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THE RULE OF NINE

Simplified and Expanded

An easy, speedy way
to check addition,
subtraction, multiplication
and division

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FOREWORD

The Rule of Nine is a fast, easy method of checking the mechanics of arithmetic. It is not new. This pamphlet is merely a clarification of it.

The individual who understands this system automatically checks every step of his work. Many young people, who have been uninterested in Arithmetic, will tackle it with new zeal, knowing that they have in this rule the means of obtaining higher grades. It can also be used to great advantage by adults in all walks of life.

The Author

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SECTION I

Introductory Facts

For readers who do not already know, a single digit is any single number from 0 to 9. Numbers like 8, 5, 7, and 2 are single digits. However, 86 is not, nor is 10, nor 12. They are combinations of single digits.

The basic rule of the Rule of Nine is that *all digits* which go to make up a number must be *added together* until only a single digit remains. Example: The numeral 12 becomes 3 because 1 and 2 added together become 3; the numeral 14 becomes 5 because we add the 1 and the 4 to get 5; the numeral 25 becomes 7 because the sum of 2 and 5 is 7.

Likewise, 56 becomes 2. We arrive at the 2 in the following manner. First we add the 5 and 6 to get 11. Because this 11 is not a single digit, we must add the two 1's together to get 2. The numeral 2 is a single digit. Because we can add no further, we have satisfied the rule. The numeral 78 would add up to 15, which in turn would add up to 6. The numeral 683 would add up to 17, which in turn would add up to 8.

You should, by now, understand why we say 56 equals 2. Do not let this statement, which may at first seem to contradict what you have been taught, worry you. For our purposes in this pamphlet you will find that we add by two different methods, the normal one, and by the Rule of Nine. The numeral 56 equals 2 is a very good example of addition by the Rule of Nine, a form of addition that will be used very frequently from here on.

Here is a *very important fact* about the Rule of Nine. The adding of 9 to any number does not change its single digit value. Notice why: 7 plus 9 equals 16. The 1 and the 6 must now be added together and come out 7, or the very same number you started with before you added the 9. Let us write this down so you can see it. Notice that each of these examples begin and end with the same number regardless of the addition of the 9.

$7+9=16=7 \qquad 2+9=11=2 \qquad 5+9=14=5 \qquad 9+9=18=9$

You have just learned above that the single digit value of a number is not in the least affected by the addition of a 9.

Here is one more closely allied step. The addition of any combination of numbers whose sum equals 9 does not affect the single digit value of the number to which it is added any more than would be addition of the 9 itself. The numeral 276 equals 6, because the sum of 2 and 7 equals 9, and this 9, when added to the 6, does not change it because the total of 6 and 9 is 15, which in turn adds up to 6. Add up 635 and you will find it will come out 5. There will be more on this later.

This may seem difficult to adults, but remember that youth is more adaptable to new processes than is maturity. A ten-year-old can probably comprehend this Rule more readily at first than his parents. He is used to dealing with things new to him. Nevertheless, a bit of work with a pencil and paper will give anyone mastery of the preceding rules which, when learned, form the basis of the whole system.

SECTION II

Multiplication

Let us take multiplication for our first application of the Rule of Nine. Before we proceed, let me point out something you will observe for yourself, when you are more fully acquainted with this system.

The basic underlying idea of the Rule of Nine is to reduce all numbers to one digit so that we may do with the single digits whatever we do with the whole number. The answer we get by manipulating the whole numbers (whether by addition, subtraction, multiplication, or division) that answer when reduced to one digit, should equal the single digit answer arrived at by manipulating the single digits in the same fashion.

We all know that 12 times 12 is 144.

```
  12
×12
 144
```

Here we multiplied the 12's to get 144. To check the 144, our answer, we reduce each 12 to 3, and, because we multiplied the 12's together, we do the same with the 3's. 3 times 3 is 9. Now let's see how this looks.

```
 12  adds  up  to  3
×12   "     "   "  3   (3×3=9)
 144  "     "   "  9
```

Since the answer 144 also adds up to 9 the answer is correct.

