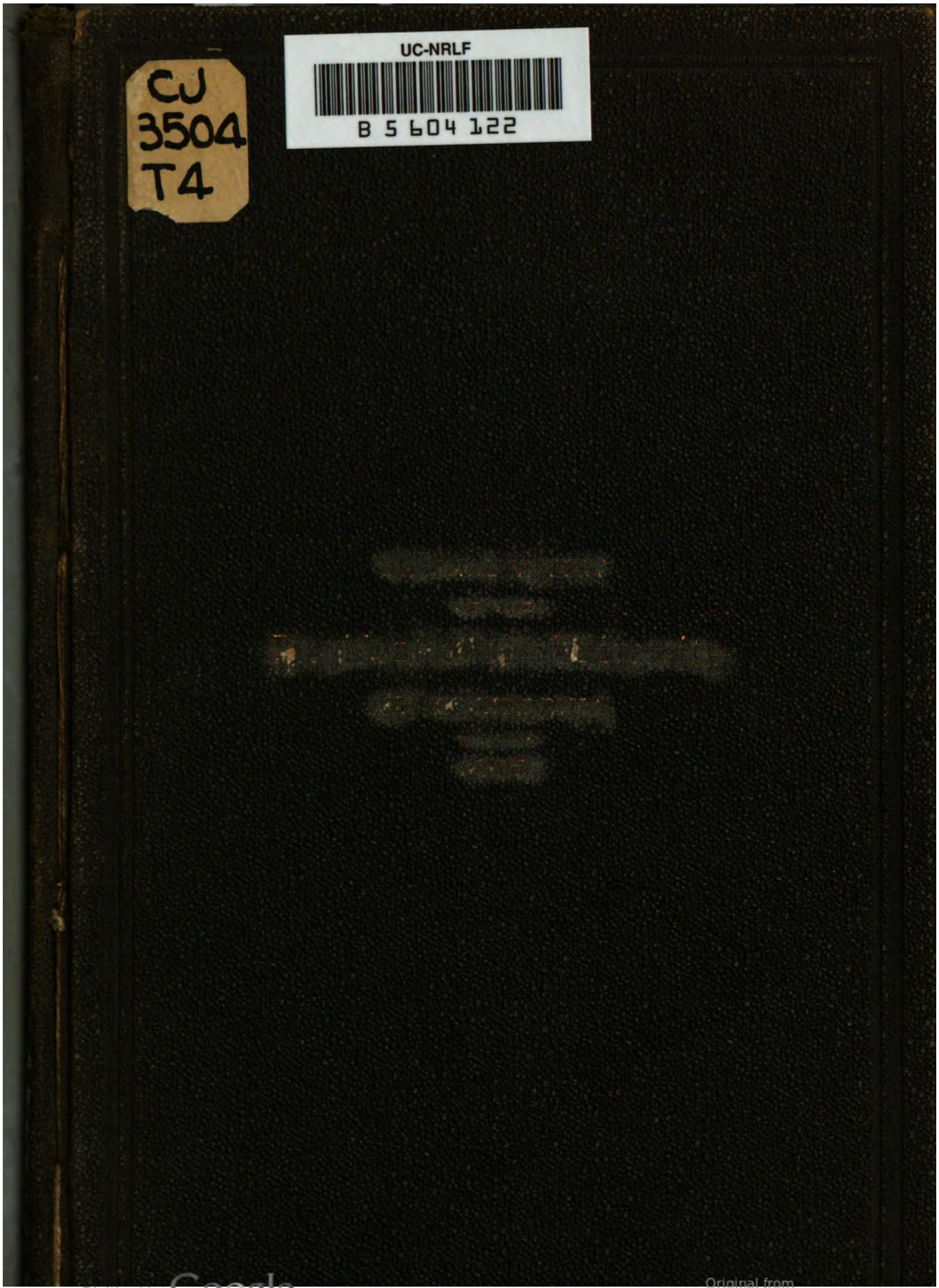
The old numerals, the counting-rods and the swan-pan in China. By Prof. A. Terrien de Lacouperie ...

Terrien de Lacouperie, -1894.
London, 1883.

http://hdl.handle.net/2027/uc1.b5604122



Sir Walter Medhurst
from the author

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THE OLD NUMERALS, THE COUNTING-RODS AND THE SWAN-PAN IN CHINA.

BY

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REPRINTED FROM THE NUMISMATIC CHRONICLE,
VOL. III., THIRD SERIES, PAGES 297—340.

LONDON:
1883.

THE OLD NUMERALS, THE COUNTING-RODS AND THE SWAN-PAN IN CHINA.

