to follow it up to a demonstration. In the book which I published on the Quadrature, in 1851, I have treated all these things diffusively.

But for our present discussion, we must pass over the labored demonstrations, showing the mechanical errors of geometry in its application of straight lines to curved lines, and come at once to some of the principal demonstrations which show our ratio to be the true one, and our principles of reasoning sound. The method now in use by mathematicians being found to be impracticable, it is necessary to try some other method. I have therefore taken up shapes in their distinctive qualities, as a basis of reasoning, irrespective, to some extent, of the straight lines which serve to form those shapes. Those who contend for the geometry of the schools, will perhaps think this is not mathematical. But I am authorized to do it by truth and nature, for it will be perceived that the first step of nature in the material creation is the production of shapes, and lines are nothing more (I mean the imaginary lines of geometry) than the dimensions, boundaries, and divisions of those shapes; therefore shapes are primary things, and hence a true basis of mathematical reasoning. We proceed then with a proposition that shall lead us directly to the demonstration of the Quadrature.

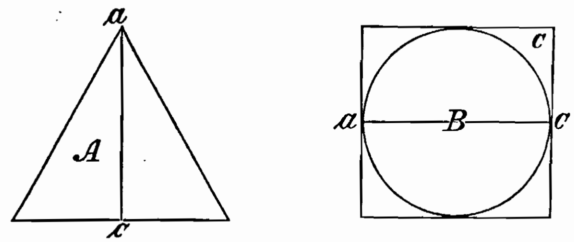
Proposition Second.

The circle and the equilateral triangle are the primary of all shapes in nature; they are opposite each other in all the elements of their construction, and hence the square of diameter being made the standard of measure, the circle and the equilateral triangle in their fractional relations to the square are opposite each other in ratio of the squares of their diameters.

I believe it is admitted by all, and therefore needs no argument to prove, that the triangle is the primary of all shapes formed by straight lines, because no shape can exist having less than three sides; therefore the equilateral triangle is the primary of all shapes formed of straight lines, and equal sides and angles; and the circle is the ultimatum of nature, in the extension of the number of its sides; it is infinite, so that all its sides are blended in one. In this respect, therefore, they are opposite each other in the first and chief element of their construction. For the convenience of reasoning, I am obliged to depart from the elegance of a mathematical rule, and to consider angular shapes as having circumference and diameter, and the reason for this is, that circumference and diameter are the only dimensions by which all regular shapes may be measured alike; therefore we treat the three sides of the triangle, or the four sides of a square, as the circumference, and a line perpendicular to the centre of either side as the diameter.

Pursuing this examination, then, we see that the equilateral triangle has the greatest circumference and the least area of any possible shape formed of straight lines, and of equal sides and angles, and the circle has the greatest area, and the least circumference of any possible shape; therefore, in this also they are opposite each other in their elements of construction. The triangle is measured by half its circumference, and the radius of an inscribed circle, and is thus found to have the least radius of any possible regular shape, of equal circumference; and the circle has the greatest radius and the least circumference of any possible regular shape of equal area. If we take the aggregate of circumference and radius in each shape, they are most equally divided in the circle, and most unequally in the triangle; in every case that which is greatest in the triangle is least in the circle, and that which is least in the triangle is greatest in the circle, and in every particular the two shapes are at the extreme and opposite boundaries of nature, being the greatest and the least that is possible. Therefore being opposite to each other in all the elements of their construction which go to make area, and the square of diameter being the standard of measure, the circle and the triangle in their fractional relations to the square are opposite each other in the ratio of the squares of their diameter.

Now let the area of the triangle A and the circle B, each equal one, then the diameter of the triangle ($a\,c$) is found to be $\sqrt{\sqrt{3}}$. Therefore the diameter of the circle B being in the opposite ratio $= 3^2 \times 3^2$, viz., $3 \times 3 = 9 \times$



$9 = 81$. Therefore 81 is a fractional diameter of the circle by which the circle and the square are equal to one another.

We come now to another law of the circle and the square which it is necessary to consider in carrying out to demonstration the opposite ratio of the triangle. It is this:

Proposition Third.

The true ratio of circumference to diameter of all circles is four times the area of an inscribed circle for a ratio of circumference to the area of the circumscribed square for the ratio of diameter.

Hence the diameter of the circle B in its fractional relation to the square C, being 81, therefore $81 \times 81 = 6{,}561$, which is the area of the square C, circumscribing B; therefore by the two last propositions the area

of the circle B shall be some definite and certain number of 6,561 parts of the square circumscribing it. It is known to be greater, much greater than 5,152, and less, much less than $\frac{5154}{6561}$ parts of the circumscribed square, therefore (reductio ad absurdum) it shall be 5,153, because it can be nothing else, there being no other 6,561 parts between 5,152 and 5,154; and because by the last proposition the ratio of circumference is four times the area of the inscribed circle, therefore, $5{,}153 \times 4 = 20{,}612$, and 20,612 for the ratio of circumference to 6,561 for the ratio of diameter is the true ratio of circumference to diameter of all circles.

The Quadrature of the circle is therefore demonstrated, and agreeably to the axioms laid down at the commencement the point in a line of decimal figures where this ratio differs from the approximation is seen to be in the sixth decimal place, viz.:

Approximation	= 31415926
My ratio	= 31415942

And this difference arises wholly, *First*, from the essential difference in the property of curved lines, which enables them to enclose a greater area than any possible number of straight lines in a polygon; and *Secondly*, from the mechanical error, by which the circumscribed polygon of geometry at a great number of sides is made to coincide with the extent of the area of the circle instead of thoroughly enclosing it as is due. This last proposition is also a law of the circle unknown to mathematicians, or, if known, is nowhere treated of as an element for demonstration; and by this law we may find as many true ratios of circumference as there are numbers in existence; for example, let the area of the circle $\dot{B} = 112$. We then find the area of the square circumscribing B by simply multiplying 112 by 6561, and dividing the product by 5153, the last product is then 142.6+, then $112 \times 4 = 448$, therefore 448 to 142.6+ is a true ratio of circumference and diameter, only it happens that the diameter in this case is an infinite decimal fraction, just as an imperfect square has an infinite decimal fraction for its root.

The three propositions here given are laws of nature, and laws of the circle as well. They are but a small part of those which may be adduced in support of the mathematical truth which they assert, and they cannot be disproved by any possible means. Of the second proposition, the opposite ratio of the triangle and the circle, it may be said it is the "particular fact" that governs the relation of straight lines to curved lines. The triangle is seen to be *the first departure of nature from curved lines.* To produce shapes by means of straight lines, and, to find the relation which each bears to the other, we go back to the point where nature begins, and in doing so we are rewarded with success.

I shall make no further demonstrations of the circle to-night, but devote a few moments to collateral subjects, such as I think may be entertaining or useful in this connection.

From a variety of facts and circumstances which have fallen under my observation, I have been induced to believe that the Quadrature of the Circle, such as I have demonstrated it to be, was known to the ancient Egyptians, and it will be matter of interest to inquire whether such be the fact, and if it was known, to trace the circumstances, when, where, and by what means it became lost. It is not my purpose or province to discuss the complete history of any branch of science, but I think a few words respecting the probable antiquity of the Quadrature, as one of the long-lost sciences, and a sufficient reason given why it has never been reached by modern Geometry, cannot be without interest at the present day.

One of the first and strongest reasons for this belief is, that the quadrature, as I have demonstrated it, and the order and principles of revolution indicated by it, are rightly adapted to be the parent of the Ptolemaic system of astronomy in an age when our own Earth was supposed to be the centre of the Universe, and all else in space revolving around it.

Or, perhaps, I might with more propriety say, that the system of astronomy as then understood was exactly calculated to reveal the properties of the circle such as we have shown it to be, in its relation to time and the revolutions of the great astronomical circles. As the ancients had not the advantage of instruments for observation such as we now enjoy, they must have had some means of calculation to supply the deficiency, and the circle alone could supply it.

Perhaps it is not amiss to state here, that by the aid of the quadrature we are able to calculate a sidereal lunation, the mean year, the sun's distance, etc., with perfect accuracy, each by a simple problem, without the aid or requirement of observations of any kind whatever.

Among the minor evidences which we find, that favor our belief, antiquarians inform us that the number 6561 was held to be a charmed number among the Egyptians, and this number we find to be the foundation of the circle in its relation to the square; its power and influence in the quadrature is certainly sufficient in my estimation to justify such a superstition, and it is natural to suppose that among an ignorant people the superstition should remain, after the learning which had discovered its properties and influence had been lost.

Professor Ingraham, in his fiction of the "Pillar of Fire," has related the Egyptian fable of the Phœnix, which every 651 years returns to its funeral pyre of the sun, and there consumes itself, but immediately rises again from its own ashes to renew its flight for another period. By a very pretty fancy the bird is made to represent a star, and to answer to the transit of Mercury across the sun's disk, in which the planet is lost to sight, and appears to consume itself in the sun's rays during the transit, but emerging to a new life as soon as it is past. I am not aware whether the transit of Mercury has been calcu-

lated for so long a period as 651 years. They occur frequently, but irregularly, generally once in about thirteen years, sometimes in a lapse of only seven years. Herschel says that they return in nearly the same order every 217 years; of course, then, they would return in the same order every 651 years, that period being three times 217. I know nothing of the authority from which the Professor claims to unite the period and the star with the fable, but it is well known that the fable of the Phœnix is a legendary superstition of the Egyptians, and I can easily associate his version of it with the superstitious remnants of the learning once possessed by their ancient Magi. The quadrature, as we have demonstrated it, reveals the fact that 651.4409+ years, is the Earth's synodical period in revolving round the sun, exactly answering to the Moon's synodical period in revolving about the Earth; a period, I believe, unknown to astronomy, except in vague and uncertain theory; and that period is, as I believe and affirm, also the period of the revolution of the Magnetic Pole around the Geographical Poles of the Earth; and here are two facts which, being fully established, are worth a life of study to learn. The subject is treated in my second lecture on Polar Magnetism, read before the Geographical Society, and published in the volume of their Transactions for 1869–70.

The circle was also used as an emblem by the Egyptians. It was sculptured over the doors of their temples as an emblem of Deity, power, and duration, in which they showed a just estimate of it, for the circle contains and controls all magnitudes and all space—it is at once the greatest and the least of all created things.

The Pyramids of Egypt, since they have been visited by scientific men, are understood to have been built for the purpose of creating permanent standards of measure, and in reference also to some astronomical truth. A gentleman of Cincinnati has examined the construction and the measurement given of one of these pyramids, and he finds my ratio of circumference to correspond with the measure of the Egyptian cubit so nearly, that he thinks it must be the same. He quotes the value of that measure in British inches, as given by different persons who have scientifically examined the subject with a view to ascertain its exact value. The following are the results of their examinations expressed in British inches:

Cubit of Elephantine..................	20.625+
" of Memphis (Jommard)...........	20.473+
" of Turin...........................	20.578+
" of Sir Isaac Newton...............	20.604
" of the French measurement in 1799.	20.611+

As my ratio of circumference is 20,612, it is easy to see their agreement.

The same gentleman reports to me that the application of my ratio enables him to reconstruct the Pyramid of Gizeh on paper, with all its chambers and in all its dimensions.

That which most challenges our belief is, that the quadrature of the circle being once known should ever have

been lost, being, as it is, an elementary truth, and the foundation of all growth, magnitude, and extension in our world; and we propose now to trace the circumstances that shall reveal to us when and how this has happened.

Pythagoras, the earliest of the Grecian philosophers who taught mathematics and astronomy, before establishing his school of philosophy, 570 years before Christ, went into Egypt and sojourned there for the purpose no doubt of learning something of Egyptian philosophies, preparatory to setting up for himself in his own country. This seems to have been the habit of all or nearly all of the ancient philosophers of Greece, and is one of the evidences of the early civilization and learning of the ancient Egyptians. It is more than probable, however, that this civilization and learning had been long in its decadence at the time when Pythagoras visited Egypt, otherwise we should have had through him and others something more of the connecting link between the ante-pyramidal learning of Egypt and the more modern Greek school than we now have.

It is fair to presume, from the evidences we have, that the chief learning of the ancient Egyptians was in mechanics, mathematics, and astronomy. They probably had little of history or other literature; or if they had, whatever remnant was then left of it was destroyed by the conflagration of the great Alexandrian Library in the wars of Julius Cæsar. I have always supposed that the destruction of that Library lost to us all knowledge of ancient Egyptian history and literature; and that if it it had still existed, we should not now be ignorant of the origin of the Pyramids, or unable to decipher the inscriptions on their sarcophagi. The then rising Greek school sought only from their Egyptian neighbors the more solid sciences, and in that age of the world had little object in preserving the history and light literature then probably declining in Egypt.

The establishment of the Alexandrian Library is accredited to Ptolemy Philadelphus; but it is much more probable, that, as a liberal and enlightened prince, he made great additions to it, and that the Library had long been the depository of ancient learning, and perhaps also of the archives of the nation in days of its highest glory and prosperity. The Library was supposed to contain seven hundred thousand folios or parchments, and in those days of slow production of manuscript and copy it would not have been possible for any prince in one reign or in a dozen reigns to have accumulated so vast a Library. The destruction of the Library, and the corruption and vice of the people which culminated in the reign of Cleopatra, finished the degradation of the Egyptian mind, which had long been sinking under the influence of luxury degenerating into license.

Pythagoras brought with him from Egypt into Greece a system of mathematics consisting chiefly of the mechanical properties of numbers, which he taught to his disciples, and his pupils carried the system so far, that

they professed to demonstrate spiritual truths by numbers only. They will readily be forgiven their error, if we first examine effectually the mechanical properties of numbers, and then remember that they had not the restraints upon their imagination which modern Geometry imposes.

Abated of this extravagance, the system which Pythagoras taught is one of the evidences that what he learned of mathematics in Egypt was but a sequence of the study of the mechanical properties of numbers adapted to the principles of revolution, and was the basis of calculation in the early periods of the Ptolemaic Astronomy.

I hold the Ptolemaic system of astronomy, like the Library, to have been only revised, and perhaps enlarged, but not invented during the reign of either of the princes of that name; I believe it to have had its origin by slow degrees from the remotest periods of time, when men gazed into the heavens and beheld the motions of the stars.

While Egyptian learning had been long declining, the Greek school in mathematics began its rise. Dating from the time of Pythagoras onward, from time to time it received the contributions of noble minds, until at length culminating in the master mind of Archimedes, and the production of Euclid, the system of Geometry and Mathematics was established, such as it remains at this day, with little addition, and without a single change.

This was the age which consummated Grecian renown in science and the arts, as previously in war. Centuries had been occupied in producing results which we have embraced in a paragraph. A people, poor, proud, ambitious, passionate and brave, the Greeks flourished and grew refined and powerful in a state of war, such as Egypt, plethoric with wealth, effeminate from luxury, and cowardly and negligent from that effeminacy, sank under, till the light of her science had gone out in a darkness that became proverbial.

Archimedes, by his system of demonstration, rejected the mechanical properties of numbers as taught by Pythagoras, and made numbers secondary to the measurement of straight lines and angles. He thus dropped the only connecting link between the Egyptian and the Grecian school of mathematics, and rendered it forever impossible by any means which his system afforded to find the equality of straight lines to curved lines. From that time may be dated the loss of whatever might have been before known of the circle in Egyptian science. If Archimedes, in his system, had permitted the reasoning from shapes in their whole and distinctive qualities, he might probably have preserved the knowledge, which without it has been lost for two thousand years. His system was less theoretic, more practical in its operation, more palpable to the senses, and as a consequence more definitely comprehended than that of Pythagoras or the Egyptians. It was therefore, in one branch especially, a very great advance in practical geometrical science. In comparison of all former systems, it was like modern telegraphy, compared with Franklin's attracting the lightning from the clouds. But it should

be observed that, without Franklin, telegraphy would probably never have been known, and without Pythagoras, Archimedes would probably have died a Helot.

The excellence of the system of Archimedes soon gave it the preference over all others; but it had one grand defect, with the value and importance of which the world is yet to become acquainted. It contained not a single element, having the least or the remotest affinity to the properties of curved lines in their intrinsic relation of value to straight lines; and curved lines, not straight lines, lie at the foundation of all growth, magnitude, extension and area in our world. The universe, our world included, is built and moves upon curved lines, and nothing else.

Archimedes himself knew the deficiency, but with all his knowledge he knew not how to supply it. He invented a problem, however, by which he showed an approximation, and in doing so he in fact acknowledged that he knew nothing of the distinct properties of the circle, or of its exact relation to the extension of area as measured by the square.

But now there approached a period in the history of the world, which men have agreed to call the "dark ages." Following the crucifixion of our Lord, states and empires were overturned, science languished, art nearly perished, literature faded away, and for more than 800 years, no improvement in the world's condition was manifest; only a few of the productions of mind during the period of a higher civilization were preserved, consisting chiefly of some of the Greek poets, but along with them the invaluable works of Archimedes. At the end of that period, Egypt had become a forgotten country, and all her science had lapsed into the obscurity of utter darkness.

But light and truth have never perished from the earth. While our ancestors of Western Europe were yet sunk in barbarism, Rome, even in her declining power, sent her legions there to conquer the country. They carried with them some of the evidences of the civilization which had existed in the east; the seed was sown in ground not before occupied, and learning began to dawn in the then Western hemisphere; the system of Archimedes and of straight lines took possession of the mathematical branch of learning, and all the learning which Europe or America enjoys to-day in mathematical science is directly the result of the works of Archimedes, which have been accepted and adopted *with the grave and inherent defect of an absence of all knowledge of the properties and value of curved lines.*

The problem which Archimedes invented and used for an approximation is the same that is in use to-day in every mathematical book, and by every mathematician in Europe and America, and their continued use of it is a sufficient acknowledgment that they know no better method; that in fact they are ignorant of any of the exact properties of the circle, or of curved lines, and this is fully attested by their own standard-bearers.