Here is another:

```
 341  adds  up  to . . . . 8
 ×26   "     "   "  . . . . 8   (8×8=64=10=1)
2046
682
8866 adds up to 28 which equals 1 (2+8=10=1)
```

From here on, the single digit appearing to the right of certain numbers as shown above indicates their single digit value as determined by the Rule of Nine.

And here are two more:

```
  242 . . . 8                       7325   8
 ×14 . . . 5   (5×8=40=4)          ×234   9   (8×9=72=9)
  968                              29300
 242                              21975
 3388=22=4                       14650
                                 1714050=9
```

That is the basis of it. We will come back to it later, and simplify it much more after we have taken addition.

Problems

371	7685	372	4825	4921
×23	×37	×33	×139	×72

SECTION III

Addition

In this section you will see there are two methods of adding, the usual one you now use and by the Rule of Nine. You should learn to add by one equally as well as by the other. The regular method says that 6+5=11. When using the Rule of Nine, we say that 6+5=2 (because the 11=1+1=2).

When adding a string of three or more figures by the Rule of Nine method, we add all sums to one digit as we progress from figure to figure. Take the following example: 6+5+3=5. In arriving at this 5 we say 6+5=2, then carry our single digit forward to say 2+3=5.

Look at this sequence: 8+7+5+6=8. We arrive at the answer 8 as follows: 8+7=(15) . . . 6, then 6+5=(11) . . . 2, and then 2+6=8.

The above numbers within the parentheses are at first a necessary mental step. Soon, however, when adding the above figures, you will automatically say 8+7=6, and then 6+5=2. Finally you will add the 2 to the 6 to get 8. This adding to a single digit, as you progress from figure to figure in a line of addition, is one of the most important things to learn—and also to remember.

Here is another thing to remember. When adding, we not only completely ignore 9's, but combinations equaling 9. You will find that the addition of a 9, a group of 9's, or a combination equaling 9 never changes the number to which it is added.

To prove the above, add the three groups of figures below without the numbers enclosed in the parentheses, and then again, this second time including the previously excluded numbers. Your answer will in each case remain the same:

$7+5+(9)+5=8$; $7+6+(2+7)+5=9$; $(6+3)+(5+4)+3=3$.

In adding by this method, whenever you reach the sum of 9 drop it . . . forget it, and start again with the next number. For example: $7+5+6+8+4=3$

Here the first three numbers, the $7+5+6$ becomes 9 if you have added them to a single digit each time. Reaching this sum of 9 you should ignore it, forget it, and go on to add the $8+4$ which becomes 3, the single digit answer for this group.

In the following column of figures, adding downward, every cross line indicates that 9 has been reached and the count started again.

6
7
5
—
8
5
6
8
—
2
7
—
5
—
59 5

Do not forget to add to one digit as you move from number to number! You would add the first three numbers in this column, starting downward, as follows: $6+7=4$, then $4+5=9$. Forget the 9 and recommence with the 8. If you continue correctly, when you have reached the third crossline and dropped the

9, the 5 which remains is the single digit value of the column as ascertained by the Rule of Nine.

Notice that the correct answer to this column is 59, which also adds to the single digit 5. The Rule of Nine therefore checks your addition as correct.

When we have groups of numbers which must be added together, we approach the problem in a different way. After finding what we think is the correct answer to our problem, we check it by adding the individual lines crosswise, one at a time, by the Rule of Nine. The example below may clarify this for you:

63724	4
58462	7
39653	8
25778	2
187617=3	3

Look for easily recognized combinations totaling 9. The ability to do this readily will increase your speed immensely.

Having added the four lines across, we now add together the single digits we arrived at. Eliminating the 7 and the 2, a 9 unit, we find only the 8 and the 4 left, which total (12) . . . 3. If, when we total the column by the conventional method and reduce our findings to a single digit, should this digit be three, the answer is correct.