1. Researches in the literary productions of the Chinese present considerable difficulty in arriving at the precise date of the original invention or introduction of anything; as in many cases, where we consult a native author, we find reference to some still earlier work, and almost on every point we find a statement connecting the matter with the deeds of the sages of antiquity. The high veneration of the Chinese for the works of the ancients has made them more desirous of elucidating these, than of seeking fame in unbeaten tracks; and some of their most important statements have reached their present shape by an almost innumerable series of increments. Their records, as compiled by later writers, are for this reason open to a criticism of peculiar interest. In their ultra-reverence for the sages of their primitive period, who are supposed to have been the models of all virtue, the possessors of all knowledge (lost after them and sometimes found again), the Chinese cannot admit, nor even suppose, that these sages may have been deficient on some point or

another, or may have been ignorant of anything whatever.[1] A protracted effort has been made by the commentators, in order to find supposed allusions, in the early books, to all that has been known afterwards to them, either by some progress of their own, or by acquisition from foreign countries.

2. The result has been that the commentaries gradually come to occupy in their esteem a position almost equal to the original text. Often by the fault of the transcribers under the Han period (about the Christian era), when the ancient texts were rewritten in a style of writing more ideographical than the old one,[2] the original text being not clear, these commentaries are invaluable, but at the same time they have opened the way to the strengthening of mere suggestions and guesses made according to the wishes and knowledge of the writer, though indifferent to the primitive text. Things to which there are no allusions in the ancient classics or commentaries are not considered worth studying; they are left in the dark, and it is often impossible to find any record concerning them.

3. Besides the difficulties inherent in Chinese literature, we have, sometimes in Europe, to face some *ready-made solutions* which have passed into currency from the unwarranted assertions of some early writers on China unaware of these difficulties, solutions which are still accepted and repeated everywhere. For instance, in regard to the subject of the present note, we read that

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[1] *Vid.* on such a view concerning paper-money, my article *Paper-money of the Ninth Century and supposed Leather Coinage of China*, p. 4 (London, 1883, 8vo. reprinted from *The Numismatic Chronicle*, 3rd series, vol. ii.)

[2] *Vid.* my paper, *The oldest Book of the Chinese*, § 26 (London, 1883, 8vo. reprinted from the *J.R.A.S.* vols. xiv. and xv.)

the Chinese have used the *Swan-pan* from time immemorial,[3] and that they ignore the value of position,[4] two statements which are erroneous and against the weight of evidence gathered in the following pages.

4. In Europe, the Chinese Abacus does not seem to have been known to the learned much before Martini and Spicelius, who gave a description and picture of it in comparison with the European *abacus*.[5] De La Loubère in the valuable relation of his embassy to Siam has also described the implement.[6] And in many other works we

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[3] Joseph Hager, in 1801, thought that the "*suon-puon* had preceded the use of writing in China. The fact (he says) is, that this instrument represents again nothing more than knotted cords, as may appear by the figure . . . But as the processes of subtraction or of addition would have made it necessary to untie the knots at every instance, or to form new ones, moveable knots, or sliding beads, were contrived, which being put on strings of wire, instead of cords, are to this day employed by the common people in China" . . . *Cf.* his book, *An explanation of the elementary characters of the Chinese*, p. x (London, 1801, fol.). But this hypothesis is not justified, and we might say is not true so far as regards the Chinese. Among the MS. papers of Father Brotier, from the early Jesuit missionaries in China, the following is not without an interest of some kind for our subject: Confirmatio systematis temporum prophetic. petita ex magicâ constructione et mysticis numeris figurae *üe Kipan tu*, seu Abaci majorum latruncularum a diluvii tempore usque ad praesentem aetatem in hieroglyphicâ Sinarum traditione servatae. In fol. pp. 18. *Vid.* H. Cordier, *Bibliotheca Sinica*, col. 509.

[4] I have at hand, for the first statement, P. Perny, *Grammaire de la langue Chinoise*, vol. i. p. 108 (Paris, 1873, 8vo.); L. Rodet, *Le Souan-pan et la Banque des Argentiers*, in *Bulletin de la Société Mathématique de France*, vol. viii. 1880, Paris, 8vo.; and for the second, Sir John Davis, *The Chinese*, vol. iii. p. 66 (London, 1844, 32mo.) repeated everywhere.

[5] *Vid. De re literaria Sinensium*, p. 215 (Lugd. Bat. 1660). P. Martin Martini two years previously had described it as invented about 2600 or 2700 B.C., in his *Sinicae Historiae Decas prima*, lib. i. published at Munich in 1658, and at Amsterdam in 1659.

[6] In his second volume, with a figure.

find it properly described and figured,[7] but we must except John Barrow, in his *Travels in China*,[8] who has represented it increasing from left to right instead of the contrary, repeating the same error as a writer in the *Philosophical Transactions* 131 years previously.[9] Properly speaking, the two descriptions of the increase from the right or from the left are inaccurate, as the increase from the right is the true one. In order to avoid any misconception, we shall quote the short description given by E. Bridgman: "The *Swan-pan* consists of an oblong frame of wood, with a bar running lengthwise forming two compartments; through this bar, at right angles, are usually placed seventeen (but sometimes more) small pins, having on each seven balls, five on one side, and two on the other side, of the bar. Any ball in the larger com-