Playfair, in his Elements of Geometry, a standard work

in the schools, admits the deficiency, and says that geometers "know no better method;" and Torrelli, the learned commentator of Oxford, on the works of Archimedes, distinctly says, "that geometers by their approximation" determine nothing whatever relative to the properties of "curved lines."

Under these circumstances, without a single element of truth on which to ground his opinion, Legendre had no need of a demonstration to prove the impossibility of squaring the circle by any method known to him; it ought to have been self-evident to him before he attempted any demonstration, and if it were not so, it only proved that, in this particular, he did not comprehend his own science, which he certainly did not, if he has spoken the truth of his own mind.

When Legendre made the declaration, which he did in his geometry, that he had "determined the quadrature of the circle as accurately as the root of any imperfect square could be determined to the same number of figures," and that "no person having the least pretension to geometrical science would ever make the attempt to find it more exactly," he was utterly and unmistakably ignorant, not only of the first principles embodied in curved lines, but of every element of their construction, and every special property belonging to them. The Academy of France, however, endorsed his dictum, and thereupon passed a resolution, that no paper purporting to be on the subject of the Quadrature should forever thereafter be received or considered by the Academy; and so far as I can understand they have ever since adhered to this determination.

The Royal Society of London followed soon after with a similar resolution, and Montucla's garbled and prejudiced history of the Quadrature was adopted to sustain these societies in their bold and presumptuous tyranny. The influence of this tyranny was to make it disreputable for any person to speak of the Quadrature as a thing attainable. He was at once branded as an imbecile, an enthusiast, or a lunatic, and such is the common reproach which is to-day heaped upon any one who shall even intimate the possibility of its discovery, and such epithets have not unfrequently been applied to me by persons of pretended education in geometrical science.

It seems impossible that a man of Legendre's power should have been entirely ignorant of his error. In his profound investigation of the Principles of Geometry, it would seem that the absence of any element of truth regarding the properties of curved lines could not have escaped his notice, and hence his declarations above quoted, and his concurrence in the resolution of the Academy of France, are all the more remarkable. Unfortunately the knowledge of the error would not reveal to him the truth, and in his pride he was unwilling to acknowledge the one without supplying the other. With the natural impulsiveness of his countrymen, therefore, he threw the whole aside, as without the pale of demonstration.

The action of the Academy and the Royal Society,

however, cannot be so easily justified. Their resolutions have enjoyed more than three-quarters of a century of usurped domination over the minds of men touching the Quadrature, and apparently with the consent if not the abject submission of the mathematical world. The very learned President of the Smithsonian Institute at Washington wrote me more than twenty years ago, in reply to some papers that I sent him, and referring me to Legendre's remarks, before quoted, said, that "he presumed, on my seeing what that great geometer had said, I would think no more about it." I had already seen what Legendre had said, and had satisfied my own mind that Legendre knew nothing at all of the properties of curved lines.

All that Legendre or the Academy could have been justified in saying, was, that the system of geometry with which they were acquainted contained no element of truth in regard to the value of curved lines, and that they knew nothing better than the Archimedean approximation. Their error lay in the egregious conceit that their system of geometry contained the whole truth of nature, when, in fact, it is nothing more than a *method* for finding out the truth.

The resolutions which the Academy and the Royal Society passed were in blind ignorance of what they were doing, without comprehending in the least the nature or the importance of the question they were deciding, and worse still, as we have already seen, without the knowledge of a single principle of truth involved in it.

It will no longer be a mystery, therefore, that the Quadrature of the Circle, which might have been and probably was known to the ancient Egyptians, has never been reached by the modern geometry of the Greek school, for the one plain reason that in the construction of the *system itself all the elements of the Quadrature were rejected, set aside, and left out.*

But it is time that the institutions of America should reply to the Academy of France and the Royal Society of London in terms fitting the tyranny they have so long usurped.

If there are still those who think the Quadrature a useless question, I would suggest to them that if the study of the elements and properties of straight lines and angles has been productive of so much knowledge and usefulness in the world as we see daily developed in astronomy, navigation, works of architecture, engineering, etc., may not a still further development be expected from a knowledge of the properties of curved lines, which are the first elements of Nature in the production of shape, magnitude, and motion throughout the natural world?

The Quadrature of the Circle is as plain, as simple and comprehensible a problem, when the properties of curved lines are considered, as anything contained in Euclid. It is simply that the circle and the equilateral triangle (the two first shapes produced in Nature) are opposite one another in the elements of their construction, and hence, in their fractional relations to the square, they are opposite one another in ratio of the squares of their diameters.

JOHN A. PARKER.

POLAR MAGNETISM:

A PAPER READ BEFORE THE

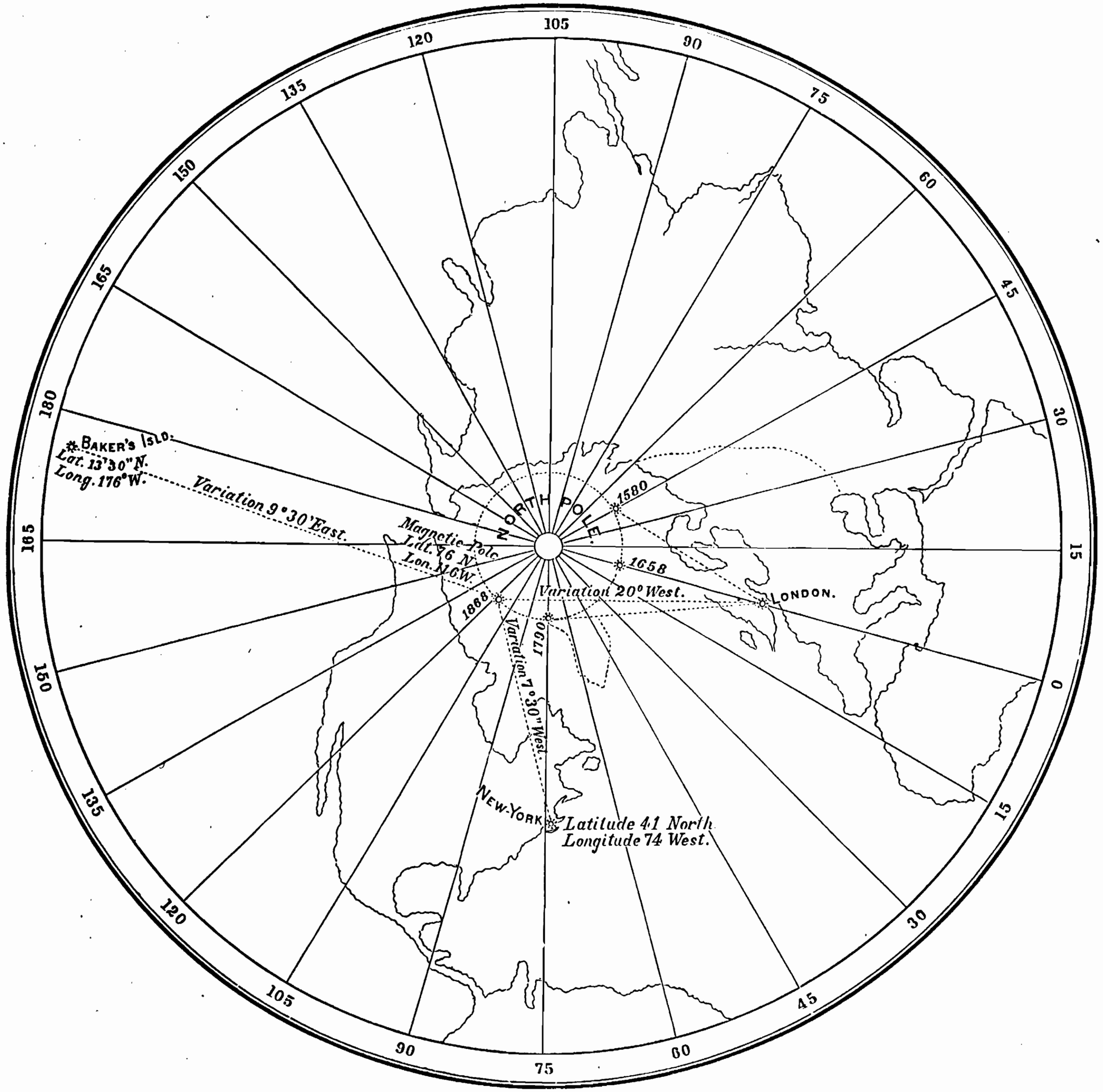
AMERICAN INSTITUTE,

ON

THE CAUSE OF POLAR MAGNETISM; THE ATTRACTION OF THE NEEDLE TO THE POLE; THE VARIATIONS OF THE COMPASS, AND THE PHENOMENA INCIDENT TO THE SAME.

BY JOHN A. PARKER.

NEW YORK:
JOHN WILEY & SON,
15 ASTOR PLACE.
1874.



By a mistake of the artist in the above Diagram of the Northern Hemisphere, the Magnetic Pole is placed too near the North Pole; as a consequence, the

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POLAR MAGNETISM.

With the consent of the society, I propose this evening to read a paper which I have prepared, on the subject of polar magnetism—the attraction of the needle to the pole—the variations of the compass, and the phenomena observable as incident to the same.

The subject stretches over a vast area of natural truth, and therefore on an occasion like the present I must necessarily be very brief on each particular point. I do not propose to examine, criticise, or discuss any of the written theories on the subject, but to limit myself to my own personal observations and reflections, and to draw conclusions only from such phenomena as are manifest to our vision and sense, irrespective of all previous theories and speculations in regard to them. The subject has not only a vast scientific importance, but also a great commercial value, and it is in this latter connection chiefly that, in the first instance, I have been led to consider it.

Some knowledge of polar attraction is necessary to the study of astronomy; it is of national importance, inasmuch as the direction of the needle is oftentimes the arbiter of the lines and boundaries of our national domain, as it is also of our private estates; we rely upon it when, in traversing the continent, we plunge into the depths of the forest, and it is our only safe guide over a trackless ocean, through darkness and tempest, to the haven we seek. Tens of thousands of lives, and hundreds of millions of property throughout the world, are every hour dependent on it for safety; any inquiry therefore into its causes, and the laws which govern it, is invested with an interest second to no other; and, notwithstanding this, it may safely be said that, in comparison with the whole that may be known, very little is at present positively known concerning it.

The existence of polar magnetism was first revealed to us by the discovery of the compass. There is some doubt as to the exact time of that discovery, but it is supposed to have been first put to practical use by the early navigators in their commerce on the Mediterranean Sea. At that time, and long afterward, the needle was supposed to tend always toward a fixed point in the north; and therefore, when Columbus undertook his voyage of discovery, he was greatly surprised and not a little perplexed to find that, as he sailed westward, the needle gradually changed its direction; and his crew became so much alarmed by it, that all the steadiness of mind which that great navigator possessed was necessary sufficiently to calm their fears to prevent an open mutiny, and thus defeat his enterprise. In later times, by the observations

of later voyagers, explained by charts, together with improved instruments for observation, and the means invented for determining the variation at any point, navigation has been made quite safe and certain to the careful and skilful navigator. But, so far as I am informed, the *cause* of these variations of the compass, and the laws which govern them, are wholly unknown to science.

I shall not detain you with a detail of the course of reasoning by which I have, in my own judgment, arrived at a full conclusion in this matter, but state broadly, and at once, what that conclusion is, and then explain some of the evidences on which it rests.

The CAUSE, then, of the variations of the compass, which some have supposed to proceed from the oscillations of the Earth, is, in my judgment, THE REVOLUTION OF THE MAGNETIC POLE AROUND THE NORTH POLE.

By the Magnetic Pole we mean that point on the Earth's surface within the Arctic circle to which the needle points. By the North Pole, we mean of course the polar axle of the Earth on which she turns in her diurnal revolution. And by the variations of the compass, we mean the divergence of the needle's point east or west, from the true north point, with its variableness at different times and in different places.

The revolution of the Magnetic Pole occurs gradually, through a long period of time, and, according to the best data which I can obtain, is completed only once in about six hundred and forty years. The time may be found to be somewhat longer or shorter when settled by accurate observation.

The exact position of the Magnetic Pole has never been accurately known. It is known, however, to be at present situated on the North American Continent, in a high latitude, and considerably west of the longitude of New York. This was made manifest by the observations of Captain Ross in his polar expedition. He placed the Magnetic Pole in about latitude 70° 30′ north, and longitude 96° west from Greenwich. It has changed considerably since that time, and is at present both farther north, and farther west, than he placed it.*

The variation of the compass from the true north point at London or Greenwich is at present westerly. At New York it is still westerly in a less degree, and at the islands in the Pacific Ocean it is easterly. Now, if the accurate variation at each of these points be taken (all local attractions being absent), and lines in the direction

* That Captain Ross did not quite reach the Pole, is, I think, self-evident, because if the situation of the Pole had been in 70° 30′ north, and longitude 96° west, it would have given a greater variation at London than existed at that time. But that his was the nearest approximation to the truth ever before known, is nevertheless quite certain. Almost at the same period of time, Humboldt placed it in his estimation in latitude 79 °north, and longitude 27° west from Greenwich. This was certainly very far from the truth, as the observations of Captain Ross, and all other authentic facts, conclusively show. For myself, I am unable to perceive on what basis of known truths Humboldt could have grounded his opinion.

of the needle be produced northerly, they will meet at a point not far from 76° north latitude and 118° west longitude from Greenwich. And that point, wherever they do meet, may safely be affirmed to be the present position of the North Magnetic Pole. It is not necessary for our present purpose that we should fix the exact point, nor are the means at hand to do it if needed—it must be done by careful and repeated observations at the same time on different and widely separated meridians, so as to exclude all possibility of error.

The only point necessary for us to determine for our present purpose is, to show that which has already been shown, viz., that the Magnetic Pole is situated at a considerable distance from the North Pole, and that being proved, we must now look for the evidence that it REVOLVES ABOUT THE NORTH POLE, which we will proceed to do.

In the year 1658, as shown by the records at Greenwich, the needle pointed due north from that position.* It is, then, certain, that in 1658 the Magnetic Pole was situated on the meridian of Greenwich and between Greenwich and the North Pole, or, coinciding with the North Pole, or in a line beyond it, at 180° west. No other supposition is possible. From 1658 the needle began to have a westerly variation at Greenwich, which continued to increase till the year 1818, a period of 160 years, when it had obtained its greatest variation. Now, on the suppo-

* Brand's Dictionary of Science, article "Magnetism."

sition of the revolution of the Magnetic Pole, it is evident that it would attain its greatest variation when it had passed over 90° in the circle in which it revolves, and that in its progress of revolution the variation would then become less. Accordingly we find that in the year 1818 the westerly variation at Greenwich began to grow less, and from that time to the present has continued to decrease, which is in accordance with the necessity of the case in the supposed revolution.

From 1818 to 1868 are fifty years, in which time, supposing the period of one hundred and sixty years to have been the exact time in which the Magnetic Pole, by its revolution, passed over ninety degrees of longitude in the circle in which it revolves, in fifty years it would pass over twenty-eight degrees more, which would place it at this time in longitude 118 degrees west, where it is found to be, as nearly as can be determined.*

Coming now to the longitude of New York, although we have no record here going back to 1658, yet we know the fact that the westerly variation is increasing at New York, while it is decreasing at London or Greenwich, and this also is a necessity of the case in the supposed revolution; and if the hypothesis of revolution be true, and the period from 1658 to 1818, when it passed from zero to

* In placing the Pole at 118° west, I give it the same ratio of progress which it appears to have had for 160 years, viz., from 1658 to 1818. But if we consult the present variation as observed at New York, it would appear to be not quite so far west, or at about 116° west.

its greatest westerly variation at Greenwich, be perfectly accurate, then it will continue to decrease at Greenwich till the year 1978, when the needle will again point due north at Greenwich; while at New York (longitude 74° west) the westerly variation will continue to *increase*, and will not attain to its greatest point till the year 1950. And corroborative of all this, supporting the hypothesis of revolution, it is known, that about the year 1790 the needle pointed due north from New York, as it should do, while to-day it has a westerly variation of nearly eight degrees, which again is in perfect accordance with, and indeed an absolute necessity in, the supposed revolution. The period from 1658 to 1790 is one hundred and thirty-two years, being the *pro rata* time necessary to pass over 74 degrees of longitude; and here it will be apparent that in 1790, when the needle pointed due north at New York, the magnetic pole was then situated on the meridian of New York, and it is now 44 degrees west of it; thus showing the progress of revolution.

Going still farther west to the islands in the Pacific Ocean, we find there, that the variation is *easterly* and growing less, still fully supporting the hypothesis of revolution.

I will mention but one other fact in proof of my position. Dr. Bowditch in his Navigator mentions, that in 1580 (a period 78 years earlier than any yet mentioned) the needle at London then pointed *eleven degrees and some minutes* EAST. The Magnetic Pole must then have been situated in about latitude 76 degrees north and 45 degrees *east* from London, and between that time and 1658 it had moved up to the meridian of London. Now, if we add to this the facts already proved, that from 1658 to the present time the Magnetic Pole has moved from the meridian of London to 118 degrees west of it, and having passed over the meridian of New York, the proposition, that the MAGNETIC POLE REVOLVES ABOUT THE NORTH POLE once in about 640 years, is then, I think, fully demonstrated; and there is nothing lacking in the demonstration but the exact time of the revolution, which, as I have said before, must be determined by future careful and accurate observation. It no doubt may be determined with nearly as much accuracy as the periodical return of an eclipse.*

Two other important facts are also proved by what has been shown. First, that in 1658, when the needle pointed due north at London, the Magnetic Pole was then situated on the meridian of London, and between that and the North Pole, and not on the opposite side of the North Pole, at 180 degrees; and *secondly*, that the revolution is from east to west. Because, had the Magnetic Pole then been situated on the opposite side of the North Pole from London at 180 degrees (and it must have been on one or the other of these points), and revolving in the ratio as we have seen, in order to produce a westerly variation at Greenwich or London, the revolution must have been from west to east. And although in that case the variations at London would have been precisely the same as they have been, yet in passing over 118 degrees of longitude it would place the Magnetic Pole to-day in longitude 62 degrees west, where we know it is not, and the variation at New York would have been at present easterly and increasing, instead of westerly and increasing in that direction as we now know it is. It is therefore certain that the revolution is from east to west.