Problems

35927	4473	64828	4721
58472	2382	31309	3259
28335	4937	30199	3521
+73211	+3219	+21154	+2347

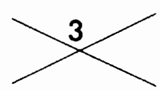
SECTION IV

Division

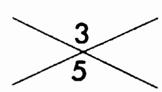
Division is beautifully adapted to checking by the Rule of Nine.

```
      2363
21/49625
   42
    76
    63
    132
    126
      65
      63
       2
```

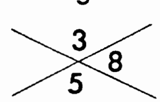
To prove this problem in the usual manner, we would multiply 2363 by 21 and then add the remainder 2. By the Rule of Nine we will follow the same procedure, but not until after we have reduced each of these three numbers to a single digit. First we will reduce the 21 to 3 and put the 3 in the top of an X as follows:



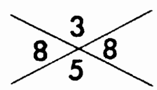
Next, we will reduce the answer, 2363, to 5 and put the 5 in the bottom space of the X.



Now we multiply the 3 by the 5 and get (15) . . . 6 to which we add the remainder 2 making 8. This 8 we place in the right hand space of the X.



If our answer is correct, we should find that the number 49625 will add up to the single digit 8 which of course it does. This 8 we will put in the remaining space in the X.



When the figures in the right and left spaces of the X agree, the answer is correct.

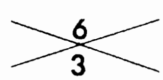
Before continuing, please try an example of your own, using the above explanation. It will probably save you time in the end.

Here is one more, just so that you can recheck yourself. This would be a tough one to multiply out for proof, but see how simple it is to check with the Rule of Nine.

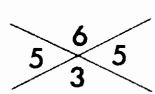
```
            1,902
834 / 1,586,489
        834
        7524
        7506
         1889
         1668
          221
```

The 834 adds up to 6

The 1902 adds up to 3



Multiply 3 by 6 and you get (18) . . . 9, which must be added to the remainder 221. 221 adds up to 5, which, when added to 9, still remains 5. This 5 we now put in the right hand space in the X. 1,586,489 adds up to 5 for the left side of the X.



Once you see that the numbers to the right and left of the

X match, you know your answer is correct. If they do not, it is wrong.

Problems

426 / 2,362,132   23 / 154,322   129 / 284,756

SECTION V

Subtraction

The Rule of Nine as a check in subtraction is not so vitally necessary. We all know how to check subtraction by addition. The Rule of Nine, however, is very often much faster, especially when once you have acquired the habit of casting out 9's and combinations of 9's, at a glance.

Let us take a problem:

$$\begin{array}{rr} 2970 & 9 \\ -1805 & -5 \\ \hline 1165 & 4 \end{array}$$

Here we look at the top line, and see at a glance a 9, and a combination of 9. Since we must put down some number, we put down 9. The second line contains a combination of 9, leaving us but the 5. Because this is subtraction, we subtract the 5 from the 9 and get 4. We now reduce the answer to our problem to one digit, and, because it also comes out to 4, our answer is correct.

When it happens that the single digits are not subtractable because they are both the same, or the upper one is smaller than the lower, you add 9 to the upper one and then subtract in the usual way. Here are two examples:

$$\begin{array}{rrr} 347 & 5 & +9=14 \\ -131 & 5 & -\ 5 \\ \hline 216=9 & & 9 \end{array} \qquad \begin{array}{rrr} 3{,}504 & 3 & +9=12 \\ -1{,}655 & 8 & -\ 8 \\ \hline 1{,}849=4 & & 4 \end{array}$$

A few trial problems will show you that this is quite correct.

Here is a short cut to remember. When both of the single digits are the same, the difference will always be 9. Why?

Because if both single digits are the same and you add 9 to the upper one and then subtract, it is obvious the difference between them must be 9.

4236	6
−1356	6
2880	9

Problems

43782	95357	39236	75632	529498
−27931	−73219	−33836	−23957	−493223

SECTION VI

Multiplication Expanded

Let us look now at the further possibilities of the Rule of Nine with multiplication. This is a procedure *everyone* should *always* follow. You have learned that it is possible to check multiplication by reducing the numbers to be multiplied to single digits, and then multiplying these single digits by each other. The most this will tell us, however, is whether the answer is right or wrong. The Rule of Nine can be of much more help than this. It can point out exactly where the mistake is, if there is one. For instance, suppose you have just completed multiplying 1356 by 732 and found your answer to be incorrect by the method shown in Section II. You have no alternative but to commence all over again and it is possible you may have to continue until you find your error, of all places, in the addition. Let us see how the following variation in the Rule of Nine can put its finger on the error for you.