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[7] *Vid.* Sir John Davis, *The Chinese*, vol. iii. p. 66, with figure turned wrong.—Du Halde, *Description de la Chine*, vol. i. p. 276. Staunton, *Embassy to China*, vol. ii. p. 95.—*Le Souane-pane, ou Machine arithmétique des Chinois*, in *De la Chine* by L'Abbé Grosier, liv. x. § 5, Paris, 1819, vol. v. pp. 154–158.—*Comptes Rendus de l'Académie des Sciences*, 16 Juillet, 1860, *Lettre de M. d'Escayrac de Lauture sur le Souan-pan des Chinois.* Purely descriptive.—*The Chinese Abacus*, by J. Goschkewicz (in Russian), Works of the Russian Mission, vol. ii. 1853, or *Ueber das Chinesische Rechnenbret*, in *Arbeiten der Kaiserlich Russischen Gesandtschaft zu Peking über China*, Berlin, 1858, 2 vol. 8vo. Transl. C. Abel, *Vid.* vol. i. pp. 293–310. An able descriptive paper.—A. Westphal, *Ueber die Chinesisch-Japanische Rechenmaschine*, fasc. viii. pp. 27–35. *Ueber das Wahrsagen auf der Rechenmaschine*, ibid. pp. 48–49. *Ueber die Chinesische Swan-pan*, fasc. ix. pp. 43–53; in *M. d. D. G. f. N. und V. Ostasiens*, vol. i. Yokohama, 1873–1876, 8vo.—*On the Abacus of China and Japan*, by Mr. A. Van Name of Newhaven in *J. Am. Or. Soc.* vol. x. Proc. 19th May, 1875, pp. cx.–cxii.

[8] 1804, in 4to. p. 296. The description in Wells Williams, *The Middle Kingdom*, t. ii. p. 146, is also faulty.

[9] Vol. xvi. London, 1688, p. 85.

partment, being placed against the bar, is called unity;[10] and on the left of this they increase, and on the right they decrease,[11] by tens, hundreds, etc; the corresponding balls in the smaller compartment,[12] increase or decrease by fifths, fiftieths, etc."[13] It is on the whole a convenient instrument.

5. The frame with beads on wire used in this country for school children learning to count has come from the French *boullier*, used for the same purpose, and introduced from Russia and Poland[14] at the beginning of the present century by General Poncelet, who had learnt during his captivity how to use the *tchotu*. The Russians had received it from the Mongol conquerors at the end of the middle age.[15]

The *Swan-pan* has been communicated to the various countries which have received the Chinese culture and inventions, but we have no information as to the date of the introduction, except in Japan, where the oldest mention is comparatively modern and dates only from Mori Shigejoshi, a well-known mathematician of the sixteenth century.[16] And a treatise on Mathematics, the *Sampô daizen*, published in 1825, states that the implement was introduced

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[10] Any one in the smaller compartment is called 5.

[11] As in our system of numeration.

[12] *Chinese Chrestomathy* (Macao, 1841), p. 378.

[13] The Swan-pan is used flat down, the wires perpendicular, and the division nearest to the calculator is the largest with five beads.

[14] *Vid.* M. Chasles, *Développements et détails historiques sur divers points du système de l'Abacus*, § 10, p. 17, n. 1; and also Th. H. Martin, *Recherches nouvelles concernant les origines de notre système de numeration écrite* (*Revue Archéologique*, Janvier, 1857), p. 601.

[15] *Cf.* P. Larousse, *Grand Dictionnaire Universel*, vol. i. p. 636.

[16] *Vid.* A. Westphal, *Beitrag zur Geschichte der Mathematik in Japan*, in Mitt. d. D. G. f. N. u. V. O. vol. i. fasc. ix. pp. 54–55.

into China about the middle of the fourteenth century, and passed thence into Japan.

6. No Chinese records of the Abacus or *Swan-pan*=Counting-board, exist as far as we know, in Chinese literature, and its origin is shrouded in mystery. The small collections, the *Poh wuh tche*[17] of the third century, the *Suh poh wuh tche*[18] in ten books of the twelfth century, the *Kwang poh wuh tche*[19] in fifty books, issued in 1607, as well as the great cyclopedias the *Tai ping yü lan*[20] in 1000 books, issued in 983, the *San tsai t'u huey*[21] in 106 books, issued about 1590, the *Yuen kien ley han*[22] in 450 books, issued in 1710, the *Pei Wen yun fu*[23] in 106 books, issued in 1711, and the *Kin ting T'u shu tsih tch'eng*,[24] the monster cyclopedia, in

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[17] 搏物志 by Tchang Hwa, in 10 books.

[18] 續搏物志 by Li Shih; supplement to the preceding.

[19] 廣搏物志 by Tung Sze Tchang; an extension of the first.

[20] 太平御覽 by Li Fang and others; made up of quotations from 1690 principal works and many others. An edition of 1807 is in the library of the Royal Asiatic Society. A most valuable work.

[21] 三才圖會 by Wang K'i. For a rather unfavourable appreciation of this work, *vid.* A. Wylie, *Notes on Chinese Literature*, pp. 149–150.