* The only evidence which I have seen going to disprove my hypothesis of revolution, is a remark of Dr. Bowditch in his Navigator; while at the same time giving some dozen or twenty statements of the variations of the compass at different times and different places, all of which are perfectly consistent with, and corroborative of, the truth of my position. He also says that in 1708 the variation of the compass in Massachusetts (probably at Cambridge) was 8° west—in 1742 6° west—and in 1780 2° west. This is not possible with the truth of my position, and one or the other must be in error. If he had said east instead of west, it would have been in perfect accordance. It may be an error of print, or the difference may have been caused by local attraction; but I am more inclined to think that the difference arose from the fact that the observers in these cases *reversed the poles*, and that their meaning was, that the variation of the true north from the needle's point was so many degrees west. If we suppose this to have been the case, and that the variations, as we now define variation, were east and not west, their record would then be in perfect conformity with both the theory and progress of revolution as we have explained it. Such a supposition is not improbable when we consider that one hundred and sixty years ago the whole subject was but little understood or attended to in this country; and when we reflect that from 1658 to the present time the Magnetic Pole has occupied two hundred and ten years in passing from the meridian of London to its present position, it could not have been in a situation to give a westerly variation at Cambridge in 1708, and the foregoing solution of the Cambridge record is the only one possible.

Having now gone so far, and as I believe demonstrated beyond a doubt, the fact of the revolution of the Magnetic Pole, the subject rises to a higher sphere, and the questions naturally suggested to the mind are, what is Polar Magnetism? and what is the *cause* of this revolution?

These are lofty questions indeed when we consider the source and influence of the things we are to inquire about, and they are only to be approached with caution and reverence. I will, however, as in the former case, answer these questions at once, as I believe to be the true solution of them, and then give a synopsis of the reasons which direct my judgment to such conclusions.

I regard magnetism as a universal principle, pervading all space, and impressed on all matter, and one of the forces employed to regulate and control the universe; and I consider the revolution of the Magnetic Pole as being *caused* by magnetic attraction *to the highest centre or system, to which the Earth in her various revolutions is immediately related.* The attraction is from centre to centre, and the magnetic needle balanced in its horizontal position becomes an *indicator only of the line of attraction*, and directs itself always to that point on the Earth's surface which is in a line with the centre of attraction. There is therefore no absolute Polar Magnetism, or Magnetic Pole. It is ideal, not real, and like the Earth's axis it is only imaginary, but necessary to be considered in order to illustrate a truth.*

* In saying that there is no absolute polar magnetism, I must be understood to mean that there is nothing there, at the Pole, to cause the attraction of the needle; its motion is governed by a higher law, and by a force with which the locality of the Pole has nothing to do.

I have said that I regard magnetism as a universal principle, and I so regard it because it is everywhere present in our world—in every place ever visited, in every imaginable position, it is there, possessing the same attributes and exerting the same influence. That which we call a "magnet," however, is not magnetism, any more than an electrified body is electricity—it is simply a magnetized body or substance, capable only of retaining its magnetism for a limited period. Both magnetism and electricity are latent and hidden principles in nature, the very existence of which is a mystery, and of which we know nothing except as we can witness their effects. They are natural forces, and although latent in themselves, they are yet capable of rising to a force little short of infinite, whenever circumstances combine according to their nature to call them into action. They are not the same, although they may be different phases of the same principle; but I doubt if enough has been learned of either, to justify a decision in that particular. They have many affinities in common, such for example as the attraction of iron—also the power of some bodies to arrest and turn away their current, and of others to receive and retain for a longer or a shorter period an impregnation of their qualities, which is again imparted to other bodies when placed in contiguity or brought into contact. The loadstone is only such a substance, to which magnetism has a strong affinity, and which has therefore the power of absorbing and retaining longer than any other known substance the magnetism it has received.

Both magnetism and electricity may be excited to activity by motion or revolution. Everybody has seen electricity excited by a revolving machine, and if you stand under a revolving belt in a manufactory, you will feel magnetism enough to raise the hair on your head; but no one will for a moment suppose that either magnetism or electricity is created by these motions—they are simply roused from inactivity, as latent heat is rendered active by motion, concussion, or attrition. The difference between the two I conceive to be, that the force of electricity is eccentric, diffusive, and equalizing; that of magnetism is concentric—attracting to the centre.

From all the phenomena which I have been able to see or observe, I have come to the conclusion that what we call polar magnetism is the result of a magnetic force rendered active by revolution. As before stated, the force is concentric, attracting to the centre. It is therefore centripetal, and, considered as a universal principle of nature, it identifies itself with that force which astronomers call the "attraction of gravitation," a force known to exist, but for which no satisfactory cause has ever been assigned; and we are here led to ask, what is that force, if it be not magnetism? By whatever name it may be called, we believe it to be the same force which directs

the needle to the Pole, and which we habitually call "magnetism." That it is a natural force derived from a latent principle, and put in motion by a forward revolution, I think fairly deducible from what is well known to every astronomer, viz., that the magnetic attraction, or "attraction of gravitation," call it by what name we please, in revolving bodies is the opposite of that centrifugal force created by their revolution, *and always equal to it.* And since the motion of the heavenly bodies is sometimes faster and sometimes slower, as proved by Kepler's law, that in passing round their elliptical courses, they pass over equal areas in equal times, therefore the centrifugal force created by their motion is sometimes greater and sometimes less, which must always be met by an equal development of the opposite and concentric force to keep the planet in her orbit. And it is well known that such development is always obedient to the rate of motion of the body revolving. It is evident therefore, I think, that the activity of the magnetic force is produced and regulated by FORWARD REVOLUTION.

That the attraction of the needle is to the centre of the Earth, I think is shown by a variety of circumstances. *First*, the shape of the globe is such that direct rays falling upon its surface, necessarily concentrate at its centre, and the attracting force would seem to be of that character; and *secondly*, if a needle be magnetized in but one end, that end will point downward towards the centre, though not at all increased in weight; but if both ends be magnetized, and the needle balanced to a horizontal position, it then points *in the direction of a line with the magnetic current.* For the same reason that the attraction is to the centre, if a compass be placed near the magnetic pole and compelled to keep its horizontal position, it refuses its duty and will turn every way at random; but if left to itself its point tends downward towards the centre of the Earth, and this is what is called in navigation "the dip of the needle," which increases always as you approach towards the Pole.

From these circumstances it has been sometimes assumed that the interior of the Earth is a powerful natural magnet; and even with my theory it is so, though not in the sense as has been supposed; for it will be seen at once that, if the attraction were to the centre of the Earth, by a fixed magnet, then there could be no change of variation in the same locality; and as we have seen that such change does take place, that theory is at once exploded.

We prefer, then, to follow our own choice, and assume that the direction of the needle to the Pole is only an indicator of a line of higher attraction; and that the immediate motive power is a concentric magnetic force, rendered active and *in*volved by revolution. Let us see then what will be the result.

We will suppose that the revolution of the Earth on her axis is the governing cause, or controlling force, the attraction being to the centre of the Earth; then the needle being balanced to a horizontal position, would always, and in all places, point due north and south, in a line with the centre of motion; which it never does, except in two places at a time (the antipodes of each other and through their lines of longitude); and there could then be no revolution of the magnetic pole, and no variation or change of variation. We must, therefore, look further for our evidence in support of the truth of our position.

Let us suppose, then, that the earth's revolution round the sun is the governing cause, the line of attraction being from the sun's centre to the centre of the earth.* It will be seen at once that in the earth's revolution round the sun, or with the sun round any other centre, her axle on which she performs her daily revolution, does not, and can not lie in a line with the line of attraction from centre to centre, but must lie at an angle to it. And how is this? If we draw a line from the north pole to the south polar axle through or over the Atlantic Ocean, and parallel to

* We make no question here of the laws of gravitation as demonstrated by Newton, or the character or degree of its force, which is always directly as the masses of the heavenly bodies, and inversely as the squares of their distances; the only question is the identity of the gravitating force with the magnetism which directs the needle to the pole, by whatever name we may designate either. It is worthy of remark here, that Mr. Barlow, in his experiments to correct the influence of iron in the ship on the compass, found that the attraction of iron on the magnet was of the same force and character as the "attraction of gravitation," although he does not appear to have observed it himself. He found that a hollow globe of thin iron had the same influence on the magnet as a solid globe of the same surface dimensions. In other words, he found the attractive force to be directly as the mass, and inversely as the square of the distance.

the earth's axis, we find that the north magnetic pole lies west of such line at an angle to it, and the south magnetic pole lies east of such line at the same angle to it, and the result so far answers to the theory. But here we meet with difficulty; for if the earth's revolution round the sun should be the cause of polar attraction, the line of attraction being at an angle to the earth's axis, then because one body revolving about another body also in motion gains one revolution of the body around which it revolves, therefore the magnetic pole should revolve about the north pole once in every year, which it does not. Again, we have another difficulty —the earth does not revolve about the sun with either pole turned towards him; but the tropical zone is forever turned towards the sun, and never the polar regions; and the line of polar attraction is seen to be at a wide angle to the line of the sun's attraction. Confirmatory of our theory, however, there is an inferior polarity in the Indian Ocean, north of the equator, sufficient, in passing over it, to disturb the needle; and another in the Pacific Ocean, south of the equator, the two at about the same angle to the equator, as the north and south poles to the earth's axis. These polarities may answer to the sun's attraction, and the two, that is to say, the equatorial polarities and the north and south magnetic poles, are, as I believe, travelling together in perfect unison with each other. The south Pacific polarity was, I think, discovered by Captain Cook, and has since its discovery, as laid down by him, moved some twenty or more degrees west, thus keeping pace with the north magnetic pole on this continent.

It is evident, I think, from what has been shown, that the north and south are the superior polarities, to which all others are secondary, because all others are in their influence purely local, while the influence of the north and south is paramount and universal.

We must, therefore, look to a yet higher source for the origin of that magnetic attraction which shall *cause* the revolution of the magnetic poles in the period of time in which we have seen that they do revolve.*

We have already seen that magnetism is a universal principle, and one of the forces which nature employs in the government of the universe; we have seen, too, that the force which it exerts is centripetal, and identifies itself with the force which controls the motion of gravitating bodies; we have seen that the line of attraction from any centre around which the earth may revolve must be at an angle to her axis on which she daily turns, and that the polar attraction *is* at such an angle; we have seen that if the attraction of the needle to the pole be governed and controlled by the attraction of magnetism from a centre around which the earth revolves, and such attraction is at an angle to the earth's axis, then, that point on the earth's surface to which the needle is directed, and which we have called the magnetic pole, shall revolve about the north pole in the same time in which the earth revolves about the governing centre of attraction; and we have seen that

* The North and South Poles revolve together, each being always opposite to the other, and at an angle to the Earth's axis.

the magnetic pole does revolve about the north pole once in about six hundred and forty years, taking the period of one hundred and sixty years to be accurate, in which it has been seen that ninety degrees of the revolution has been performed.

All these conditions are necessary to the truth of our theory (and more might be both cited and proved), and they are conditions, all of which we have seen to exist; and one of these conditions, viz., the revolution of that subtile influence and immaterial point which we have called the "magnetic pole," could not exist or be performed by any other means than the magnetic attraction to the centre around which the earth revolves.

Our limited astronomy has given us the data of no higher revolution of the earth than that around the sun, although it is inferred from the order of things that she may revolve in company around a higher sphere, and to that we must look for the attracting force that will fulfil the necessity of the case. I shall here be found trespassing slightly on some of those creations of astronomical science which imagination has built on too slight foundations of truth, and which time will, I think, sweep away by discoveries in revolution as simple as that which swept away the ancient systems of astronomy from the faith of mankind. The simple turning of the earth on her axis was found to perform all those wonders, and account for all the phenomena, which men's minds had for centuries supposed employed

the might of Heaven to move the whole universe around us for the accommodation of light to our comparatively little world.

The earth, we know, is balanced within a planetary system, revolving about the sun in perpetual order. We must then admit one of two things, to wit: That either the sun, with his system, revolves about another and a higher system, which is beyond our power of immediate observation; or, that the sun is fixed in the centre, and that his system embraces the whole visible and invisible heavens—which we have good evidence that it does not. The supposition is contrary to reason, and contrary to the order of revolution. We therefore assert, as a necessity, that the sun, with his system, revolves about another and a higher system, carrying the earth with him in the same manner as the earth revolves about the sun, carrying the moon with her. This is not a disputed proposition, and I only state it in this form to connect the thread of my argument.

The fact being admitted, it is reasonable to conclude that this is the highest revolution or source of magnetic attraction to which the earth—separately considered—is immediately related. First, because the attraction will necessarily be from system to system; and, secondly, because the earth being situated in the midst of the solar system, a part of it, and only a point in it, the line of attraction would be always nearly the same; or if affected at all by any other revolution, in a planet of the earth's magnitude relatively to these great systems, the effect would be a scarcely perceptible vibration. *

If these premises be all true, then it will follow that what we call the magnetic pole shall revolve about the polar axles of the earth in the same time in which the earth moving forward in company with the solar system performs a complete revolution relatively to that system around which the sun himself revolves. Let me not be misunderstood; this is not the sun's period; that of course is very much greater. The earth is only a satellite of the sun, and in the revolution of which we speak she has but repeated exactly, but on a grander scale, the same phenomenon which our own satellite, the moon, performs monthly; accompanying the earth in her orbit, at the re-

* I have not time or room here to attempt the demonstration of the truth, but I have no doubt whatever that, although the higher revolution is the governing cause of all, there is yet a daily and a yearly revolution which it would not be difficult to trace. Dr. Bowditch mentions a daily vibration (which is no doubt a revolution), amounting to some minutes of a degree, and I recollect that a few years since the same thing was observed by M. Leverrier, Director of the Observatory at Paris, which for want of another reason he ascribed to expansion by the sun's heat; but although magnetism is more or less intense in a high or low temperature, I cannot accept this as a sufficient cause for a change of variation. It may be caused by the sun's attraction, he being sometimes east and sometimes west of us; but the return of the needle during the night to its position of the previous day is too gradual to be caused by temperature alone. The yearly revolution, when traced out, will, I think, explain satisfactorily why the motion round the great circle is sometimes faster and sometimes almost stationary, or even slightly retrograde.

turn of every full she has performed a complete revolution, *relatively to the sun's centre*, around which the earth herself revolves. So also the earth, moving onward in company with the solar system, performs a complete revolution, *relatively to that centre around which the sun revolves in his orbit.* We have seen that the magnetic poles (for the north and south poles revolve together) perform a complete revolution round the north and south polar axles of the earth once in about six hundred and forty years, and hence we infer that in THE SAME PERIOD OF TIME, the earth, still keeping her place in the solar system, performs a complete revolution relatively to another and higher system around which the sun himself, with all his attendant train, revolves. This revolution is accomplished by the simple fact that, because the sun is in motion in an orbit, therefore the earth, in her yearly revolution around his centre, performs more than a complete revolution of one primary circle in space; and consequently, in a series of revolutions round the sun, gains one complete year relatively to that centre around which the sun revolves. It is a demonstrable truth that every period of time marked by the revolution of the heavenly bodies is greater than one primary circle, for the reason that each and all of the heavenly bodies are themselves also in motion. Our theory then is seen to be that the magnetic attraction to that high centre is the force which directs the needle to the pole—that our revolution around that centre is the cause of the revolution of the so-called magnetic pole, which last is again the cause of the variations of the compass. The reasons given for the truths shown are sufficient for the effect—the effect is purely mechanical and plain to the sense; in harmony with the mechanism of the heavens and the order of revolution, and, until a better reason is shown, I am compelled to believe in its entire truth. *

The cause of gravity has never yet been explained or understood, notwithstanding that much is known of the laws which regulate it. Newton discovered that a falling body was governed by certain laws, and that these laws were applicable to the motions of the heavenly bodies, and this he called gravity, or the "attraction of gravitation." But neither he nor any one else, that I am aware of, ever explained in any satisfactory manner, even to himself, the *cause* of this gravity. But if my positions

* In limiting the period of the revolution to 640 years or thereabouts, it will be observed that the correctness of that period depends entirely on the accuracy of the observations and record of the variations of the needle from time to time. And as a consequence of the sensitiveness of the needle to diverting causes from local attraction, and the slowness of the motion, which renders it difficult to note the exact period of its passage of any particular meridian, together with the probable imperfection of the instruments in early use for observation, on which the records depend, even if all we have here said be perfectly true in principle, the actual period when found may prove to be somewhat longer or shorter than the period here named. Astronomers have suggested an indefinite period of six or seven hundred years, which I have sometimes seen mentioned, and which marks an epoch with them. They may, perhaps, find little difficulty in harmonizing it with this revolution.

be true, it will appear that one law governs polar attraction, centripetal force, and the gravitation of bodies, viz., a latent magnetism set at liberty, rendered active, and *involved* by a forward revolution.

The earth attracts all bodies to its centre, and a stone, because of its greater density, will fall quicker through the atmosphere than a feather; but I regard density as only an element of velocity in overcoming the resistance of a medium, and the atmosphere being withdrawn, the feather, attracted by the magnetic force, will fall as quick as a stone.*

There is one other result that must follow the truth of my hypothesis, too important to be omitted here.