We were speaking of multiplying 1356 by 732. Notice now what happens if we do this as three separate problems:

A	1356	6	
	×2	2	(2×6=12=3)
	2712	3	

Our first problem checks. Now for the second:

B	1356	6	
	×3	3	(3×6=18=9)
	4068	9	

That checks also. Now for the third:

C	1356	6	
	×7	7	(7×6=42=6)
	9492	6	

Suppose now we were to put the whole thing together, but still treat each line of multiplication as a separate unit. As we have seen in A, B, and C, we would have the following problem.

1356	6	
×732		
2712	3	(Refer to example A.)
4068	9	(Refer to example B.)
9492	6	(Refer to example C.)
992592	9	See next paragraph

To check our answer by this method, we add together the single digits of the three lines, A, B, and C (3+9+6) and get 9. Our answer to be correct must also add up to 9. By using this method, a student is able to check his work line by line as he proceeds with it.

Suppose that in the second line of multiplication, line B, you had made a mistake. The Rule of Nine would immediately spot it because the line would not add up to the single digit 9. If, on the other hand, all three lines of multiplication checked out correctly by the Rule of Nine, but the answer failed to check as correct, the error would then have to be in the addition.

By the use of this system, you are able to locate, almost at a glance, just where the trouble was without having to rework the whole problem.

Notice this. Had you been going to check only the answer as you were shown how in Section II, you would have arrived at its single digit value by adding the multiplier (7+3+2) to get 3. You would then have multiplied the 6, the single digit value of the top line, by 3. This would have given you 18, which would in turn add up to 9.

Here is another interesting and important fact.

You will remember that 9 multiplied by any number, or vice versa, will give you a product whose single digit value is 9. Because of this fact, if the upper line in the preceding problem had added up to 9 instead of 6, lines A, B, C, as well as the answer, would have added up to the single digit 9.

Look at this problem:

6012	9
×245	2
30060	9
24048	9
12024	9
1472940	9

It is a nice short cut to remember.

In spite of the fact that the method of checking only the answer—as shown in Section II—may look easier, *do not use it!* Use the method you have just learned. Check *line* by *line.* It will take but seconds longer, and will prevent any chance of a combination of mistakes which might nullify the accuracy of the Rule of Nine.

Here's what could happen if you fail to check line by line as shown on page 11:

Correct Multiplication

786	3
×56	
4716	9
3930	6
44016	6

Incorrect Multiplication

7 8 6	3
×5 6	2
4 7 (2) 6	
3 (8) 3 0	
4 3 0 2 6	6

Notice that one of the circled digits is one digit too large and the other one digit too small. Each of these errors has cancelled out the other so that the single digit 6 in the right hand column actually "proves" this incorrect problem as correct. Don't let this happen to you. Check line by line.

Problems

3623	5628	84294	48221	7221
×254	×229	×3275	×327	×749

SECTION VII

Division Expanded

There is one more step in the Rule of Nine in relation to long division which is a must for the beginner, especially if he is uncertain as to his multiplication. This procedure, if faithfully followed, should prevent you from making the usual multitude of mistakes, mistakes which in the aggregate might nullify the accuracy of the Rule of Nine: Let us see how this works:

```
     (7)    3
   835/257164
       2505     A
         66     B
```

The (7) over the divisor is its single digit value, and may be either retained in the mind or put down as above.

Now to check line A, it is only necessary to multiply (7) by 3 to get 21, whose single digit value is 3. If in adding the 2505 of line A to its single digit value we find it to be 3, line A is correct. Why? Because we are really doing this:

```
   835     7
    ×3     3     (3×7=21=3)
  2505     3
```

Our next step is to bring down the 6. This makes our next number to be divided 666, still too small to contain, or be divided by 835. We therefore show this fact by putting the 0 over the 6 we just brought down. This particular problem was especially selected to illustrate this particular use of the 0. Please, therefore, remember this—as we will refer to it again at the end of this section.