[22] 淵鑑類函 by several authors. A precious work. The index has been published by Prof. Summers in his *Descriptive Catalogue of the Chinese, Japanese and Mandchu Books in the Library of the India Office* (London, 1872), pp. 11–12. And there is a notice by Mr. Herbert A. Giles, *A Chinese Encyclopedia*, pp. 753–764, in *Time* (a monthly magazine) for Oct. 1882.

[23] 佩文韻府 Compiled under the special superintendence of the Emperor. The largest collection of compound expressions and quotations of phrases where they occur in literature. A notice of the work, by the late W. F. Mayers, is in *China Review*, April, 1878, vol. vi. pp. 288–290.

[24] A valuable description of this wonderful work, now in the British Museum, of which one hundred copies only were printed, has been published by F. W. Mayers, *Bibliography of the Chinese Imperial Collections of Literature*, in *China Review* (Feb.–

10040 books, issued in 1726, do not contain any entry about this ingenious implement. And as each of these large works may be considered as a compendium of Chinese knowledge, this absence is rather startling. It is only in a late work the *Kih tche king yuen*,[25] a cyclopedia of arts and sciences, in 100 books, issued in 1735, that we find a few lines under the title *Swan-pan*. But curiously enough they do not bear on the instrument, and refer only to *swan*, a word meaning "to reckon with counting rods."

7. In the absence of direct records, we have no other line to follow than the negative process. We must try to reach the truth by ascertaining the time when it did exist in China and the time when it was still unknown. And for that purpose we shall consider successively the various data that linguistics and palæography, numismatic notation, mathematical and historical evidence can offer for the elucidation of this interesting problem in the history of the progress of general civilization.[26]

A.—Linguistic and Palæographical Evidence.

8. The Chinese, who profess for their writing a peculiar reverence, almost like a worship, have been so careful in preserving the old forms of their characters, that when

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Apr. 1878), vol. vi.; *cf.* pp. 218-223. The title is translated: "Compendium of Literature and Illustrations, Ancient and Modern, drawn up under Imperial Authority," in Chinese 欽定古今圖書集成.

[25] 格致鏡原 by Tch'in Yuen-lung. It is divided, as justly says Mr. A. Wylie (*Notes on Chinese Literature*, p. 151), into thirty sections; the origin and history of every subject being traced by a long series of quotations from the native literature, ancient and modern.

[26] See below, §§ 42-43.

these forms are available, it is almost always possible to obtain valuable information from their study. In case of inventions or introduction of a new thing, a new character was made, or an older one (with a slight modification) was applied to it. Now the *abacus* has no name in Chinese, and if it was an ancient invention of the Sons of Han, we should certainly find a special group or ideogram in the writing describing it. And if the introduction had taken place at an ancient period such as the Han period, or even later, we should again find for its appellation an older character modified, for the purpose, as usual, by the addition of an ideographic determinative. But such is not the case, and the double name or qualificative *swan-pan*[27] 算盤 "counting dish or board" shows plainly that it is a comparatively modern acquisition for the Chinese.

9. We have nothing to say of the second word *pan*, which is the common expression *(pan-tze)* for "dish, plate." The first word *swan* is the proper one, meaning "to reckon, to plan"[28] *(swan-shu, swan-tu)*; it is sometimes translated in Sino-European dictionaries (Morrison, W. Williams, Eitel)[29] by *abacus* or *swan-pan* by an abuse of extension of meaning; abuse much too frequent, which prevents the proper understanding of the language, and for which the Chinese lexicographers themselves are sometimes responsible; because two words when associated have got a certain meaning, it does not follow that each of them *ipso facto* has gained the meaning when separated.

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[27] The expression *Swan-pan* itself is more modern than the knowledge of the implement. See below § 41.

[28] *Cf. K'ang-hi Tze-tien*: Pu 118+8 str., f. 26.

[29] Medhurst, Glemona, Stent have remained faithful to the Chinese definition.

The official dictionary, the *K'ang-hi Tze-tien* (1716), positively states that *swan* means "to calculate with the counting-tallies," and quotes various passages from the three Rituals in support.

This is conclusive against the supposed high antiquity of the *Swan-pan*. Now let us see if something more can be learnt from this line of study.

10. No conclusion about the antiquity of *swan* can be drawn from the three rituals; the *Tcheu-Li*, the *I-li*, and the *Li-Ki*, as we now have them, are spurious compilations of the Han period; what parts of them are not genuine, we know not. That the greatest part of their contents existed during the last period of the Tcheu dynasty (fourth century B.C.)[30] seems pretty sure, but what existed at an earlier period is doubtful. In the case of the *Tcheu-Li, i.e.* "Institutes of Tcheu," it is commonly said that the authorship is attributed to the Duke of Tcheu (eleventh century), a man of great ability, who was the first lawgiver of the new dynasty which he had largely contributed to establish; now we have a check against this supposed antiquity in the discrepancies presented by this work with the *Tcheu Kwan, i.e.* "The Officers of Tcheu," a genuine chapter of the *Shu-King, i.e.* "Book of History." The explanation is to be found in countless additions and improvements successively introduced; but the proof that such has been the case detracts a great deal from their authority in matters of precise investigation on special points. As the

[30] The supremacy of the Royal or Central Kingdom, ruled by the Tcheu dynasty, over the other states of the Chinese agglomeration, was no longer recognized at that time. The period called that of the Civil Wars extends from 481 B.C. to the foundation of the Empire 255 B.C.

very passages may be interpolations which have crept in from later commentaries, and have got mixed up with the primitive text, their authority, to be trusted, requires confirmation from other quarters.