It is well known that the motion of any body revolv-

* The subtilty and force of magnetism was illustrated by Laplace, under the name of gravitation. He says, "that if a new planet were thrown into space, it would instantly feel the force of attraction from surrounding bodies whatever their distance, and the velocity of that force, from one body to the other, would be many million times greater than that of light;" but I am not aware that he ever associated magnetism with gravitation, or treated of it as a universal principle and an active force in nature. Certainly he never considered the two as identical. Most of the inquiries of scientific men into the principles of magnetism have, as I believe, been expended in experiments on *magnets*, instead of *magnetism*. I can very well believe Laplace as to the velocity of magnetic attraction. We have an example of velocity in the Atlantic Cable. The electric influence is computed to have passed over a distance of 2,000 miles in $\frac{6}{10}$ of a second,—but even that time was probably employed by the thought of the observer in marking the time, and the electric current passed in no appreciable space of time.

ing about another body, also in motion, is necessarily spiral, and therefore the motion of the magnetic pole, revolving about the north pole, and being governed and controlled by the attraction of a body in motion around which the earth is revolving, shall also be spiral, never returning to exactly the same point. And hence because one law and one force governs both the motion of the earth and the revolution of the magnetic pole, therefore the polar axle of the earth *shall change* its position with every revolution of the magnetic pole, in such manner that the present poles of the earth's axis, in her diurnal revolutions, will, or may, at length reach the present line of the equator, and the equator will then become the poles; and thus the idea suggested by geology, that the present poles of the earth have, at some time of the earth's existence, been an equatorial region, becomes a problem solved, and reduced to a simple and undeniable truth. Not a doubt exists in my mind that such has been the fact and will be again; and I am not less satisfied that the Glacial theory of Professor Agassiz, so far as the fact is concerned, that parts of our own land, and of others in Europe and elsewhere, have at some time been the country of glaciers, is no longer a speculative idea, but a *mathematically demonstrable truth.**

* There is a truth known to astronomers, which I think strongly confirms this point of my argument. It is this: In the longitude of Athens it is known that the sun in Cancer does not come so far north, by nearly a degree, as it did two thousand years ago. Astronomers have inferred therefrom that the tropics are narrowing, and the earth consequently is drawing nearer to the sun. But this I think is a mistake, and that the phenomenon mentioned proceeds from the change in the line of the equator, as I have explained it; and that if we had had the opportunity, two thousand years ago, of observing south on the meridian of Athens, it would then appear that Capricorn has receded south to the same extent as Cancer, and *vice versa* at the Antipodes. It would also depend on what distance the meridian of Athens may be from the point where the former line of the tropic would bisect the present line, whether on some other meridian, distant from Athens, the sun's recession may not have been much greater than is seen at Athens. But for all evidence of this kind, the observations of astronomers are totally wanting.

The early system of astronomy was, I think, incompetent to the solution of a question like the revolution of the solar system; and modern astronomy has done so little towards it, that it must be considered as yet open to examination and argument. I have hope, therefore, that the suggestions here made may prove a step towards a more perfect knowledge of it.

Years ago, when I first thought of the subject here treated of, it was only in relation to the variations of the compass as observed at sea, and its practical application in commerce and navigation. But I perceive that I have risen to much higher themes, and perhaps some may say, meddled with things too high for me; but I am unable to perceive that I have at any time gone beyond a just inference from the truths shown.

The simplification of the vast speculative ideas of geologists and others, which this examination has suggested to my mind—my thorough conviction of their essential truth, and that the truths here shown may be availed of for the increase of knowledge and the improvement of science, are my reasons for making public my reflections on these matters. I submit them to the fate which future developments may award to their truth or falsity.

NOTES BY THE AUTHOR.

I RESPECTFULLY desire the reader of the foregoing to regard the subjects treated of, as in *two divisions.*

FIRST.—The revolution of the magnetic pole round the north pole, which is based on observations of known data, and of record; and if the record be true, there can be no difference of opinion about the result. It is of *itself*, independently of what follows, a truth of the highest importance to science.

SECONDLY.—The origin and nature of polar magnetism, and the *cause* of the revolution of the magnetic pole.

This part of my subject is based on reasoning from natural laws, and the conclusions are the inferential consequences of such laws. It is, therefore, more open to criticism than the first division; but the only debatable question is, do the reasons sustain the conclusions? If they do, that ends the discussion, till other and stronger reasons are opposed to it.

On page 26, I have asserted the gradual changing of the poles of the earth, "in such manner that the equator may at length become the poles." It is not probable that in the progress of such change, the north pole, for example, can move southward, in any direct line. That also may revolve, and perhaps surround the globe spirally, following the plane of the ecliptic, and to that opinion I am inclined, as the most natural, and the mechanical result of the force exerted.

Astronomers have observed some change in the poles of the earth, relatively to the north star, but I am not aware that they have considered it as an actual change of the position of the pole itself, and without that, their efforts to account for the geological formations, and the glacial epochs, are, to my mind, unnatural, laborious, exceedingly complex, and *wholly insufficient.*

It has been suggested to me, that my assertion that "there is no absolute magnetic pole," and constantly treating it as real, is contradictory and unintelligible; and I admit that it requires a better explanation than has yet been given. It is unfortunate for us, that to make ourselves understood we are obliged to use the imagery of material things to express an idea of immaterial things; magnetism and electricity are without the pale of matter such as we can comprehend, and therefore, to describe the subtile and peculiar influence concentrated at the magnetic pole, we must help our thoughts by a figure. I would describe the magnetic pole then, not as a moving point, but as a focus of mingling rays, the centre of which is the daily revolution. If we could be present at that centre we should neither see nor feel anything different from what we do here, but the magnet would be sensitive to any approach to it, and probably feel its influence far off, like the gravitating force of water on its approach to the Maelstrom.

In a foot note, page 22, I have said that I have no doubt but there is a daily revolution of the magnetic pole, which has been considered by astronomers as a mere vibration, caused by expansion from the sun's heat. But let it be understood that it is a revolution as we have explained it,

and caused by the revolution of the earth on her axis. It will be recollected that while the earth turns round once on her axis, she at the same time moves forward more than one million six hundred thousand miles in her orbit. Now if we can determine or assume the diameter of this daily revolution, or breadth of the vibration, whichever we please to term it, it is a basis for calculating the distance of the attracting force which governs the revolution. And the distance of the attracting force, which is such as to reduce the angle, which has over one million six hundred thousand miles for its base, to a few minutes of a degree on the earth's surface, is thousands of millions of miles from us; and, incomprehensible as it may seem, that distance may yet be known.

The idea is attributed to Sir David Brewster, that "magnetism is most intense in a low temperature." That is, no doubt, quite true by itself, and yet, the truth may reach much farther than that. What if it should be found that variations in the intensity of magnetism are the *cause* of changes of temperature? There is much reason to suspect that to be the case. It is seen that the temperature on the northern part of this continent, where the magnetic pole is at present situated, is subject to greater extremes than on the shores of Europe, or the coast of the Pacific in the same latitudes. I have reason to believe also, that the Polar Sea is more open by an eastern route round the North Pole, than by the route of Hudson's or Baffin's Bay, and next to my surprise that the voyage round the pole should ever be undertaken, is, that it should not be undertaken by the eastern route or by Behring's Straits. Four hundred years ago, when the magnetic pole was situated on the opposite side of the North Pole from Greenland, history tells us that that country was literally a *green land*. Since that time the magnetic pole has passed near to or over its northern portion, and it has become a country of ice and snows. The south magnetic pole is situated in about longitude 62° east (opposite the north), and if I am not mistaken, in that region, the barriers of ice obstruct the southern progress in a lower latitude than in other parts of the Southern Ocean. These things are highly worthy of a careful investigation by the learned.

A friend of very acute perceptions, and who has taken a great interest in the problems which I have endeavored to solve, asks me the question, "What will you do with the flattened poles of the earth,—does the earth change her shape with the changes of the pole?"

The question is pertinent and of force, and if required to answer it, my answer for the present would be this: Admitted that the poles are really flattened as they appear to be. It is known that both the north and south poles are situated in the midst of a wide and deep sea. The breadth of these seas is such that, admitting the truth of my hypothesis, the poles must have been within their area for many centuries past, and will continue to be so for many centuries to come. Let us suppose that the extreme radius of the flattened surface is fifteen degrees of the earth's surface on all sides of the pole; at that distance from the centre (the polar axle) the velocity of the earth's revolution on her axis will become such, as may tend, by its centrifugal force, to raise the surface of the water, or so to lessen its gravity towards the centre of the earth as to raise the surface somewhat. By a natural consequence, this force will increase towards the equator till you reach that point, or the greatest diameter of the earth east and west. By the same natural consequence, the centrifugal force will diminish as, from the radius of fifteen degrees distant, you approach the pole, where the centrifugal force is entirely lost. The water at the poles, then, being undisturbed in its gravity by the centrifugal force, will press outward from the centre and the poles become flattened. I see nothing irrational in this supposition; and it will be obvious at once that when the poles shall have passed over these seas, and be centred on the firm continents, the waters will assume their even gravity, and the earth assume her perfect globular shape.

Or, if this reason is not satisfactory, perhaps it may be thought to be a tidal influence, caused by the attraction of the sun and moon, which gathers the waters towards the equator, and draws them away from the poles; the effect would still be the same in causing the flattening of the poles, and restoring the earth to her globular shape, when the poles have passed the area of the wide seas in which they now exist.

There are many evidences to show that the waters of the Atlantic Ocean, and of its bays and sounds, are fuller now than they have been at some other time long past—such as the washing away of the headlands and islands of our sea-coast. Witness the northern shore of Long Island and the numerous bluffs now half sunk in the Sound; witness the numerous islands in Massachusetts Bay and their steep crumbling sides and diminished areas; go across to the British Channel and examine the Cliffs of Dover and the bottom of the sea around them, and abundant evidence may be found, both that many centuries have been employed to undermine and pull down their lofty crests, and that time was when they were not so as they are at present, and when the sea did not reach their base. There is a bluff on my own farm on Long Island sixty feet high, which has been washed away full eight hundred feet inland, and the boulders have fallen down, and are now covered with the tide. Once or twice every year the storms and high tides rise up over the beach and lap up and carry away a furrow at its base, which is filled up again by the washing of the rains, the same to be repeated year after year. At the rate the process is now going on, say about two inches of the whole face yearly, it would occupy four thousand eight hundred years to accomplish what has been done, and examining the channel outside of it, I would say that the water is now twelve or fifteen feet higher than it *could have been* when the headland was perfect.

The problems of new and exceeding interest which would result from the acceptance of the truths which I claim to have demonstrated, would be, I think, almost without bound; but I must leave them to those whose leisure and legitimate pursuits will allow them to follow out in minute particulars that which I have only sought to grasp as a whole.

JOHN A. PARKER.

SECOND LECTURE

ON

POLAR MAGNETISM:

ITS ASTRONOMICAL ORIGIN, ITS PERIOD OF REVOLUTION,

AND THE

SYNODICAL PERIOD OF THE EARTH IDENTICAL.

BY

JOHN A. PARKER.

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POLAR MAGNETISM.

ITS ASTRONOMICAL ORIGIN; ITS PERIOD OF REVOLUTION AND THE SYNODICAL PERIOD OF OUR EARTH IDENTICAL.

In a paper published by me, as read before the Society of the American Institute in March last, on the subject of Polar Magnetism, I showed, from the records of the variations of the compass for nearly three hundred years past, that the Magnetic Pole, in its movement westward, had in that period passed over more than 160 degrees of longitude in its revolution round the North Pole or Geographical Pole of the Earth.

The reasoning on this subject appearing to be conclusive, that the Magnetic Pole revolves about the North Pole, we were naturally led to the inquiry, What is Polar Magnetism? and what is the cause of this revolution? Those who have read my published paper on this subject with care, will know that our solution of those questions was in effect: that the forces of magnetism and gravitation are identical,—that the force which directs the needle to the Pole is wholly astronomical,—that the source of the governing attraction is from the highest centre to which the Earth in her various revolutions is immediately related,—that the revolution of the Earth relatively to that centre is the cause of the apparent revolution of the Magnetic Pole,—that the period of the revolution, as indicated by the recorded variations of the compass, is about 640 years, more or less, to be determined,—that the Earth in that period, as a satellite of the Sun, performs a revolution round the Sun relatively to the central point of the Sun's orbit, exactly similar to that which our own satellite, the Moon, performs in her synodical period, in which she revolves about the Earth relatively to any given meridian, from opposite the Sun's centre (the central point of the Earth's orbit) to opposite that centre again.

These are our main positions, which we are bound to prove by direct and unequivocal evidence, and also to defend and sustain against all contrary evidence which the ingenuity and skill of scientific men may bring against it.

If these things shall prove to be true, a Reviewer has said, "they will rank among the most daring and felicitous in the annals of scientific discovery;" and if I may be allowed the expression of my own judgment in the matter, taken with all their adjuncts, they will prove as useful for the extension of knowledge as any scientific discoveries ever made.

Before we proceed further, however, to illustrate and prove our own theory, it may be well to examine the

position which the science of the schools at present occupies on this subject. We shall do so disembarrassed of the technicalities with which scientific men have surrounded the general subject, and which, as I believe, are a bar to their own progress and discovery. It must be recollected, however, that we are not discussing the general principles of magnetism, or its abstract quality, nor the particular properties of magnets, or the numerous experiments to which they have been subjected, and which have occupied so large a share of the attention of scientific men; our subject limits itself and all inquiry to that paramount law, which, local and disturbing influences apart, at all times and in all places claims the direction of the needle to the Pole. We must therefore confine our remarks to what is at present understood in the schools in respect to this branch alone of magnetic phenomena.

From the earliest date of discovery of any change in magnetic attraction, it became a subject of inquiry by what law these changes were governed. For a very long time knowledge was confined to the observations of a few individuals in limited spheres; and these served little purpose except to establish the fact that changes were taking place, but without being able to assign any reason for it. Explorers and navigators finally became the largest contributors to the knowledge of the world on this subject. By visiting almost every accessible part of the globe, and reporting their observations at home, a vast amount of facts were furnished for the use of investigators; as these discoveries increased, the manifest importance of the subject enlisted the aid of governments in the interest of the schools, and the last half century has witnessed a more extended effort than ever before, if possible to discover the laws of polar attraction. For this purpose the observations of all time have been brought together, and the minds of eminent men have been engaged, to see what results could be obtained from the facts thus collected.

These results can be soon stated. It has been discovered that there is a line surrounding the globe, bisecting the terrestrial Equator at an angle of twelve or fourteen degrees, which they have called the Magnetic Equator. It is that line where the needle has no inclination or dip, but rests perfectly horizontal. It requires the minutest possible observation to be able to locate it within a degree or more of latitude.

As at present indicated, it passes over the wide continent of Africa where no observations at all have been had. It passes over the widest portion of South America, where very few observations have been had, and it passes over the kingdom of Siam and the Island of Borneo, where there have been no surveys at all. To supply the deficiency of observations over all these regions and wide intermediate tracts of ocean, other means were resorted to in their endeavors to locate the Magnetic Equator throughout. It was discovered that the inclination or dip of the needle increased in nearly a regular progression as we proceed

northward from the line of the Equator towards the Magnetic Pole, and this fact was made use of with apparent good judgment to assist in determining the locality of the Magnetic Equator, and consequently also of the Magnetic Pole. But when, by careful calculation from all the premises and ascertained facts at hand, the Equator was supposed to be determined, it was found to be no Equator at all—that it was wholly irregular and without symmetry, and hence of no use whatever in determining any truth relatively to itself or to the Magnetic Pole. Nothing discouraged however by the want of success, the eminent men and scholars engaged in the work have enlarged their labors—they have collected together all the observations made to this end in every part of the world, both those which are recent and for a long series of years past—they have divided the globe into eastern and western hemispheres, in reference solely to the east and west variations of the needle, and they have covered the globe as it were with the lines produced from the observations so collected—but all without success to discover a law which should govern polar attraction and thus point to the Pole itself.

The same course of reasoning is still being pursued, and scientific men in Europe and America are engaged in collecting new observations, and drawing lines as deduced therefrom;—lines of declination, of inclination, and intensity—lines parallel, convergent, and divergent—lines in fact of every imaginable name and description, with a view to determine accurately the position of the Magnetic

Equator at any given time with its corresponding Pole, and the laws which govern their changes as observed from time to time: and I need not say that all this has been without success for the objects intended. The only positive knowledge which these exertions have given us, is a knowledge of the facts which explorers have communicated, but with no law whatever to govern them;—no two theorists agree in their results. Humboldt, who may be supposed to have been among the most learned in theory, placed the Magnetic Pole by calculation in latitude 79° north and 27° west, almost at the same time when Ross found it by practical observation to be situated approximately in 70½° north and 96° west. Where there is so great a discrepancy between fact and theory, there is necessarily a fundamental error either in the theory itself or in its manipulation.

Whatever of truth and sound reason there may be in the use made by scientific men (and there is, no doubt, very much of both) of the facts in their possession in this regard, their inevitable failure from inherent causes to arrive at true results is, I think, manifest.

First.—Because the facts from which they reason, for want of means to distinguish, necessarily include the disturbances due to the sensitiveness of the needle, but which do not belong to the law of polar attraction.

Second.—The changes going on are such that, with their method of applying facts, that which was true a year ago is not true at the same point to-day.

Third.—Many, and indeed most of the facts collected together, and which form the material of their work, are facts determined by observations made by different persons, and at different times, so wide apart, that although all of them were perhaps true at the time and place of observation, yet only one of them could be true at the same time, and not one of them is exactly true to-day.

Fourth.—It is self-evident, I think, that no true result can ever be arrived at by the method of imaginary lines, unless the facts on which such lines are based could be determined in all parts of the world at the same time, which has never been done and does not admit of a possibility.