We now bring down the 4 and are ready to proceed with our problem:

```
     (7)   307
    835/257164
        2505      A
         6664     B
         5845     C
          819     D
```

Line C's single digit value is arrived at by multiplying (7) by 7 which gives us 49, or the single digit value 4. We now add the figures 5845 of line C to their single digit value, and if this equals 4 we know line C is correct. The remainder, line D, is checked by adding it with line C to get line B. Your problem is now completed.

If you now check your answer and find it correct by the Rule of Nine, your answer should be correct with one possible exception. This exception would occur only if you had failed to place the 0 after the 3 to show that 666 was too small to be divisible by 835. The Rule of Nine is unable to spot this omission of the 0 inasmuch as 307 and 37 both add to the same single digit 1.

Realizing that is just about his only possibility of error, the average student will be so intently on the lookout for it that he will seldom if ever again fail to place the necessary 0 in his answer.

Now here is the second and quicker way to check long division in which the X is not used. To prove long division, the regular way, we multiply the divisor by the answer and add the remainder. This will give you the dividend which is another name for the number you divided into.

To prove our answer by the Rule of Nine, we follow the same course. That is, we multiply the single digit of the divisor by the single digit of the answer and add the single digit of the remainder.

Looking at the following problem:

```
×———————(8)
|         242
|        ____
(8)   35/8486=8
|        70
|        ___
|        148
|        140
|        ____
|         86
|         70
|         __
|         16
+———————(7)=8
```

To prove it, we would multiply 242 by 35 and add 16. This would give us 8486 because our problem is correct.

To prove it by the Rule of Nine, we once more follow the same course, only this time we multiply the single digit value of 242 by the single digit value of 35, and add the single digit value of 16. So we have 8 × 8 which equals 64 which equals 1; to this 1 we add 7, which gives us 8. If the problem is correct, this 8 should match the single digit value of the dividend, the number we divided into. We now look at 8486 and realize that its single digit value is 8. Our problem is therefore right.

Here is why this works:

```
   242=8
  ×35=8=(8×8=64=1)
 ____
 1210
 726
 ____
 8470=1
  +16=7
 ____
 8486=8
```

Should our dividend, 8486, have added up to any other digit than 8, then the problem would have been wrong. Since we have checked our multiplication step by step, the error could only be in subtraction. The thing to do next is to begin checking the subtractions. Beginning with the first one, in our problem it would be where we subtracted 70 from 84, and work on down until you find your error. When you have corrected it and refinished your problem, check it once again by the Rule of Nine.

When solving a long division problem of which the divisor adds up to the single digit 9, if there is a remainder its single digit value will always equal the single digit value of the dividend.

```
      207                        202
45/ 9325=10=1          333/ 67325=23=5
    90                        666
     325                       725
     315                       666
      10=1                      59=14=5
```

Problems

265/45321   731/32957   572/43282   321/43912

SECTION VIII

Addition, One More Idea

The fastest way to add a column of figures is to carry the single digit value of the first line down to the second and so on, dropping 9's as you go. Do not fail to learn this!

```
→ 285
  358
  837 = 4
 1480 = 4
```

The first line adds to 6 which is carried down to the line below and added to the 8, 5 and 3 to get 4. This 4 is carried down and added to the 837 to get 4, the single digit value of the answer.

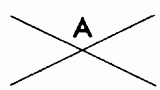
Problems

3275	5329	4331	7823
7329	9122	7821	1135
3221 =	5624 =	5432 =	5421 =

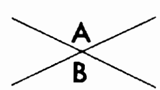
SECTION IX

Division, Some Short Cuts

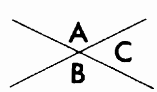
In this section, some short cuts in division will be shown. You will remember that in checking our long division we made an X. We put the single digit value of the number we divided by in the upper space of the X. In this chapter we shall call the digit A and put it in the top of the X like this:



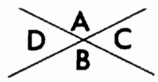
The single digit value of the answer (we shall call it B) we will put in the lower space, opposite the A this way:



We shall let the letter C indicate the single digit value of A×B+remainder. This C we will put in the right hand space of the X in this way:



In the space to the left we shall put a D which represents the single digit value of the dividend, the large number we divided into:



Now here's one more thing to remember before you go on. You may not know it, but any number multiplied by 9 will give you an answer which will add up to the single digit 9. Try this on your multiplication table, or better still, with some large numbers.