11. The character 算 *Swan* does not teach us much. It does not seem to be older than the fourth century B.C., when it occurs in the works of Meng-tze (372-289 B.C.) and it seems to be a simplification made at that period of more complicated characters,[31] because of the apparent picture of "hands disposing something" which it had assumed in the writing, a picture which could not have been fancied in the older forms in the previous style of writing. Hü Shen, the learned author of the *Shwoh Wen* (first century A.D.), who was rewarded only a few years ago by a shrine in the temple of Confucius,[32] was unable to find an older form, and gives[33] an etymology, suitable only in the style of writing *Siao-chuen* preconised in his work, which is not supported by the palæographic form of the principal character to which he refers;[34] but he indicates another character 筭 of the same sound and meaning, with which it is sometimes interchanged.[35] This character 筭

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[31] 算, 祘, of which we have Ku-wen forms. *Vid.* Min tsi kih, *Luh shu t'ung*, k. vii. f. 2; viii. f. 4. *Cf.* also k. viii. f. 7.

[32] In 1875, *vid.* T. Watters, *A Guide to the Tablets in a Temple of Confucius*, pp. 98–100 (Shanghai, 1879, 8vo.)

[33] *Vid. Shwoh Wen tchen pen*, edit. of 986, reprinted in 1598, k. x. f. 39*v*.

[34] The sounds agree in no case. The derivation indicated by Hü Shen is 竹 and 貝; the older form of the latter is not the same as in the characters quoted above n. 29, *cf.* Tung Wei Fu, *Tchuen tze Wei* (1691), *s.v.*, they were composed of 廾 and 鼎 or 貝, whereas in the others we recognize 竹, 目, and 廾. *Cf.* Min tsi kih, *l.c.*

[35] *Cf. K'ang-hi Tze-tien*, Pu 118+7; f. 20.

is more interesting for our subject. The description given by Hü Shen himself[36] at once attracts the attention, as more definite in its obscurity than anything else we have seen: "Long of six inches, to calculate calendar and numbers."[37] This points undoubtedly to the "counting-rods," as no other implement would be suited by these words. The character 筭 is apparently of the same period as the preceding; it has no pedigree in the older style of writing, and no other form is known than that given in the *Shwoh Wen* and substantially the same as that in the modern style of writing.

12. Finally there is another character 祘 of the same period described by Hü Shen: "to see anything clearly and take an account of it,"[38] which has a very curious shape. It is described by the learned lexicographer as composed of two characters 示 "reveal;" this looks more like a graphical than an historical etymology; but as we have no example of an older shape nor of its form in an older style of writing, must we be satisfied with it and consider as of later date the definition of "six-inch measure" found in modern books? It is not unlikely that there is a shade of truth in the last statement. Hü Shen,[39] besides the definitions here reproduced of this character and of the first 算, adds in each case: "same as 筭," the second character we have described, and

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[36] *Shwoh Wen tchen pen*, ibid. f. 40 *v*. 筭.

[37] The description continues as follows: made of 竹 tuh *reeds* and 弄 lung *to play*; sound as 常 (*tch'ang*), 弄 (*lung*) is not pronounced, *ibid.* In Sinico-Annamite 算, 筭 are read: *tsan* and 常 *thuong*; in Mandarin: *suan* and *tchang*. 筭 means ideographically "reeds to play with."

[38] *Vid. Shwoh Wen tchen pen*, k. ix. f. 9*v*.

[39] *Vid. O.C.* at the passages referred to above.

we have seen that this one 筭 indicates "slips of wood as counters to reckon with."[40] Now the whole matter shows that the Chinese hierogrammatists have combined the ideographic value of a previous character with its apparent picture to frame a new character having the external pictorial appearance suggesting either the six-inch length of the rods, or the separation required to make an account of anything. A quotation given by the *Shwoh Wen* from the *Yh Tcheu Shu*:[41] "The Sage separates the people to take an account of them, he adjusts and divides to take an account," seems to show that the last view is the proper one.

13. There is another character 籌 *Tch'eu*, which has some claim to be mentioned here; it means notably "to reckon" and also "tallies, counters." But in the *Shwoh Wen* it is only described as "pitching arrows into a jar,"[42] a favourite game or sport from very ancient times. By a most natural extension of ideas, the meaning of the character has come to that stated above.

To resume, we learn from palæography that the Swan-pan is a modern introduction into China, and that previously to this convenient implement, the Chinese used "counting-rods," which were known about the fourth century B.C., and do not seem to be much older.

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[40] *Cf.* Wells Williams, *Syllabic Dictionary of the Chinese Language*, p. 833.