In all the experiments of scientific men, they have regarded the Earth as the source of magnetism. Not a half century ago, many of them considered the centre of the Earth as a fixed magnet, and all magnetic influence was referred to that cause; but the changes observed contradicted that idea, and they have latterly imagined magnetic veins running through the Earth, which they have endeavored to trace by lines drawn upon its surface. To account for the changes going on, these veins have been supposed to be shifting, so as to accommodate the veins to the changes. Some have gone so far as to imagine a constant circulation of the molecules or atomic principles composing the Earth, thus producing the changes of the magnetic tendency. The celebrated Arago treated this idea with seriousness.

It has been observed that temperature, the aurora borealis, and other natural phenomena, have an influence on the magnet, and it is doubtless true; but these influences, being a part of the general law of magnetism, they do not touch that paramount law which claims the direction of the needle as soon as these influences are removed.

The Sun's heat is supposed to cause the daily vibration of the needle—the periodical return of spots on the Sun have been supposed to have an influence on the declination of the needle, and some one has suggested that the Moon has such influence also, but by *no one, before myself, has it ever been advanced that the cause of polar attraction is wholly astronomical.*

I have thus given as full and true a statement of the condition of science in the schools on the subject of polar magnetism as our limited time and space will permit. I would remark that revolution as a cause or accompaniment of magnetic attraction was not heard of among them till since the publication of my pamphlet in March last. Since that time the *Cornhill Magazine* of June, the *Scientific American* of October, and *Silliman's Journal* of November, have each of them published an article indicating revolution, but without any acknowledgment of my paper, though it was sent to all of them.

We now turn to our own hypothesis, as explained in the opening paragraph of this paper:

In all inductive reasoning on physical laws, there are truths in nature which may be laid down as axioms, from which the reasoning admits of no departure, and I there-

fore here lay down as axioms—*First*, that the laws of mechanics are the laws of nature. *Second*, that the construction of the universe, and the relations of the heavenly bodies to each other, and of our Earth to them, *is Nature herself*, and consequently in the relations of the heavenly bodies to each other, and of our Earth to them, we shall find the perfection of all mechanical laws. I think there can be no objection to these principles.

The question has been asked me by some intelligent persons, whose perceptions were not quite clear on the subject, "by what mechanical process the revolution of the Magnetic Pole around the North Pole can be accomplished by astronomical attraction?" I will therefore give an illustration, which I have given before at a private reading of my first paper on the subject, and which I think explains it.

Let us suppose that the globe before us is transparent in such a way as to admit a line of light passing through it. Let us then suppose that a single ray of light, emanating from some distant centre, shall strike the globe at twelve or fourteen degrees from either Pole, and passing through the centre of the globe at an angle to its axis in a line to the opposite side. It is seen that the ray of light will make a point on either side of the surface of the globe, each at the same distance from the geographical Poles, but at opposite angles to the globe's axis. Now let the globe revolve eastward on its axis: it is seen that the ray of light is stationary, but the points of light on the surface appear to revolve westward, while the globe is revolving eastward.

Now if you let the ray of light represent the line of attraction from the great centre around which our hypothesis assumes that the Earth is revolving, you will have a perfect mechanical idea of the revolution of the Magnetic Pole around the geographical Pole, caused by astronomical attraction, with this difference only, that whereas the ray of light appears to revolve with each revolution of the Earth on her axis, the Magnetic Pole only revolves in that period of time in which the Earth gains one complete year in moving eastward round the Sun, just as we gain one entire day when we travel eastward round the Earth, and just as the Moon, in moving eastward round the Earth, gains one revolution of the Earth on her axis, and just as the Earth, in moving round the Sun from opposite a fixed star to opposite that star again, gains one sidereal day.

Now let us observe that if the cause of the revolution of the Magnetic Pole be such as I have described, viz., attraction to a higher centre relatively to which the Earth is revolving, then *because* the Earth moves eastward on her axis and in her orbit, therefore, by the mechanical law which is our axiom in the case, the apparent revolution of the Magnetic Pole SHALL BE WESTWARD—and that the Magnetic Pole is known to have been moving westward for nearly three centuries, is a positive proof as far as it goes of the truth of our hypothesis; whereas if the Earth be the source of magnetism, as the schools teach us, this effect would be

mechanically impossible. This is one of the many phenomena that confirm the truth of our theory, and we may safely ask the question, can anything be shown to the contrary sustained by an equal mechanical test of its truth?

The irregularities heretofore observed in the variations of the needle (and I would include *inclination* with *declination*) which have baffled inquiry, and led to the conclusion by many, that it was not governed by any law definable by reason, are, in my judgment, when examined upon their merits and aside from local and disturbing causes, a perfect evidence of the revolution of the Magnetic Poles, their equable motion through space, and the astronomical influence controlling their apparent motion. I will mention a few facts illustrative of that view, and in doing so I will keep to the axiom of mechanical truth and necessity.

London is situated in latitude 51° north, and the present declination of the needle there is about 20½° west. The Island of New Nantucket or Baker's Island in the Pacific is in longitude 176° west, nearly on the opposite side of the globe from London, and in latitude 13′ 30″ north,—the variation or declination there in 1866 was 9° 30′ east, or less than half that at London. Why then, being so nearly on the opposite meridian to London, has it not the same amount of variation east that London has westerly? Chiefly for the reason that being farther south, if a circle be described from each centre (London and

Baker's Island) through the Pole, the radius of the circle described from Baker's Island is more than twice as long as that described from London, and the length of a degree in the circumference of such circle is more than twice as great, and consequently, the Poles being nearly fixed points, the variation is less than half as great at Baker's Island as at London.

Again, we suppose the Magnetic Pole to revolve in about 76° north. London is situated in 51° north and New York in 41° north. We at New York are therefore more than one-third farther from the line traversed by the Magnetic Pole than London is—consequently, for the reasons explained above, the variation or declination at New York can never attain to a greater amount than something less than two-thirds of the greatest variation observed at any time at London.

The greatest westerly variation ever observed at London was in about the year 1818 equal to 24½°—the greatest therefore it can ever attain at New York will not exceed about 15 or 16° west. But if we move northward on the meridian of New York to the latitude of London, the variation there (local causes aside) will always be precisely the same as at London whenever that point stands in the same position as London relatively to the Magnetic Pole, and will attain to the same extent as the greatest observed at London (24½°) whenever the Magnetic Pole shall reach the meridian of 90° west of it.

Again, it will be evident that although the Magnetic

Pole moves through its orbit with a nearly equal motion (and entirely so except the small daily and yearly revolution), yet the variation of the needle changes much more rapidly at some times than at others at the same point, and very unequally at different points at the same time. The cause is easily seen.—Whenever the Magnetic Pole is on any particular meridian, as of London for example, it moves almost at a right angle to that point, and the variation increases rapidly; but before the Magnetic Pole reaches the meridian of 90° from the point of starting it moves at an angle so small relatively to the point of observation, that any increase of variation is scarcely perceptible, and as the change of variation becomes slower at London it becomes faster at New York. We have but to trace the motion of the Poles on the lines of the globe to have ocular demonstration of all these truths.

Similar apparent discrepancies exist in regard to *inclination* as well as the "variation" or declination, and from not dissimilar causes.

It has been quoted to me for example that my theory of the revolution of the Magnetic Pole cannot be true, because from the records of the observed *inclination* at London, it had changed but little more than 6° in 140 years, and at that rate it would take 7900 years to complete a revolution; but if we examine the mechanical effect of inclination upon the principle of the revolution of the Magnetic Pole, we shall soon see what inclination has to do with the latter. Inclination in north latitude is simply the downward tendency of the north end of the needle, which increases in a ratio not exactly determined as we proceed northward from the line of no inclination towards the Magnetic Pole. At the Pole the inclination is 90°.

Now, on the supposition of the revolution of the Magnetic Pole, it is self-evident that the inclination at London will be greatest when the Magnetic Pole is nearest, and least when it is farthest off. The Pole will be nearest when it is on the meridian of London, and farthest off when on the meridian of 180° from London.

As the Pole moves westward from the meridian of London, variation or declination will begin at zero and increase, and *inclination* will begin from its maximum point to diminish until the Pole reaches 90° of longitude, when variation will begin to diminish, and both *variation* and *inclination* will go on diminishing till the Pole reaches 180°. Thus we see that in obedience to the requirements of revolution, inclination has been diminishing at London from 1740 (the earliest record) to the present time. It must continue to diminish till about 1983, when the Pole shall reach 180° west from London. It will be seen also, that from the maximum to the minimum of inclination indicates half a revolution of the Magnetic Pole, and from the minimum to the maximum the other half; but the maximum at London can never reach 90°, because London is never at the Magnetic Pole, and the minimum can never descend to zero

because London is never 90° of a circle of the globe from the Magnetic Pole.

These remarks might be continued almost indefinitely, and with appositeness to the case; but I trust that I have said enough to show that the seeming irregularities which have heretofore baffled inquiry, are in fact a necessity, and perfectly regular when applied to the revolution of the Magnetic Pole, and their existence is therefore the best proof we can have of the truth of our theory, so far as a perfect explanation of their apparent irregularities is concerned.

But our principles involve much higher truths than the apparent irregularities of magnetic attraction which it is proper that we should state here. They claim that magnetism and the attraction of gravitation are identical *in fact*. They are known to be so in the *degree* of their forces, and since nothing whatever is professed to be known of what causes gravitation, and since by its equality of force magnetism is seen to be capable of doing all that gravitation is supposed to do, the evidence is all in favor of their identity.

Secondly.—Our principles involve the necessity that all planets are magnetic, and hence their attraction one to another, and hence that attraction towards the Earth which governs the revolution of the Magnetic Pole.

The Earth being a planet and magnetic, it is justifiable to assume that other planets are magnetic also. I think their relative motions and reciprocal influences demonstrate this truth, and if magnetism and gravitation are identical, it is impossible to be otherwise.

Our Earth, on which we reside, is a lower order of planet—it is a centre to itself and to its own satellite, but to none else. All motion from that centre is therefore necessarily outward from that centre, onward and upward in an ascending ratio from the lower to the higher; as, of the Earth on her axis, the Moon about the Earth, the Earth and Moon together about the Sun, and the Earth, Sun and Moon about a higher centre. In obedience to the axiom of mechanical law, the ascending ratio must also be in strict geometrical progression, for geometry is neither more nor less in this case than exact mechanical proportion. Geometry is pure mechanics, and the one cannot be separated from the other. The period of the Earth, Sun, and Moon is the third in the ascending ratio as above named, and consequently the third also in geometrical progression. These facts being admitted, we are prepared to prove the period of the revolution of the Magnetic Pole by proving the period of the Earth's revolution relatively to the highest centre to which in her various revolutions she is immediately related.

In my work on the Quadrature, published in 1851, after demonstrating the circumference and diameter of one primary circle to be as 20612 to 6561, and the motion of three gravitating bodies to be as four to three of one primary circumference, I then establish the following geometrical proportion, viz.: "As one primary circumference of a circle is to the Moon's time round the Earth, so is the Moon's time round the Earth to the Earth's time round the Sun (see the Quadrature, page 114 in practical questions, and proposition 4, page 130); and the proportion stands thus: (the Moon's time round the Earth being 27.482666 +) 20612 : 27482666 :: 27482666 = 366.43555, which, pointing off three figures for units in consequence of the increase of diameter of the circle, is the number of revolutions the Earth performs on her axis in revolving about the Sun (the time being circular time), and this period it will be seen is the second in the order of ascension as we have before described. The *third*, which *is our period* of the revolution of the Magnetic Pole, will then stand as follows: 20612 : 366.43555+:: 366.43555+ =651.4409+, the revolutions *being years* instead of *days*, and this I believe to be the exact period of the revolution of the Magnetic Pole around the Geographical Poles of the Earth.

The brevity of this demonstration will not probably suit the judgment of those gentlemen learned in astronomical science as at present taught, who, following the angular system of their predecessors of more than twenty centuries, have developed nothing definitely relative to any period such as we have here given. We must, therefore, for our own satisfaction as well as theirs, examine this period a little more minutely, that we may see what is its significance apart from and beyond its government of the motions of the Magnetic Pole.

As I have said in my last published paper on this subject, "this is not the Sun's period—that, of course, is much greater." The question then comes home with force, what period is it? and I answer, that it is the EARTH'S SYNODICAL PERIOD—a period never calculated, if indeed at all considered by astronomers—a period in which the Earth as a satellite of the Sun repeats on a grander scale exactly the same phenomenon which our own satellite the Moon performs every time she makes a complete revolution of the Earth relatively to the Sun's centre.

In this period the Moon having the Earth for the centre of her orbit, and starting from a point where she conjoins the Sun's centre (which is the centre of the Earth's orbit) with any given meridian on the Earth, she pursues her journey eastward round the Earth till she conjoins with the Sun's centre and that same meridian again; and this is her synodical period. She conjoins with no two meridians however at the same instant, but with every meridian on the Earth's surface in the course of one revolution of the Earth on her axis; and this is the highest revolution of the Moon which is exclusively her own, and once completing this, she goes on repeating it forever, following the Earth round the Sun.

So also the Earth having the Sun for the centre of her orbit, and starting from a point where she conjoins with any given meridian crossing the Sun's equator and the centre of the Sun's orbit, she pursues her journey

eastward round the Sun till she conjoins with the centre of the Sun's orbit, and that same meridian on the Sun's surface again. And this is the Earth's synodical period in revolving about the Sun. She conjoins with no two meridians however on the Sun at the same instant, but with every meridian in the course of one revolution of the Sun on his axis; and this is the highest revolution of the Earth which is exclusively her own. After this she goes on repeating the same forever, following the Sun in his orbit round his great centre.

We have given the Earth's synodical period as 651.4409 + years.* The Moon performs her synodical period round

* From the solution of the problem of three gravitating bodies as shown by the Quadrature, pages 100 to 116, there results a series by which the Earth's synodical period is reached as the fourth in order from one primary circumference, the same as it is the third in order by geometrical progression.

For example, let C be the primary circumference of a circle (20612.), then C plus $\frac{1}{8}$=D plus $\frac{1}{8}$=E plus $\frac{1}{8}$=F plus $\frac{1}{8}$=G. Then G=651.4409 + which, pointing off three figures to the left for years, is the Earth's synodical period.

To show the direct connection of this series and this period with solar time, take the following example: Let C=1, then C plus $\frac{1}{8}$=D plus $\frac{1}{8}$=E plus $\frac{1}{8}$=F plus $\frac{1}{8}$=G, then G=3.160493 + to infinity. Now G is found to be exactly equal to the circumference of one solar day in its relation to the circumference of one diameter; hence G × 5153, the area of one primary circle, and ÷ 5184 the unit of one solar day as evolved by the multiple 6 will give 3.1415942 + to infinity, equal to the decimal circumference of one diameter. It is therefore competent from this series to produce the circumference of one diameter from any number whatever, and to any number of places of figures required, and with the use of any equal multiple or divithe Earth in a little more than twenty-nine and a half revolutions of the Earth on her axis from opposite the Sun's centre to opposite that centre again, or twenty-nine and a half solar days (it is 29.530588+).

If therefore we give to the Sun the same number of revolutions on his axis, relatively to the centre of his orbit, from one conjunction to another (and he can have neither more nor less without breaking the order of revolution), it will prove that the Sun revolves on his axis relatively to the centre of his orbit once in twenty-two years and twenty-one days. Any fixed spot on the Sun's disc will therefore return in eleven years and ten and a half days from the time of its disappearance, or in half a revolution of the Sun on his axis relatively to the centre of his orbit. Herschell in his Astronomy gives the periodical return of the Sun's spots as eleven and eleven one hundredths years—equal to eleven years and thirty-nine days. Herschell did not claim to know anything of the period which we are endeavoring to prove; but the coincidence of time of the return of the Sun's spots abundantly proves its truth. But as the Sun's individual spots are not clearly identified, and some may doubt the entire

sor of the primary circle and the solar day. This series may also be made a perfect test of the truth of my ratio of circumference and diameter, and of the deficiency of geometers' ratio which it is not in the power of numbers or geometry to disprove. Many of these things, which are considered by mathematicians as mere curiosities of figures, are in fact mechanical truths of the very highest order.

accuracy of time, we will not rest our evidence here,—we have a more sure testimony in what follows.

The Sun appears to revolve on his axis relatively to us in twenty-six or twenty-seven days. It is therefore certain that the above revolution of plus twenty-two years relatively to the centre of his orbit must be accomplished by a small daily precession over mean or sidereal time. Herschell gives the Sun's synodical period on his axis (*i.e.* relatively to the Earth and the Moon) as $27^d\ 5^h\ 5'$. The Moon's sidereal time round the Earth is $27^d\ 7^h\ 43'\ 3''$; at that rate therefore, according to Herschell, there is a precession of the Sun on his axis of $2^h\ 38'\ 3''$ in a sidereal lunation. We have thus the right amount of the Sun's precession in his axial motion relatively to us to conform to the period of 22+ years relative to the centre of his orbit. More exactly I think the amount of that precession is $5'\ 43''\ 32'''$ daily, equal to $2^h\ 36'\ 25''\ 57'''$ in a sidereal lunation, differing from Herschell's period but $1'\ 37''$ in a lunar month.* At that rate of motion, in plus twenty-two years the Sun by his precession will gain one complete

* Whatever difference there may be in the relative motions of the Sun on his axis and of our Earth, it being the primary of all relative motions, it can be neither more nor less than unit, and the unit of circumference irrespective of magnitude is 20612, which expressed in the divisions of time = $5'\ 43''\ 32'''$, which I assume, therefore, is the true difference in the relative motions of the Sun and our Earth on their respective axles; and I prefer this as a principle of motion rather than the result of Herschell's instrumental observation, which, like all other observations, is at best only a very close approximation.

synodical period of the Moon and one precession added; and the Earth having passed twenty-two times round the Sun will have gained of solar time twenty-two times the amount of one precession of the Sun on his axis, the whole together equal to $32^d\ 0^h\ 51'$. This is just as it should be; the motion is perfectly mechanical, and the time agrees with Herschell's demonstration of the Sun's precession within a few minutes in plus twenty-two years. Neither one of these phenomena could exist without the other—they are in fact creative of each other; the existence of the period creates the precession, and the existence of the precession proves the necessity of the period.