Now here are some short cuts in checking division.

All problems without remainders will, if correct, show a 9 in spaces C and D if a 9 should appear in A or B. Once you note this fact you have but to skip the intervening steps and check for a 9 in the D position.

This happens because any number into which 9, or any combination adding up to the single digit 9, will go evenly, that number will itself add up to the single digit 9. A little time will help you digest this and see how simple it really is.

Below are a few examples without remainders where the divisor adds up to 9. Note that in every case the dividend (the large number we divide into) also adds to 9.

8	9 5	9 76
9/72 9	63/315 9	612/46512 9
72	315	4284
		3672
		3672

Here are some others without remainders whose answers add up to 9.

801 9	72 9	36 9
8/6408 9	65/4680 9	413/14868 9
64	455	1239
08	130	2478
8	130	2478

If you work a few of these problems out with the X you will see that the space C in the X must always be 9 because the digit which occupies it is the product of A times B. In the above examples either A or B is 9 so their product must also be 9.

Now, in problems with remainders whose answer or divisor adds up to nine, we take a different short cut.

We know that the product of A times B will give us a 9 and that in the regular process of checking we would add that 9 to the remainder to get the single digit we put in space C. Since we know that the addition of 9 to any number does not change it one bit, what we finally put in space C is the single digit value of the remainder.

Here then is your short cut.

In examples with a remainder, when a 9 appears in spaces A or B, you place the single digit value of the remainder in the space C and check for D.

Another short cut. Whenever your divisor adds up to nine, add up your dividend next. To be correct, your remainder must match it. However, if the dividend adds up to 9, there may be no remainder.

Study the examples below and see if this is not correct.

```
         9
        36
   54/1983   3
      162
      ---
       363
       324
       ---
        39   3
```

```
   9      247
   72/17846   8
      144
      ---
       344
       288
       ---
        566
        504
        ---
         62   8
```

```
          9
        162
   25/4063   4
      25
      ---
      156
      150
      ---
        63
        50
        --
        13   4
```

Problems

425/64325   538/43261   35/3425   321/45632

SECTION X

SUBTRACTION

Subtraction, the Unbelievable

Do not attempt this until you are in complete mastery of the "Rule of Nine" and would appreciate more speed in subtraction

Now we advance to subtracting the single digit value of the middle line from the upper before we convert the upper to a single digit. Start your calculations with the middle line.

2358 1655 703	The middle line adds up to 8 so we brush the 8 in the upper line away with our eye and add the 2, 3 and 5 together to make 1 which is the single digit value of the answer.

Sometimes, the eye can brush away two or three numbers in the upper line. Here's an example of two:

6526 3851 2675	The middle line adds up to 8 so the eye brushes away the 2 and 6 and merely adds the 6 and 5 to get 2, which is the single digit value of the answer.

At other times, the middle single digit will be larger than any digit in the upper number. Also, no upper level digits may combine to exactly equal it. In this case, add any two together and subtract the single digit value of the middle line. Now, add the digit thus obtained to the remaining digits in the top line to find the single digit value of the answer. Here is an example of this:

6544 3751 2793	The middle line adds to 7 which we subtract from 8, the sum of the two 4's, leaving 1, which we add to the remaining 6 and 5 to give us 3 for the single digit value of the answer.

When the upper line has only one digit, and this is smaller than the single digit value, of the middle line, then add 9 to the upper digit and subtract.

2000 1385 615	The lower line adds up to 8. Now add 9 to the 2 in the top line and subtract the 8 to get 3, the single digit value of the answer.