[41] A Record of the Tcheu dynasty. *Vid.* on this work, A. Wylie, *Notes on Chinese Literature*, p. 23. If the quotation in the *Shwoh Wen* is from Hü Shen's pencil, this work cannot have been found in the tomb of the Wei princes, along with the "Bamboo books Annals" in the year A.D. 279, after having remained buried some 575 years, unless Hü Shen had access to a copy which disappeared after him.

[42] Or jar-pitching arrows. *Cf. Shwoh wen tchen pen*, k. x. f. 38. *Cf.* also, Min tsi kih, *Luh shu tung*, k. iv. f. 46.

B.—Numismatic Numerals Evidence.

14. The evidence adduced from the coinage is more momentous than any other, as the coins are the most reliable witnesses of history. In the case of the Chinese coinage of the centuries before the Christian era, their evidence will be found of an exceptional interest, as it is the first time, we believe, in scientific research, that their testimony is called for. This might be, if required, our excuse for the brief explanations, that the novelty of the case requires to be introduced here.[43]

15. Barter in China, as everywhere else, preceded coinage. Gold, silver, copper, silk-cloth, tortoise-shell, precious stones, grains and shells of some kind, were used for that purpose, according to certain regulations afterwards introduced for the measures and equivalents of weight. Various sorts of small implements or tools in bronze, more convenient to pass from hand to hand, were soon preferred to the other materials. Tradition attributes the casting of that kind of objects in ancient times only for the sake of the people impoverished by droughts or

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[43] The only works of any value on the history of Chinese Coinage in ancient times, the paper by Ed. Biot, *Mémoire sur le Système monétaire des Chinois* in *Journal Asiatique*, IIIe ser. vols. iii. and iv. (Paris, 1837, 8vo.); and W. Vissering, *On Chinese Currency, Coin and Paper Money* (Leiden, 1877, 8vo.), are utterly worthless for the coinage previous to the Han period (200 B.C.). They have followed blindly one authority, that of Ma Twan-lin in his *Antiquarian researches* (*Wen hien tung k'ao*), an immense work, wonderful in the variety of its subjects, but which has been much too highly praised by Rémusat, who had not in his time the possibility of verifying its accuracy. We must take a more sober view, and admit that this Chinese author was utterly deficient in criticism. What Dr. Bretschneider says of his Geography, we can repeat of his numismatic notions, that they are full of blunders and confusion.

otherwise. Small spades, adzes and knives,[44] improper for the work for which their shape was intended, and later on, flat rings, were multiplied and entered into currency. Trustworthy statements are scanty for the reasons expressed above (§§ 1, 2). Strict regulations for this barter were issued after the establishment of the Tcheu dynasty (eleventh century B.C.). At the beginning of the sixth century, *Tchwang*, King of Tsu (one of the states of the Chinese confederation), attempted, without success, to make all this differently sized bullion exchangeable, indiscriminately, regardless of its weight.[45]

16. In 523 B.C. the King *King* of Tcheu (the Middle-Kingdom) issued, but without success, the bullion (then coinage), in various sizes and weights regularly proportioned. But the habit of weighing, still in use in the present day for precious metals, was already too strong to be overcome. A fiduciary coinage has never been willingly accepted in China, and the coins, whatever mark they bore, were never taken for more than their intrinsic value without great objection. The above described bullion does not appear to have been turned into a coinage by a regular stamp, before the time of the last-named *King*, and the traditions pointing to an earlier date are obviously spurious improvements on ancient texts which do not bear such a construction.

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[44] Knives were not long ago, and are perhaps still, in use as a currency on the S.W. borders of China. According to R. Wilcox, *Survey of Assam and the neighbouring countries executed in* 1825–6–7–8 (Asiatic Researches, vol. xvii.), "the Khamti and Sing-Pho were supplied by the Kha-Nung with salt and thin iron dhas, the latter forming the currency of the district." The name of *dha*, a small square knife, is obviously connected with the Chinese *tao*, the name of the knife-money.

[45] It was the first attempt in China of a fiduciary money.

Coins were not largely multiplied before the last hundred years of the Contending States period (481–255 B.C.) during the desperate struggle of the various Principalities against the encroachments of the rising State of Ts'in, of which the Ruler, having nearly subdued under his sway the whole country, established the Chinese Empire.

17. Their legends are generally very scanty; the name of a city, or more cities associated for the issue, and besides that, sometimes together or isolated, the intended weight value, and a serial number (of the issues or quantities?) are the only information to be found on the coins of that period. The serial number is, of course, the only one of real interest for the subject of the present paper.

Graphically, excepting the earliest, the figures are very loose. In fact they offer the same carelessness as the other characters of the legends. This is the result of the freedom of issuing coins almost impossible to repress when the limit is the intrinsic value more or less visible, and which has made China the home of counterfeiters. The accompanying table exhibits the figures[46] and their variants, according to the legends of the coins, spades, weeders, and knives of the fourth and third centuries B.C.