I consider the evidence perfect in itself, and it is therefore certain, I think, that plus 651 years is the Earth's synodical period, and that the Sun turns on his axis relative to the centre of his orbit once in plus twenty-two years, in which, revolving about the Sun, the Earth conjoins with any meridian on the Sun, and the centre around which the Sun revolves. The fact that this same period is or is not also the period of the revolution of the Magnetic Pole turns wholly upon the admission or denial whether or not the attraction of the needle is caused by astronomical or planetary influence. If that be admitted, it is then an absolute necessity that the superior attractions which govern the Earth's motions relatively to the Sun and the centre of his orbit, must also govern the movement of the Magnetic Pole.

If I understand correctly the principles and necessities

of combined motion, these orbs revolving together and about each other, all governed by the same laws, and their forces all equalized between themselves in respect to their relative magnitudes, distances, and velocities, can no more fail to make these conjunctions in the ascending and progressive order exactly as begun, without getting out of place in respect to each other, than the hands of a clock can fail to conjoin at 12 M. without getting out of place in respect to time. The perfect observance and exact fulfilment of this order is in fact a part of the law which keeps the Earth in her perfect balance, fulfilling her seasons in exact time, and with perfect safety.

I must here ask attention to the fact that in proving the exact period of the revolution of the Magnetic Pole, I have at the same time explained and proved the Earth's synodical period—a period not before known or understood in the science of the schools—and this is done chiefly by the aid of the Quadrature, so long condemned by the schools as a useless question.

I consider the evidence perfect in itself, yet I have no doubt but it will be rejected by those who contend for the orthodox and angular system of demonstration, which in my opinion is incompetent to discover the truth in either case. The true and immediate reason of its rejection however will be, that its simplicity and truthfulness to nature does not flatter the ostentatious pretensions of speculative learning. *Victory*, however, will not always hang in the balance, but must eventually be declared on the side of truth.

There is another truth of Astronomy with which this period has something to do—it is the precession of the equinoxes.

Two forces are always active in revolution; and in order that revolution may be continuous it is necessary that one of the forces should always precede the other a little. Wherever there is continuous revolution, therefore, there is always a precession of one of the forces which govern it. It is a universal principle and a mechanical law, without which motion cannot be continued. It may be best illustrated by the steam-engine, and it is just as observable and necessary in the motion of the heavenly bodies as in the steam-engine.

In the construction of the engine one of the first difficulties encountered was to overcome the force of gravity. It was found that when the two forces of gravity and steam-power came exactly opposite each other the engine stopped. Something was wanted to carry forward motion beyond the centre of gravity, and this is exactly the same thing as precession in the motion of the heavenly bodies. Ingenuity and perseverance at length overcame the difficulty with the steam-engine, and it is a remarkable fact and worthy of attentive consideration, that the principle by which the difficulty is overcome is precisely the same as is observable in the relative motions of our Earth and the heavenly bodies, viz.: by a slight difference in time in the action of one of the forces,—a moment's precession of one force over the other. Thus, as we have just now seen,

the Sun, the source of the governing attraction in his revolution on his axis, takes precession of the Moon in his orbit more than five minutes daily, and fulfils his revolution more than two hours sooner in a lunar month than the Moon completes her orbit relatively to a fixed star.

The Earth appears to be moving between two forces—the attraction of the Sun as the centre of the solar system to which the Earth belongs, and a force outside of the solar system which controls and limits the sidereal or mean year. But in the motions of the Earth the Sun's attraction always takes precedence, or has the precession, because, as I suppose, that the Earth is nearer to him than to the opposite force, and because the Earth undoubtedly belongs to the Sun's system.

Thus, while the Earth is turning on her axis, the Sun actually goes before, and the Earth is nearly 3′ 56″ longer in turning on her axis from opposite the Sun's centre to opposite that centre again, than she is in turning from opposite a fixed star to opposite that star again. The Earth in her daily revolution is thus carried forward beyond the gravitating centre of the two forces, the solar and sidereal, and the amount which she is thus carried forward is the Sun's daily precession.

Again, in the Earth's revolution round the Sun the Sun arrives at his solstitial points nearly forty-five seconds before the star arrives at the same meridian, and the solar year is ended before the sidereal year is ended; and this again is the Sun's annual precession at his solstitial points over sidereal time, or the mean year.

So also the Sun arrives at his equinoctial points about 3½ seconds of time and plus 51″ of a degree before the star arrives at the same meridian, and this is what is termed the "precession of the equinoxes," in which, according to *our* theory, the Sun by his precession carries the Earth beyond the centre of gravity of the two forces which govern her motion; and this precession will surround the globe in about 25,000 years, sometimes called the great year of the Earth.

We are now to see what our period of the revolution of the Magnetic Pole and the Earth's synodical period has to do with the precession of the equinoxes.

In my work on the Quadrature, pages 97 to 99 inclusive, I have shown that "all periods of time are greater than the revolution of one primary circle in space, because all the heavenly bodies by which time is measured are themselves also in motion." The revolution of one primary circle in space when reduced to time is 23^h 51′ 23″ 20‴—and the sidereal revolution is the exact mean between one circular and one solar day, less 44″ 52‴, which is the Sun's precession at his solstitial points. In other words, the excess above the mean between a circular and sidereal and a sidereal and solar day is the Sun's precession at his solstitial points.*

* The solar day equals 24 hours, difference greater than circular day........8′ 36″ 40‴

As the Earth's synodical period is the same revolution as her annual period, only continuous for a longer time, and relatively to a higher centre, which is on a scale of geometrical progression above the annual period, it is therefore evident that the Sun's daily precession in the Earth's synodical period shall be the *mean of the difference* between a circular and sidereal and a sidereal and a solar day, and any excess of the actual period above the mean of such *difference*, shall be the Sun's precession at his equinoctial points. In this we follow the exact rule as seen to exist in the annual revolution and the Sun's precession at his solstitial points.

The actual mean as described above is found by calculation to be 2′ 9″ 10‴ solar time,* and the actual time of the Sun's daily precession in the Earth's synodical period is 2′ 12″ 37‴ + ;† the difference above the mean is 3″ 27‴ +,

The sidereal day equals 23 h. 56′ 4″ 6‴, difference less than solar day	3′ 55″ 54‴
The circular day equals 23 h. 51′ 23″ 20‴, difference less than sidereal day	4′ 40″ 46‴
Excess of difference above the mean	0′ 44″ 52‴

* The difference between one circular and one sidereal day is 4′ 40″ 46‴, half equals........2′ 20″ 23‴

The difference between one sidereal and one solar day is 3′ 55″ 54‴, half equals........1′ 57″ 57‴

Together = ...4′ 18″ 20‴

Half equals........2′ 9″ 10‴

† The period 651.4409+ × 365.24225+ the days and fractions of days in

which is the Sun's daily precession above the mean as before stated; and because in performing one complete revolution round the Sun, the Earth has gained one entire revolution on her axis from the Sun to the Sun, or one solar day, therefore the Sun, on the completion of the full year, arrives at his equinoctial point preceding the star by exactly the excess (3″ 27‴+) of the whole daily precession above the mean of the DIFFERENCE between a circular and sidereal and a sidereal and solar day, as before stated,—just as at his solstitial point, the Sun precedes the star by exactly the excess above the mean between a circular and sidereal and a sidereal and solar day.

The calculations by which these results are reached are perfectly simple and reliable, but the detail of figures would be too dry for rehearsal here, although some very curious and interesting facts might be developed thereby.*

The precession of the equinoxes is therefore seen to be

the mean year, gives the product 287933.74+, equal the number of solar days in the whole period; and this product 237933.74+ divided into the mean year reduced to the smallest appreciable divisions of time (thirds, or 60ths of a second) will give the required Sun's precession daily to accomplish the whole revolution. Thus, 651.4409+ × 365.24224=237933.74 + and 365 d. 5 h. 48′ 50″ 53‴ reduced to its lowest denomination = 1893415853 ÷ 237933.74=7957.74+, which is the amount of the Sun's daily precession over mean time, the time being in thirds or 60ths of a second, and 7957‴+ expressed in the proper divisions of time = 2′ 12″ 37‴+ the Sun's actual daily precession in the Earth's synodical period.

* The elements of the exact period of the Sun's orbit are, I think, developed in these calculations, which I am induced to believe is exactly 25,000 years,

the natural and necessary result of the mechanical laws of motion, and not the result of accidental and extraneous causes, as has been taught by astronomy.

To speculate on the cause of an observed phenomenon and make a show of reason out of it is one thing, and to demonstrate the necesssity of such phenomenon, its cause and exact value by a mechanical law of universal application is quite another thing. And this is the real difference between me and the astronomers of the schools in regard to the cause of the precession of the equinoxes.

In my last published paper on this subject I said that "problems of new and exceeding interest, almost without bound, would follow the acceptance of the truth of my demonstrations." As yet I have scarcely touched upon their number or interest; others of a more startling nature than any yet mentioned, and not less clear and conclusive, are in reserve and ready to be demonstrated whenever the principles of revolution and the value of the quadrature are admitted. These demonstrations will contradict no *known* truth, but they will add many new truths to those already known. As in the case of the precession of the equinoxes, they will set aside some of the speculative causes of the observed phenomena, and substitute in their room a natural and efficient cause. They will correct false theories; and no one need be surprised if in a very short time from this astronomers should find themselves compelled to in-

but with a lap or precession at the end indicative of another revolution that will take over 13,000,000 years to fulfil.

quire whether they have not always been mistaken in supposing the unnatural existence of a projectile force.

Sir J. F. W. Herschell in his Astronomy has remarked that "Astronomy, unlike the other sciences, can never change." His meaning doubtless was, that the truths of astronomy were fixed from eternity in the laws of nature, and therein he was right, and astronomy can never change. But some who have not Herschell's perceptions of truth, have construed him to mean that the science of astronomy as taught, being already perfect in itself, can suffer no change. But this is a very great mistake. There are many things taught as truths in astronomy which are purely conjectural. I speak not now of any of the observed phenomena, but of the causes assigned for such phenomena, which are often accepted by the learned only for want of a better explanation ; and if my demonstration be true, such has been the case in regard to the precession of the equinoxes ; by attributing it to a wrong cause they have been led into another error to find the cause of nutation. I believe that if the principle be accepted, that the Magnetic Pole revolves by astronomical attraction, gradually, but very slowly changing the equator and the Poles, it will at once explain all those curvilinear phenomena about the Poles of the Ecliptic, which have led to the assumed cause of precession and nutation. Precession and the appearance of nutation will still exist, but the cause of them will be quite another affair. They are simple, natural and mechanical truths, the result of a compound motion.

I do not believe the story which the savans tell us to account for precession and nutation,—that this Earth of ours, which bears us all so kindly and smoothly on her bosom, is in fact a hump-back, and that she wags herself lewdly in presence of her lord the Sun when he approaches the equator. I am satisfied that she has no such deformity or infirmity, and that she will at length vindicate herself against all such aspersions of vain and presumptuous philosophers.*

* It is held by astronomers that the cause of the precession of the equinox is an accumulation of matter and protuberances of the Earth about the equator,—and that the attraction of the Sun and Moon acting upon the increased mass, gives to the Earth a disturbed motion, causing a vibratory movement on her axis, and this they call nutation or a *nodding*. I have no belief in such a cause.

NON-EXISTENCE OF

PROJECTILE FORCES IN NATURE.

A PAPER READ BEFORE

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BY JOHN A. PARKER,

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NO PROJECTILE FORCE IN NATURE.

In a paper which I read two years ago, before the American Geographical and Statistical Society, on the subject of Polar Magnetism, in which the argument was, that the Magnetic Poles revolve about the Geographical Poles, and that Polar Magnetism is of planetary and celestial origin, I indicated a doubt of the existence of any projectile force in nature, and its consequent absence in the motions of the Heavenly bodies.

In that paper I said something of the "universal law of precession" prevailing throughout all the Heavenly bodies, the greater over the less, as a means provided by Nature in connection with magnetic attraction, to make motion continuous.

In the paper which I shall now read, I propose to say a few words on the non-existence of projectile forces, and to give some practical proofs of the theory advanced.

It will be seen at once that if the universality of precession is the cause of the continuance of motion in the Heavenly bodies, it constitutes the first established truth, which confirms a doubt of the existence of a *projectile* force. Nature never wastes her powers by supplying two forces where only one is necessary, and if we once admit the existence of magnetic attraction (gravitation), to produce motion, and the universal law of precession to make it continuous, we have then accumulated all the powers that are necessary, and why should anything more be required?

The idea of a projectile force has its origin, no doubts in our knowledge of the fact, that our own Earth, in her orbit round the Sun, passes over an amount of space many times greater than her revolutions on her axis would measure, and at first sight a projectile movement appears to be an inevitable necessity, and to account for it rationally has exercised the science, ingenuity, skill, and judgment of the best minds which the world has afforded, in centuries past. I shall not attempt to repeat what has been said or written on the subject; it is sufficient for our present reasoning to know that by no ingenuity of skill have the believers in an active projectile force (and that belief has, I think, been universal) ever been able to bring their theory within the operation of any natural law known to exist. It is reasonable to suppose that the laws of Nature, seen in our own world, are the laws of the universe, and we have no right to assume the existence anywhere of laws which are contrary to Nature here. But then comes the certain truth, that the Earth does move through space, both positively and relatively to other bodies, faster than her diurnal revolution on her axis would carry her, and how shall it be accounted for?

The result which all who have reasoned on the subject have finally come to is in substance like this. They represent the Deity as occupying some limited space, sitting on a vast throne, high and lifted up, engaged in his work, as a man would work with his hands, creating worlds, which he throws off to the right or the left, giving them an impetus which, for some unknown reason, is to last forever, and to be their guide through illimitable space.

They thus, in fact, malign the Deity, by likening him to man, and to His own created things. They forget that right and left are only relative terms to a stationary being of limited power, and presence, and that neither height nor depth has any existence in the infinity of space. But the question comes back again, how else can they account for the vast movements of the Earth and the Planets which we know exist, and see daily performed.

The way is then open for imagination to conceive a method by which these movements can be accounted for, but it must be a method consistent with the natural forces known to exist, consistent with the known movements of the Heavenly bodies, and contradictory of no known truth whatever.

Imagination is one of the greatest, if not the greatest, power of the mind which God has given us: we identify it with thought, but it is superior to thought, and always leads it; it is creative as well as suggestive in its power; it presents to us the images of things as perfect as the realities, and, no matter from what distance they may come, her flight to bring them is instantaneous as the lightning. Although wayward at times, she is always the willing handmaid of reason, and under that guidance, *Imagination is always the pathway to discovery.* We must invoke her help, then, in our present dilemma, and go wandering for a while in the realms of space, with reason and imagination alone for companionship.

I think I can best lead your minds along with me in this wandering, by relating a story of fact. I was sitting on the piazza on the west side of my country house, to witness the splendors of a summer sunset. The air was cool and refreshing, and I sat till twilight had deepened into night, when Imagination assumed the reins of thought, and all that here follows is of her suggestion.

"What mean these glittering worlds in view, that downward slope their west'ring wheels?" They are all apparently moving—By what means? For what purpose, and to what end? But astronomers say it is we that move, not they, and astronomers must be right, else how could such journeys be accomplished? But there is no appearance of our moving—the air is still around us,—it is even moving gently in the same direction with us. If we were in such rapid motion as is supposed, the air would rush by us in the opposite direction with a force that would annihilate us. The atmosphere must then move with us. Astronomers say it does, and astronomers must be right.

How high is our atmosphere? Philosophers tell us that it is only forty miles! What is next to it? Where is the dividing line? There must be great friction on that line if the same order of nature is obeyed there as here. Philosophers have again told us that there is nothing next to our atmosphere, and that is the reason of a continuance of the projectile motion, because there is nothing to stop it. Are we then moving through space on the same principle as through an exhausted tube? In that case we should not move in an orbit, but in a straight line, until we should go crash against some other planet, shattering ourselves or them. And these same philosophers have, in their imaginations, gravely provided this mode of destruction for us. But this is not in harmony with Nature, or our own motion, and besides "Nature abhors a vacuum." It cannot be, therefore, that we are projected thus through space.

There is the Moon above us shining brightly. She is moving in the same direction that we are, and she moves faster through space than we do. She accompanies the Earth in her orbit round the Sun, and at the same time she goes completely round the Earth relatively to the Sun, every twenty-nine and a half days. She has kept the same relative position to our Earth in all her vast journeys of thousands of years. By what principle are they thus held together? Have we any evidence of the existence between the Earth and the Moon of any medium but the common atmosphere in whatever rarefied state that may be? Must we not suppose, then, that such atmospheres fill the whole space between them? Are not the Earth and the Moon, then, a complete system by themselves, revolving together from the beginning of time as one, yet forming a part of another and higher system, and that again still higher, forever upward and onward, till it shall reach the Throne of God! It appears to me, therefore, that the Earth and the Moon form a separate and independent system in themselves, though tributary to one higher. Can there be any dividing line between the Moon's atmosphere and ours? Is it not more probable that they are united and move together in mingling, flowing currents, like the ocean currents of this globe, which by their motion and mingling, are made to assist in preserving the world's balance?

In that case the Earth would then be the centre, and the Moon's path the circumference of one inseparable moving orb. By the laws which regulate motion and magnitude the Earth would then move through her orbit, not by attraction to the concrete Earth alone, but to the great luminous orb which should include the Moon's path, and all between that and the Earth, and this, perhaps, a Sun to other worlds; then it would follow that the Earth's orbit round the Sun should be measured by the revolutions of the Moon's orbit round the Earth. This is a new idea, it is a grand one,—it is startling. I must look to it and see if it is true.