When there are two digits in the upper line, the sum of which is still smaller than the single digit value of the middle number, then you dig up a a phantom 9. Subtract the single digit value of the lower line from this phantom 9 and add the remainder to the digits in the top line to get the single digit value of the answer. Here is an example of this:

3400 1385 2015	The middle line adds to the single digit 8. We subtract this 8 from the phantom 9 to get 1 which we add to the 3 and 4 to get 8, the single digit value of the answer.

It probably dawns on you now that you could use a phantom 9 in all subtraction problems, which is true, but allowing your eyes to brush away numbers is much faster.

If the middle number adds to the single digit 9, all you have to do is forget it and add the upper line. Its single digit value will be the answer.

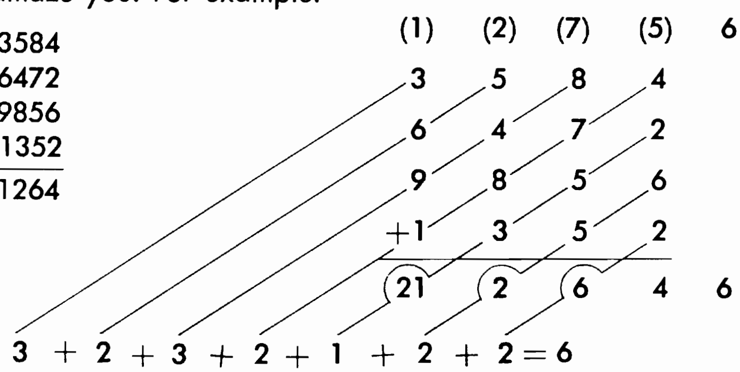
35642 = 2 13401 = 9 22241 2	Two is the correct answer here and rather easily come by.

SECTION XI

Addition, Startling Facts

In adding columns of large figures you can do things which will amaze you. For example:

```
 3584
 6472
 9856
+1352
-----
21264
```



The problem on the left above has been reproduced double-spaced on the right. In the example on the right, the digits have been added vertically, column by column, and their single digit values placed in parentheses above each column.

These upper digits in the parentheses have then been added horizontally and totalled to the single digit 6. This checks with the answer which also adds out to 6. Should these lines have been added horizontally as shown on page 6 and then added vertically to one digit, the answer would also have been 6.

Now for your own amusement add them diagonally, omitting of course the digits in the answer, starting in the lower right hand corner with the 2. Next add the 5 and the 6; next the 3, the 5, and the 2; then the 1, the 8, the 7, and the 4; then the 9, the 4, and the 8; then the 6 and the 5; and finally the 3. The total of all those digits will also add to 6.

Add the twelve outside digits together, and then the four inside ones, and you will find that the two single digits will add up to 6. Add the numbers in the example any way at all, so long as you add them all, and the answer will be 6.

Problems

	3	6	3	4	7		5	7	3	5
	4	7	5	3	4		7	2	1	4
	1	4	5	2	7		5	3	2	7
+	7	2	4	1	2	+	3	5	3	9

SECTION XII

Bankers' Use of This Rule

The queer antics of the Rule of Nine, as shown in Section XI, are the foundation of a practice used by bankers. Few, if any of them, know anything of this rule but its application. Here is the rule.

When balancing your books, should the amount you are out be evenly divisible by 9, the chances are that you transposed some digits, not made a mistake in addition or subtraction.

Remember this. It may some time save you hours of work. The following columns of figures may make this clear to you:

A			B			C		
	2184			2184			2184	
	3156			*6153			*6153	
	2289			2289			*8292	
	+8462			+8462			+8462	
	16091	8		19088	8		25451	8

We will go on the assumption that column A is the one which is correct. Column B has one set of digits transposed, marked with an asterisk. Column C has two which have been garbled, marked with asterisks. A careless person could have copied either B or C the way they are. In all three columns the addition is correct. Now see what happens when we subtract the total of A from that of B or from C.

B	19088	8	C	25451	8
A	−16091	8	A	−16901	8
	2997	9		8550	9

In each the difference adds up to the single digit 9 which, as you know, means that they are divisible evenly by 9. Therefore, when your remainder is evenly divisible by 9 the chances are greatly in favor of your mistake being the result of transposition.