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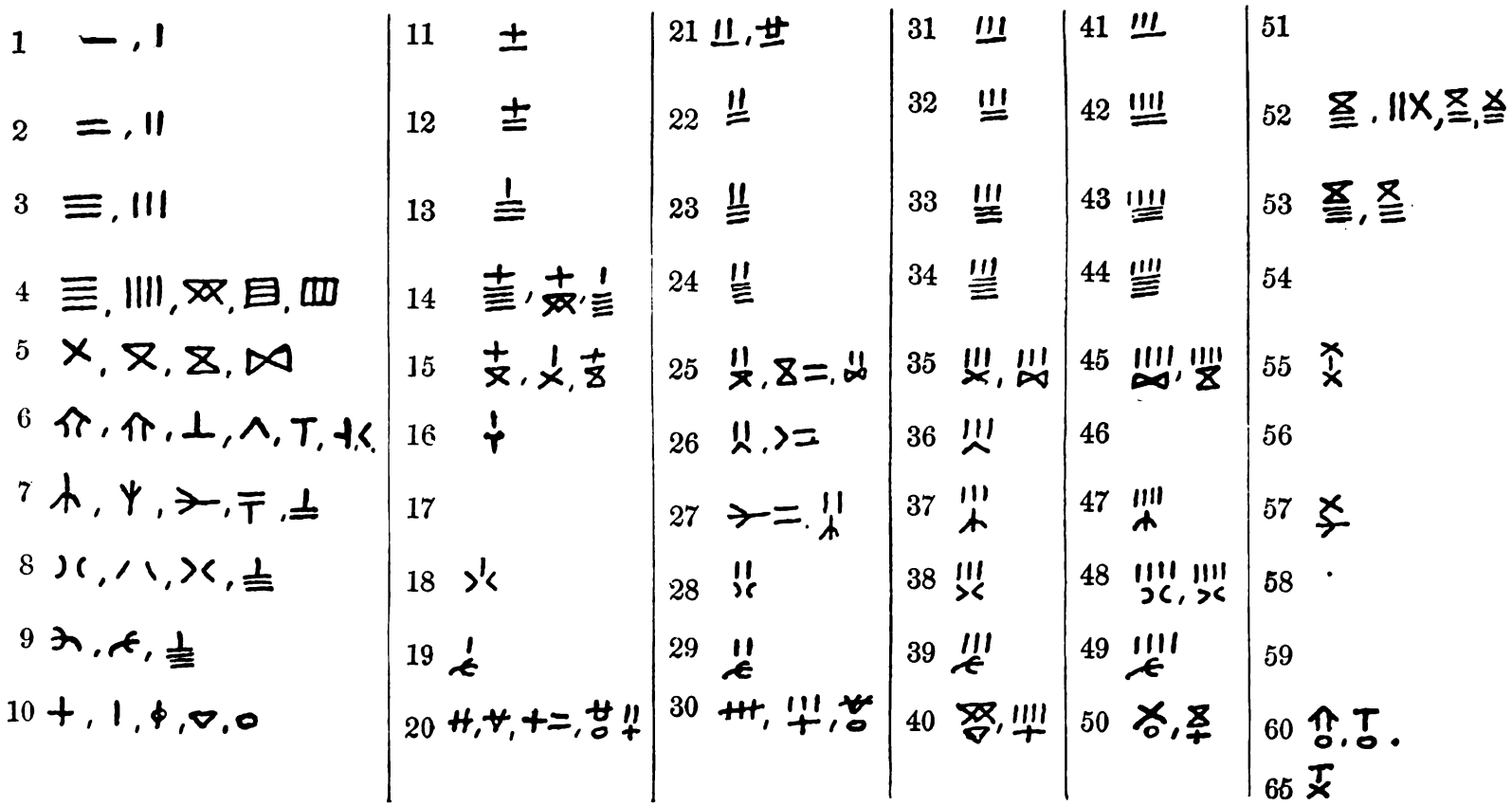
[46] These figures and numbers, which have never before been compiled, even by the Chinese, have been collected by me while preparing the Catalogue of Chinese Coins in the British Museum. They are entirely new material offered to the investigator of mathematical history. They are found on coins issued by the towns of Ping-yang, Wen-yang, Ta-yn, Ping Tcheu, Ping tcheu, Ki-shi, Shang-ching, Tze-tu, Ming, etc. *Cf.* Li Tsin Li, *Ku Tsuen hwei, yuen*, kk. v. vi. vii. viii.; *Su Tsuen hwei*, pei, ii.; and also *Ho Pu Wen tze kao*, k. ii; *Ku kin so kien luh*, kk. i. ii.

18. NUMERALS FROM THE CHINESE COINS OF THE FOURTH AND THIRD CENT. B.C.



19. A necessary remark is that the form 𝍠 for 6, which seems to be a simplification of the more complicated forms for this figure, is most likely the ancestor of 𝍡 𝍢 𝍣 which occur on later coins of the third century B.C.,[47] whereas 𝍠 occurs, on earlier coins which are marked *seriatim*, with other forms for 7, 8, 9. These new shapes for 6, 7, 8 and 9, which recall the principle of the *swan-pan* where the upper bead is worth *five*, were apparently connected with the counting-rods. *Wang Mang*, the usurper, who ruled over China A.D. 9–23, between the two Han dynasties, and to whom all sorts of wild reforms are attributed, caused the revival of a notation older than his time, and in which 6, 7, 8 were indicated by 𝍤, 𝍥, 𝍦. This is again in evident connexion with the counting-rods, and it is worth noticing that these figures composed of straight lines do not appear on the earlier coins of the fifth century, where the ordinary shape (the first of the above list) alone occurs.