Imagination has now done her work,—she has led our thoughts, she has created a figure and presented the moving image to our mind of the Earth revolving round the Sun

and measuring the distance performed by the value of the circumference of the Moon's orbit, and, having no more to do, Imagination now bids us to prove our intelligence.

I retire immediately to my room, and not to make my story too long, after a variety of defective and erroneous calculations I at length arrived at these results:

The Sun, as ascertained by my Quadrature, is distant from our Earth 92285568, of those parts of which the diameter of the Earth is 7912. We call them miles. The diameter of the Earth's orbit is then 184591136 of such miles. Its circumference is therefore 579847623+ of the same miles. In passing round the Sun the Earth revolves on her axis 365.24225 times. In so doing she gains one revolution=1. In the same period the Moon revolves about the Earth 12.36826. In doing so she gains one revolution of the Earth. Total of revolutions, 379.61151+

These revolutions are all of them solar days, each one of which is greater than the revolution of a circle in space in the proportion of 5184 to 5153 and therefore as 5153 : 5184 : 379.61151=381.895, and the circumference of the Earth's orbit being 579847623+÷381.895 gives 1518343 miles for the circumference of the Moon's orbit, and the diameter of 1518243+is 483303+half equals 241651 which is the Moon's distance from us. Now observe that this distance is of such parts as compose the Earth's diameter reckoned at 7912 miles, and, if there is any error in that diameter of the Earth, then there is an equal proportional error in this distance. And in my opinion the diameter of the Earth should be considered as 7853 + miles, and in that case the Moon's distance will be 239838 and the Sun's distance will be 91597392 miles. The reasons for all these conditions will be apparent to any one who will take the trouble to examine them.

In the problem to determine the Sun's distance, I have taken the diameter of the Earth at 7912 miles, because that is the sum set down in the books as ascertained by the measurement of a degree on the Equator. But because in the problem to determine the Sun's distance I have taken the Earth as "Unit," I am of the opinion that the diameter should be taken as .7853+ or one quarter the circumference of one diameter, and particularly this should be so, as the calculations are all made in reference to motion. This gives a difference in the diameter of the Earth of between 58 and 59 miles, and I think the last named sum (7853+) is the most accurate.

Astronomers say that the Moon's distance from us is a fraction less than 239,000 miles, but theirs is only an approximation, and they admit a liability to error equal to one or two thousand miles. And again their measurement is from the centre of the Earth, to the centre of the Moon, and it is self-evident that our measurement as first given includes the Moon's whole diameter, and her atmosphere which moves with her, and this being considered it will increase their distance or diminish ours ten or twelve hundred miles, and astronomers admit a possible error greater than that difference would show. But if we

admit an error in the diameter of the Earth equal to that above stated, we shall then approximate to the latest estimate of the Moon's distance by astronomers almost exactly. Now which shall we accept for the Moon's distance? the measurement of astronomers with its liability to error of a couple of thousand miles or so, or my hypothesis, which, if true, gives the exact distance to a mile? I think the evidence is all in favor of the hypothesis, which makes the Earth to revolve about the Sun on the value of the circumference of the Moon's orbit, this result being produced by the natural laws of motion, and the attraction of magnetism (or gravitation, if you please to call it such) with the universal law of precession to make motion continuous, and not by any imaginary projectile force which does violence to Nature.

The Sun's distance from the Earth was formerly considered to be 95,000,000 miles, some have said 96,000,000; here again astronomers admitted a possible error of one or two millions of miles. La Place thought it was within $\frac{1}{87}$ of the truth. Of late, however, they have come to the conclusion that the distance is much less, and as deduced from the angle of parallax, it has by some been stated a fraction under 92,000,000, and I hear that much anxiety is felt to ascertain the fact, by observing the transit of Venus to happen in 1874.

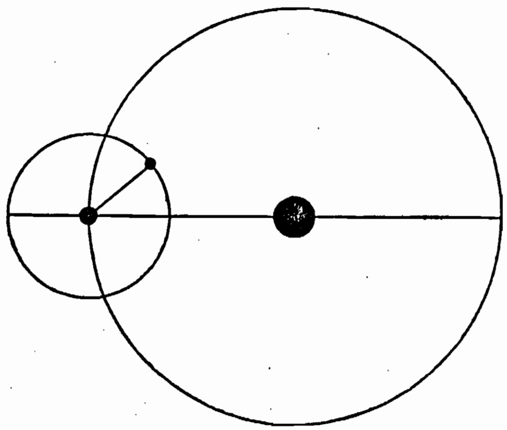
I determined the Sun's distance by my Quadrature published in 1851, to be 92,285,568 miles such as compose the Earth's diameter reckoned at 7,912 miles, and I wait the result of the coming observations of the transit without fear of contradiction. It was not until a dozen years after the publication of my Quadrature that the change was made in the estimate of the Sun's distance, and then without credit to my earlier demonstration.

But we need not wait for the transit to learn the truth. The mechanical properties of numbers, which Archimedes threw away and which all astronomers now throw away as useless, will help us out of the difficulty. If we admit the hypothesis that the Earth revolves about the Sun on the value of the circumference of the Moon's orbit (and I do not see how we can help admitting it, unless we first deny the unity of the Earth and the Moon, as constituting a perfect and distinct system, and unless we deny also the well established truth that the Heavenly bodies are governed by laws that regulate their distance and motion relative to each other, precisely according to their relative magnitudes, density, etc.), then it will follow, from the mechanical properties of numbers, that knowing how many revolutions of the Earth and Moon are performed (381,895) in the passage over the Earth's orbit, if we divide that number into the Sun's distance (which is the radius of the Earth's orbit), the product will be the radius of the Moon's orbit.

If, then, we want to know whether the Sun is 95,000,000 of miles distant from us, we will divide 95,000,000 by 381.895, the number of revolutions of the Earth and the Moon, and that will give 248,759 miles as the Moon's distance; and we know from observation, aside from our own theory, that it is not 248,759 miles, and that it does not differ much from 240,000, and by the same means we then know, also, that the Sun is not 95,000,000 of miles from us. Again, knowing approximately the Moon's distance to be about from 240,000 miles, if we take that as radius, and multiply it by the number of revolutions which the Earth and the Moon perform (381,895) in passing over the Earth's entire orbit, the product will be the Sun's distance or the radius of the Earth's orbit. And here again we see that the Sun is not 95,000,000 of miles from us, and, moreover, that its distance is not greater than the sum we have given it, viz., 92,285,568 of those parts of which the Earth's diameter is 7,912, which we call miles. Again, knowing the Moon's distance, we can calculate the circumference of her orbit, and, multiplying that by the number of revolutions, will give us the Earth's orbit round the sun.

These truths are all so perfectly simple as scarcely to need a diagram to illustrate them, but as all may not see the truth quite clearly I have prepared here a diagram to illustrate the power of numbers as applicable to our theory.

Let us consider the larger circle as the Earth's orbit, with the Sun in the centre, the lesser circle is the Moon's orbit with the Earth in the centre. Now let us suppose that the diameter of the lesser circle is 1, and the diameter of the greater circle is 12, then in the lesser revolving about the greater the lesser will make 12 revolutions. Then divide the number of revolutions 12, into 6, the radius of the Earth's orbit, the result is .5, which is the radius of the Moon's orbit. Again, if you know the Moon's



distance, and multiply it by the number of revolutions required to pass round the Sun, it will give you the Sun's distance or the radius of the Earth's orbit. And if you know the Moon's distance you can calculate the circumference of her orbit, and, multiplying that by the number of revolutions required to pass round the Sun, it will give you the Earth's orbit. If you know the Moon's distance, and divide it into the radius of the Earth's orbit, it will give you the number of revolutions of the Moon in her orbit required to pass round the Sun, and these are just the things which the Earth and the Moon together perform

every year in the fulfilment of their respective journeys round the Sun.

Divested of the lumber which Science has given these questions, it looks too simple to be believed, but you may alter your proportions and carry your numbers and diameters to hundreds of millions, as you please, and still you will find numbers true to those mechanical properties.

Admit the hypothesis, then, as in fact we have already proved it, that the Earth revolves round the Sun on the value of the circumference of the Moon's orbit, and we then know the Sun's distance, without waiting till 1874 to learn the confused results of the vast outlay for observing the transit of Venus; and what is of much greater importance, we have learned by this examination that because our Earth revolves about the Sun, on the value of the circumference of the Moon's orbit, the Earth being in the centre, the Earth therefore moves evenly through space, as the centre of that revolving orb, put in motion by the attraction of magnetism (gravitation) and kept in motion by the universal law of the Sun's precession, and therefore there is no such thing in Nature as a projectile force, and especially there is none in the motion of our Earth round the Sun.

Hence it will be seen, that accepting these premises to be true, it will result that the Earth's daily progress in her orbit round the Sun is exactly equal to the circumference of the Moon's orbit round the Earth, viz., it is exactly

1,518,343 miles, such as compose the Earth's diameter reckoned at 7,912 miles. Reduced to time, the daily motion is as follows: Revolution of the Earth on her axis, 24 hours; in the meantime the Moon has advanced, in her orbit round the Earth, 52′ 30″+, sidereal time, to which you must add the difference between a solar day and the revolution of a circle in space, 8′ 36″+.

If it is objected that neither the Earth nor the Moon revolve in a circle, and therefore those things cannot be true, the answer is, that if we suppose either the Earth or the Moon to revolve in a circle, the area of which is equal to the area of the ellipse in which they do actually revolve, then the radius of the circle is exactly the mean distance of either the Earth or the Moon from the centre around which they revolve; and it is precisely the same thing whether they revolve in a circle by an equal motion, or in an ellipse by an unequal motion; in either case they fulfil the law of passing over equal areas in equal times.

JOHN A. PARKER.

of this problem. Nevertheless he tried an ingenious method; he rolled a circle on a plane or line, and supposing that its circumference was applied to it wholly until the point which had first touched it touched it again; he therefore justly inferred that this line would be equal to the circumference. He even conceived the outline of the curve, which the point that first touched the straight line was to describe which formed the curve, since called the Cycloid. But he supposed, with Charles de Bovelle, in the following century, that this curve was itself an arc of a circle, and from this he claimed to determine it by a geometrical construction which was entirely arbitrary, resting on no real property of this movement. He also tried another method, according to which he gave the following solution of the problem: a circle being given, add to its radius the side of the inscribed square, and with this line as diameter describe a circle, in which is inscribed an equilateral triangle, the perinuter of this triangle will, says Cardinal de Cusa, be equal to the circumference of the first circle.

It was not difficult for Regiomontanus to prove that Cusa was mistaken; this relation of the circumference to the diameter fell outside the limits demonstrated by Archimedes; that is according to this relation the diameter would be to the circumference as one to a number greater than $3\frac{1}{7}$ already too large. Besides, the Cardinal learned for his age, though very much addicted to astrology, he presents in the collection of his works several geometrical tracts which are full of paralogisms.

We have just spoken of Charles de Bovelle or Carolus Bovillus, distinguished at the time by the title of noble philosopher. He signalized himself by the strangest ideas. He gave in 1507 a work entitled: *Introductionum Geometricum*, translated into French and republished in 1552 under the auspices of Oronce Finée, under the title of *Geometrie Pratique, Composée par le noble* Philosopha, *Maitre Charles de Bovelle*, etc. He claims to give there the quadrature of the circle according to the idea of the Cardinal de Cusa, which, he says, came to him by seeing a wheel moving on the pavement. But the construction by which he pretends to give the length of the line to which is applied the circumference of the rolling circle is absolutely arbitrary, and it would follow that the diameter is to the circumference as 1 is to the square root of 10, or 3.1618, which is far from the limits of Archimedes. What is also singular, is that in this same book, and in an appendix added to the first volume of the preceding works, he speaks of the

quadrature of the circle made by a poor peasant, according to which the circle having 8 for diameter is equal to the square having 10 for diagonal, that is to 50, which is false; for the circle is in this case less than $50\frac{2}{7}$, and more than $50\frac{18}{71}$, and the quadrature of Bovelle does not agree with that of the peasant, which he considers as true; for the latter gives the relation of the diameter to the circumference exactly as 1.0000 to 3.1250; the noble philosopher even wanders further from the truth than the peasant does below; and he might have been told that when one is mistaken he ought not at least to contradict himself. Bovelle says, falsely, that these relations coincide. Either he had not performed the calculation himself, or he did not know enough arithmetic to extract by approximation a square root; these works of Bovelle are pitiable; his manner of cubing the sphere is preëminently absurd.

We are sorry to find in the same class a royal professor of the 16th century, named Oronce Finée, who, by his numerous works, acquired a kind of fame. He gave in his Protomathesis a quadrature of the circle, a little more ingenious, in truth, than that of Bovelle; but which is, nevertheless, a paralogism. On the point of dying, in 1555, he urgently advised his friend, Mizault de Monthuçon, to publish his discoveries, not only upon this subject, but also on the most famous problems of geometry, such as the trisection of the angle, and the duplication of the cube, and the inscription in the circle of all regular polygons. Mizault kept his word, and in 1556 published this assemblage of paralagisms under the title of *De rebus Mathematicus hactenus desideratis, libri IV.* Most of these problems are solved in various ways by him; it happens that his different solutions of the same problem do not agree with one another, nor with those of Bovelle, and of his rural geometrician which he had approved by publishing them, it was the height of false reasoning in geometry; consequently he was easily refuted by the geometrician Buteon, who had been his disciple at the College Royal, by Monus or Nunez, a Portuguese geometrician, and several others; but still he died satisfied, fully persuaded that his name would be placed on a level with those of Archimedes and Apollonius. This scandal was renewed among the royal professors in 1600, when Monantheuil, one of their number, published a quadrature of the circle.

One Simon a Quercu (doubtless Duchêne or Van Eck) appeared on the arena a few years later, in 1585, and proposed a quadrature of the circle. His pretended discovery was much less wide from the truth than those of his predecessors and fell within the limits of Archimedes. So Peter Metius, who undertook to refute him, was obliged to seek for a closer relation of the diameters to the circumference, and found that the one was to the other as 113 to 355. The pretended quadrature of Duchêne could not stand this test, and must be named only because it led to the curious and elegant discovery of Metius; for this relation of 113 to 355, reduced to decimals, is the same as 10000000 to 31415929; which is at the most but $\frac{3}{10,000,000}$ of the diameter in excess. The diameter of the earth being only 6542816 (toises) or 13936912 yards, the error made by this relation of the circumference of a circle of that size would scarcely be 2 (toises) or 4 yards. If those who connect in their minds the problem of the quadrature of the circle with that of longitudes, knew what we have just said they would soon see their mistake; for, if these problems were connected with each other, what would be an error of 4 yards on a track around the earth? The Spanyard, Sir Jaime Falcon, of the order of Our Lady (*Notre Dame*), of Montesa, published in 1587, at Antwerp, his paralogism on the quadrature of the circle. His book is rendered amusing by a dialogue in verse between himself and the circle, which thanks him very affectionately for squaring it; but the good and model knight ascribes all the honor thereof to the holy patroness of his order. The paralogism was apparently so gross that no one took the trouble to refute it.

But a man much more famous than the foregoing challenged the attention of learned Europe by his pretensions on the quadrature of the circle; it was the celebrated Joseph Scaliger. Full of self conceit, he supposed that he had only to present himself on the field of geometry and that nothing that had baffled geometricians until then could resist a man of letters with his powers. He therefore undertook to find the quadrature of the circle, and put forward, with much braggadocia, his discoveries on this subject in a book which appeared in 1592: *Nova Cyclometria;* but he had no cause to congratulate himself on having thus wished to place himself among geometricians. For he was refuted by Clavius, Viete, Adrianus Romanus, Christman, etc., who showed each in his own way that the size which he assigned to the circumference of the circle was only a little less than the inscribed polygon of 192 sides; which being absurd, demonstrated the incorrectness of Scaliger's reasoning; but he did not surrender; and never did a man who thought he had discovered the quadrature of the circle, the trisection of the angle, the duplication of the cube, or perpetual motion,

give in to the plainest reasoning. He will sooner deny the most elementary propositions of geometry, like Moliensis Cana, who found no less than twenty-seven false propositions in the first book of Euclid. Scaliger replied with bitterness to the geometricians who had censured his quadrature; he treated them with contempt, especially Clavius who had already wounded him by an answer to his attack on the Gregorian Calender. Unfortunately for the honor of Scaliger abuse is not reasoning, and the established fact remains that Scaliger, an eagle in literature, was nothing in geometry.

Scarcely had Scaliger disappeared when one Thomas Gephirander came to take his place. But he had not Scaliger's pride; he acknowledged, even in the title of his book, that his discovery was simply the result of divine Grace. We shall see many others gifted with this same spirit of humility. The *paralogism* of Gephirander nevertheless was palpable; for it consisted in the pretension that if between two magnitudes there is any geometrical relation whatever, the same relation will exist if the same quantity is taken from each. Thus, according to this illuminate, the same relation exists between 2 and 5 as between 3 and 6, since only the same quantity, viz., unity is subtracted from each of these numbers. But scarcely any of the follies with which false reasoning, a false mind, and the conceit of never recanting one's errors inspire these visionaries have equaled those of Alph. Cano, of Molina, in a book entitled: *Nuevos descubrimientos Geometricos.* He remodels the whole of Euclid, and scarcely one of his propositions is spared by him. Yet who would believe it! he found another fool, named Janson or Jansen, who translated him into Latin under the title of *Nova reperta Geometrica*, etc. Moreover Cano admitted that he had not the least idea of geometry until the Deity, whose delight it is to humble the proud and enlighten the ignorant, had inspired him.