Problems

A	B	C
8336	8336	8336
7239	7239	2379
1358	3158	1358
+3155	+3155	+3155

SECTION XIII

One Last Word

It is never necessary to put more than one digit on paper when checking long division and multiplication. None is needed in addition and subtraction. In the latter two, do the problem first and then check the example by the Rule of 9 as shown on page 16 and finally check the answer.

In long division, put only the single digit of the divisor down and check your multiplication from it step by step as you proceed with your problem. Then check your answer as shown at the top of page 15.

In multiplication, only the single digit value of the top line need be put down. Let's assume we are multiplying 427 by 23. First multiply the entire top line by 3 and then, to prove this multiplication, multiply the single digit value of the top line (4) by 3. Now multiply the top line by 2 and then, to prove this, multiply the single digit value of the top line (4) by 2. Draw the line and add the two multiplications to obtain the final answer. To check this answer multiply the single digit value of the top line (4) by the single digit of the number in the second line (23) whose single digit value is 5.

SECTION XIV

Errors, Their Responsibility

Possibilities for error by this system exist, but they are not great. Of course the careless beginner is faced with the fact that the right combination of errors in the same problem might cause the Rule of Nine to check that problem as correct when it is incorrect. If on the other hand, he faithfully follows the procedures shown in Section VI on multiplication and in Section VII on long division, it is hard to see how even a novice can develop a multiplicity of errors.

Without the Rule of Nine:

Those who recheck a multiplication problem face all the possibilities of error that beset them in the first solution unless they resort to long division. Should they do this, there is greater chance of their division being wrong than their multiplication.

In checking his division he faces the hazards of multiplication.

In adding, he has no choice but to re-add. Mistakes in adding are often made because of mental quirks, which may, for just a few minutes, make him call 7 and 8, 14. In his recheck, he will probably repeat this error and arrive at the same wrong answer.

With the Rule of Nine:

In checking by the Rule of Nine, an entirely different method is used, one which has no likeness to the working of any of his problems. It is simplicity itself. If he reduces all numbers to one digit each step of the way as he should, he will never have to add beyond 8 + 8.

Here are some of the possibilities of error.

If in the answer to a problem, one number has been increased by a certain amount, and another decreased by exactly the same amount, this mistake will not show. Both the right and the wrong answer in this instance will add to the same single digit. It can happen. It has happened. Here is an example:

185762 . . . . 2 175763 2

This will happen so many, many fewer times than will a mistake in reproving by example by the ordinary method that it is hardly worthwhile considering.

In subtraction care must be taken because a 0 and a 9 will add exactly the same—to a 0. Notice the problem below. This possibility of error is easily checked, once it is understood. Cautioning the beginner about this weakness of the Rule of Nine will make him much more careful of this point. In pinpointing his attention on it, you will cause him to make very much fewer errors of this particular sort, and at the same time he will be on the alert to check the rule on this point.

87042	3	9	12		87042	3	9	12
–76065	6		–6		–76065	6		–6
10977			6		10077			6

If you will check, you will find this is not a very probable mistake anyway. The chances are that if the 9 were not brought down, then the 6 would be subtracted from the 7, leaving 1, which would make the answer add up to 7, not 6.

In multiplication, the shifting of one line to the right or left will not change the final single digit as determined by the Rule of Nine.

Notice this:

2842		2842	
3976		3976	
42602	5	6818	5

If the eye did not spot this, no amount of re-multiplying would disclose it. In this case division would be the only sure proof—if no mistakes were made in the division.

If a necessary cipher has been left out of an answer, the Rule of Nine would be totally unable to detect it.

With the above exceptions, the Rule of Nine is foolproof and lightning fast.

Some may harp on the possibility of error. Start doing problems and see if you can honestly turn up one of these exceptions. You may run across one of these oddities, but it will be very seldom.

The great value of this Rule lies in its simplicity, speed, and particularly, its fascination. Remember that anything which will help cut down carelessness is a good thing. The one or two times the Rule of Nine may fool the student in his lifetime will be far outweighed by the good he will reap from it.