A similar wiseacre presented at the same time in France his paralogisms upon the quadrature of the circle and the duplication of the cube. It was a merchant of Rochelle, called De Laleu. This one also pretended to have received the solution of these problems by *divine revelation*, and announced that the union of the Jews, Turks, and Pagans to the Christian religion depended upon the manifestation of this truth. In fact, according to him, the quadrature of the circle was the quadrature of the heavenly Temple, and the duplication of the cube that of the elementary, terrestrial, and aquatic altar, whence was to flow the conversion of the Jews, idolaters, etc. Accordingly some religionsections and of equations. I undertook, in 1754, to make it more plain and develop it more fully in my *Histoire des Recherches*, etc. I will have to refer to it and deem it convincing. Besides, although geometry presents numberless examples of squared curves, I know of none among the enclosed curves or curve continually receding (*retournant*) upon itself, that can be. Still D.'Alembert, in the fourth volume of his *Opuscules*, 1768, says, that he can scarcely assent to Newton's reasoning to prove the impossibility of the quadrature of the circle. I see, says he, that a similar course of reasoning, applied to the rectification of the cycloid, would lead to a false conclusion, the only difference, it seems to me, is that the circle is a receding curve and the cycloid is not. But I see nothing in Newton's reasoning which can be changed by disparity, more particularly, since the cycloid, if it is not a receding curve like the circle, is a continued curve whose sides (branches) are not separated; in a word, the reasoning of Newton rests solely on this supposition that in the circle an infinite number of areas corresponds to the same abscissa, whence he infers that the equation between the arc and the abscissa must be of an infinite degree, and consequently is not algebraically rectifiable; now, by applying his reasoning to the cycloid, I would infer that the equation between the abscissa and the corresponding arc must also be of an infinite degree, and therefore the arc is not rectifiable algebraically, which is false. D.'Alembert made the calculation and concluded by saying, it seems to me that these reflections might deserve the attention of the geometricians and induce them to look for a more vigorous demonstration of the impossibility of the quadrature and of the indefinite rectification of oval curves.

We shall now give a brief account of the principal discoveries on the quadrature of the circle, as most of them are included among the geometrical discoveries already discussed in the former volumes, I will only give them here without going into details Archimedes first discovered that the circumference is less than $3\frac{10}{70}$ or $3\frac{1}{7}$, and more than $3\frac{10}{71}$ times the diameter. Some of the ancients, as Appolonius and Philo, found nearer relations, but it is not known what they were.

About 1585 Peter Metius, in impugning the false quadrature of Duchêne, gave his near ratio of 113 to 355. It was shown above how near he was right. About the same time Viete and Adrianus Romanus also published relations expressed decimally which came much nearer to the truth. Viete carried the approximation to 10 decimal places instead of 6, and taught besides several somewhat simple constructions which

gave the value of the circle, or the circumference to within a few millionths. Adrianus Romanus carried the approximation to 17 figures. But all that is far below what was done by Ludolph Van Ceulen, and which he published in his book *de Circulo et adscriptis*, of which Snellius published a Latin translation, at Leyden, in 1619. Ceulen, assisted by Petrus Cornelius, found with inconceivable labor a ratio of 32 decimals; see V. II, p. 6. Snellius found the means of shortening this calculation by some very ingenious theorems, and if he did not excel Van Ceulen he verified his result, which he put beyond attack. His discoveries on this subject are found in the book entitled *Willebrordi Snelli Cyclometricus de Circuli dimensione*, etc. Descartes also found a geometrical construction which, carried to infinity, would give the circular circumference, and from which he could easily deduce an expression in the form of a series. (See his *Opera posthuma.*)

Gregoire de Saint-Vincent is one of those who are most distinguished in this field; true, he claimed incorrectly to have found the quadrature of the circle and of the hyperbola, but the failure in this respect was preceded by so great a number of beautiful geometrical discoveries, deduced with much elegance according to the method of the ancients, that it would have been unjust to have placed him among the paralogists we have mentioned. He announced, in 1647, his discoveries in a book entitled: *Opus Geometricum quadraturae Circuli et Sectionem Coni* libris, X, *Comprehensum.* All the beautiful things contained in this book are admired; only the conclusion is impugned. Gregoire de Saint-Vincent lost himself in the maze of his proofs which he calls *proportionalities*, and which he introduces in his speculations. It was the subject of quite a lively quarrel between his disciples on the one hand, and his adversaries on the other, Huygens, Mersenne, and Leotand, from 1652 to 1664.

If that skillful geometrician had not been mistaken, it would only have followed from his investigations that the quadrature of the circle depends upon logarithms, and consequently on that of the hyperbola. That would still be a handsome discovery, but it did not even have that advantage. This furnished Huygens the occasion of divers investigations on this subject. He demonstrated several new and curious theorems on the quadrature of the circle: *Theoremata de quadratura hyper, ellipsis et Circuli*, 1651; *De Circuli Magnitude inventa*, 1654. He gave several methods of approaching his quadrature much shorter

THE SQUARE ROOT OF TWO.

THE following pages are intended to explain the nature, uses, and advantages of the *Common Measure*, as applied to Civil Engineering, Architecture, Draughting, Machinery, Building, Painting, and Landscape Gardening.

Numerous instances could be given where it has been already tested and found to be everything that could be desired.

According to PROPOSITION 1, COR. PART I, the side of any square is to the dioganal as "*one is to the square root of two;* that is, if the side of any square be ONE (1), the diagonal *must* be the square root of two ($\sqrt{2}$); therefore, by the ordinary method, to find the diagonal of a square, when the side is known, use the following

RULE.

Take as many figures of the square root of two as will make the result sufficiently exact, and multiply these figures by the number of units in the side of the given square, the product will be the true diagonal in terms of the side of the square.

NOTE.—If the side of the square be expressed in inches, the diagonal will also be expressed in inches; but if the side be expressed in feet or rods, etc., the diagonal will be expressed in feet or rods, etc.; and if the side contain a decimal or a fraction, the result of that decimal or fraction will have its corresponding result in the diagonal. *In all cases the requisite number of figures of the square root of two must first be obtained.*

When the diagonal is given, to find the side, it is not so easy a task, and requires more time and labor than to find the diagonal when the side is known; but, for the benefit of those who still prefer the former method, I shall here insert the rule which embraces two cases.

CASE 1.

If the diagonal is expressed in integers, such as 1, 2, 3, etc., the side of the square is found by dividing these numbers by the square root of two; the result will be the side in the same denomination as the diagonal.

CASE 2.

If the diagonal contains either a fraction or a decimal, this fraction must first be reduced to a decimal and the value pointed off as in reduction of decimals. If a decimal only, it must first be pointed off, then divide this decimal by the square root of two, according to the rule for division of decimals, the quotient will be the side of the square in terms of the diagonal; *but, in all cases, the requisite number of figures of the square root of two must first be found.*

To find an *exact* common measure of the *side* and *diagonal* of the square would be equivalent to finding the exact value of the square root of two; but the square root of two is an *irrational quantity*, therefore it has no end. It is believed that the following common measure comes nearer to an *exact* common measure than any that has been heretofore found, as it can be extended *ad infinitum, and when either the side or the diagonal of any square is known the other can be found by common multiplication and division.*

Owing to the difficulty of working out the necessary figures of the square root of two, and the tax upon the mind necessary to remember the same, civil engineers, architects, draughtsmen, builders, etc., have long since felt the want of a COMMON MEASURE expressed in integers, or a series of numbers, *which would express the relation between the side and diagonal of the square;* one that could be easily remembered, plainly understood, and readily applied. Such, for example, as the ratio between the *circumference and diameter of the circle*, found by Mœtius in 1640, namely: $\frac{355}{113}$, or 113|355, which it is said will give the ratio to *six* decimal places correct, viz.: 3.141592.

The author is happy to state that such a common measure of the side and diagonal of the square has been found which may be expressed in *integers*, and he ventures to hope that when it is fully tested the old method of squaring the side, doubling and extracting the square root, will be gladly cast aside as a noble relict of "*Auld Lang Syne*," when simpler and easier methods were unknown.

The *common measure* here introduced combines all the advantages of conciseness, simplicity, and perfection, for by its aid the desired results are reached much more rapidly than by any former method and with equal exactness.

First Example.—Suppose a builder is laying the foundation for a house and wants to make each of the corners a right angle—that is, a perfectly square corner; how will he do it?

Answer.—First measure 10.5 feet on each side of the right angle in the direction of the sides, commencing at the angular point; then the distance across from the two extreme points will be 14.85 feet, or 14 feet, 10.2 inches. This result is found by multiplying the side of the given square by 99 and dividing the product by 70, which gives the desired result. See Plate 8, Figure 1.

Second Example.—Suppose a civil engineer, while surveying, wants to lay out a right angle without the use of his instruments; how will he do it?

Answer.—Measure with a tape line any distance, on either side of the right angle, say 105 feet, in the direction of the sides, commencing at the angular point. Then will the distance across from the extreme points be 148.5 feet, or 148 feet 6 inches. This result is found precisely like the former, namely, by multiplying the side of the given square by 99, and dividing the product by 70, which gives the desired result. See Plate 8, Figure 2.

Third Example.—Suppose a mechanic has a given circle, the diameter of which is 9.9 feet, or 9 feet 10.8 inches, and he wants to find the side of the largest square which it is possible to *inscribe* in the given circle; how will he find it?

Answer.—Multiply the given diameter by 70, and divide the product by 99, which will give the desired result, which will be 7 feet. See Plate 8, Figure 3.

If any person has a desire to test the correctness of these results by the former method, he is at liberty to do so, and will find them correct to the fourth decimal place.

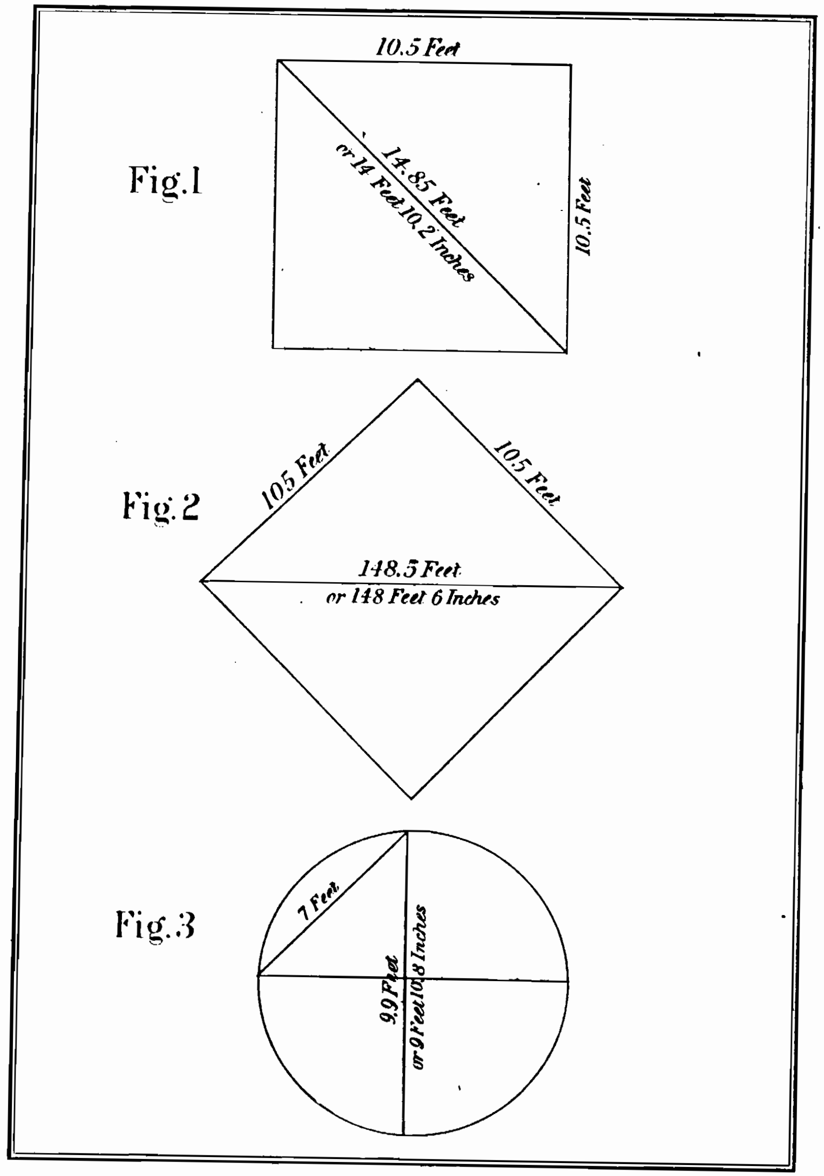
Note.—In the third example above, the diameter of the circle may be regarded as the diagonal of the given square, the side of which was required to be found; therefore the square which is so formed within the given circle is said to be inscribed within it.

From these three examples, which embrace two cases, we deduce the following rules:

Case 1.

When the side of any square is known, to find the diagonal correct to *four places* of figures.

PLATE VIII.



RULE.

Multiply the side of the given square by 99, and divide the product by 70; the quotient will be the diagonal of the square in terms of the side.

CASE 2.

When the diagonal of any square is known, to find the side.

RULE.

Multiply the diagonal of the given square by 70, and divide the product by 99; the quotient will be the side of the given square in terms of the diagonal.

NOTE.—It is believed by the author that the above numbers will answer every purpose for all ordinary measurements; besides this, they are simple and easily remembered by the most ordinary mind. If, however, still greater accuracy is required, attention is invited to the following, which the author ventures to hope will be found to be sufficiently correct to satisfy the most careful.

By *Case* 2, PART 1, it is shown that where the radius of the given circle is the *square root of two*, the side of the inscribed square will be *two*, and the side of the inscribed square, the diagonal of which corresponds with the radius, is *one*. Therefore the secant No. 2 is $\frac{99}{70}$, which, by division, gives the square root of two to five places of figures correct, thus: 1.4142; that is, if the side of the inscribed square, the diagonal of which corresponds with the radius, be divided into 70 equal parts, the diagonal of the same square will be 99 of the same parts nearly.

By *Case* 3, PART 1, it is shown that the secant No. 3 is $\frac{19601}{13860}$, which, by division, gives the square root of two to nine places of figures correct, thus: 1.41421356; that is, if the side of the inscribed square, whose diagonal corresponds with the radius, be divided into 13860 parts, the diagonal of the same square will be 19601 of the same parts nearly.

By *Case* 4, PART 1, it is shown that the secant No. 4 is $\frac{768398401}{543339720}$, which, by division, gives the square root of two to 18 places of figures correct, thus: 1.41421356237309504; that is, if the side of the inscribed square, the diagonal of which corresponds with the radius, be divided into 543339720 parts, the diagonal of the same square will be 768398401 of the same parts nearly.

By *Case* 5, PART 1, it is shown that the secant No. 5 is $\frac{1180872205318713601}{835002744095575440}$, which, by division, gives the square root of two to 36 places of figures correct, thus: 14142135623730950488016887-2420969807; that is, if the side of the inscribed square, the diagonal of which corresponds with the radius, be divided into 8350027440955-75440 parts, the diagonal of the same square will be 1180872205318-713601 of the same parts nearly.

By *Case* 6, PART 1, it is shown that the secant No. 6 is $\frac{2788918330588564181308597538924774401}{1972063063734639263984455073299118880}$, which, by division, gives the square root of two to 72 places of figures correct, thus: 14142135-6237309504880168872420969807856967187537694807317667973799-0732478; that is, if the side of the inscribed square, the diagonal of which corresponds with the radius, be divided into 197206306373 4639-263984455073299118880 sides, the diagonal of the same square will be 2788918330588564181308597538924774401 of the same parts nearly.

By *Case* 7, PART 1, it is shown that the secant No. 7 is $\frac{1555613090938580753522477984263968662546864806579817762712\text{-}6514337489817601}{1099984505550523587637197163135916400298236440516457204353 1\text{-}7842392159581760}$, which, by division, gives the square root of two to 144 places of figures correct, thus: 1.41421356237309504880168872 42-0969807856967187537694807317667973799073247846210703885038-753432764156364397719572401892916077107712236533038460062 7; that is, if the side of the inscribed square, the diagonal of which corresponds with the radius, be divided into 10999845655052358763 7-19716313591 sides, the diagonal of the same square will be 1555613-090938580753522477984263968662546864806579817762712651433 7-489817601 of the same parts nearly.

By *Case* 8, the square root of two can be found, by division, to 288 places of figures, by *Case* 9 to 576 places, by *Case* 10 to 1152 places, and so on *ad infinitum*; and, in every case, if the side of the inscribed square, the diagonal of which corresponds with the radius, be divided into the requisite number of parts, the diagonal will still be expressed

by a certain number of the same parts till it reaches the vanishing point, *when the square root of the sum of the squares of the two sides of any square can be extracted exactly*,

But if by the word "infinite," *indefinite extension* only is meant, then the operation may be continued without end, and in THAT CASE there will forever remain an indivisible unit, the square root of which never can be extracted.

If, then, the true ratio of the circumference of the circle to its diameter be as $3\frac{1}{7}$ is to 1, and the radius of the given circle be the $\sqrt{2}$, then will the area contained within the given circle be $6\frac{2}{7}$, and the side of the inscribed square will be two, while the area will be four. But the area of the circumscribed square is *double* the area of the inscribed square, therefore the area of the circumscribed square is 8.

Again, the square described upon the diameter of any circle is equal to the circumscribed square.

But the diameter of the given circle is $\sqrt{8}$, therefore the area of the circumscribed square is equal to $\sqrt{8} \times \sqrt{8} = 8$.

Again, the area of the inscribed square is *one-half* the area of the circumscribed square; therefore the area of the inscribed square is 4.

Now the area of the given circle is $6\frac{2}{7}$, and the area of the inscribed square is 4, and the area of the circumscribed square is 8; *therefore the area of the circle is to the area of the inscribed square as* 11 *is to* 7; and the area of the circle is to the area of the circumscribed square as 11 is to 14.

Therefore to find the area of any circle, when the diameter is known, use the following

RULE.

Multiply the square of the diameter by 11, *and divide the product by* 14, *the quotient will be the area of the circle;* or, *square the radius, and multiply it by* $3\frac{1}{7}$, *the product will be the area of the circle.*