20. Hitherto we have dealt only with the single figures; we have now to consider the numbers with two figures as they exhibit the most curious revelations. The collection of examples displayed in the above table shows that they are not exceptional, and that they are genuine specimens of the current system of notation. They are not open to the doubts which may arise from the possibility of having been subjected to emendations, additions or improvements through successive commentators and copyists; they are copied without falsification from coins of the period indicated above, and consequently it follows that we have nothing

[47] Specially on the coins issued by the small state of *Ki*, which protracted the struggle against Ts'in during the greater part of the third century B.C.

to do but to register what they show, *i.e.* the knowledge of the value of position, and a great step towards the use of the zero. This is as curious as unexpected, inasmuch as this progress, which is proved to have been accompanied by such great difficulties in the West, is here the natural outcome and self-improvement of the ordinary and older systems of writing the numbers in full, by dropping the appellatives of values for the mere sake of that brevity always sought for in Chinese.[48] We can see plainly how the process has been going on; for instance in 𝍠 55, the *tens* are still written, though reduced to a single stroke instead of 十 as in 14=𝍧. Too much stress as to the actual date of the beginning of the process of shortening, and then dropping, the appellative of quantities, ought not to be put on that instance, as very likely it is older; the ancient process would have been revived in that special case because the two figures for 5 being superposed could have been mistaken for a character of the writing 𝍡 *hiao* "communicate." Now other instances are to be preferred, such as 𝍢, 𝍣, where the circle (the zero?) or triangle, an abridged form of 十, exhibit undoubtedly the process of transition. These last instances are of the fourth, and the preceding is of the third century B.C. It is quite clear that the knowledge of the value of position existed at that later period.

21. As to the question of its existence in earlier centuries, we find a very curious instance of the growing process. It is in the invaluable chronicle of Tso Kiu Ming, the *Tso Tchuen*, which almost always accompanies the *Tchun Tsiu* of Confucius, that we find it. It was

[48] From a coin of *Kï*. Vid. *Ku Tsuen hwei, yuen*, k. vii. f. 3.

in the thirtieth year of the reign of Duke Siang, of the State of Lu (542 B.C.), about an old man of 73, who did not know how to compute his years, and in answer to a question says that since his birth, which happened at a first moon the first day of the cycle (of 60 days), 445 cycles and $\frac{1}{3}$ of days (or 26660 days) have elapsed. The Music master present having said that consequently the age of the old man was 73 years, the historiographer went on saying: "The character 𠆢 *hai*[49] is composed of *two* at the head and *sixes* in the body of it. If you take the *two* and place it alongside the sixes of the body (亦), you get the number of the man's days."[50] Another officer said: "Then they are 26660." The last number in the text is written in full: "2 myriads 6 thousands 6 hundreds and 6 tens" in column as the current text. But in the character *hai* the three 6 are placed horizontally one next to the other, but the text does not say if the *two*[51] is to be placed on the left (as with the swan-pan, the counting-rods, the commercial figures of the present day) or on the right (as would be the horizontal writing of any sum in full with the ordinary characters). But inasmuch as we know from the coin-instances, that it ought to be placed on the right, this uncertainty is immaterial to the value of

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[49] Now 亥 the 12th of the cycle of 12.

[50] *Vid. Chinese Classics*, ed. Legge, vol. v. pp. 552 and 556. The translator has given in brackets the disposition with the || on the left, but without stating his reasons. But the numbers exhibited by the coinage (cf. 20*c*, 25*b*, 26*b*, 27*a*, 52*b*, of the table above) show that the *two* is to be placed on the right.

[51] Also called the weight-character and written 碼字 or 馬子 or 瑪字 and also 馬式 which is the earliest and appears at the end of the sixteenth century.

position of the three sixes, one for the tens, the next for the hundreds, the next for the thousands and the following 2 for the tens of thousands. This looks more like a puzzle than anything else, and as if the dropping of the appellative of quantities was a mere supposition of the said music master. Unless supported by contemporaneous evidence, we must not see there a proof of a current knowledge of the value by position, but certainly it was a hint, which not long afterwards suggested the possibility of dropping the appellatives and letting the figures stand by themselves. We have seen how usual was the process on the coinage of the fourth and third centuries B.C.

22. Considering again these numbers from the coins, we see that the figures could be written vertically as the ordinary Chinese characters, or horizontally from right to left, the increase by tens going from left to right, contrary to our numerical notation.[52] This was also contrary to the modern Chinese cursive notation, contrary to the order in which were used later on the counting-rods, contrary to the order followed in calculating with the swan-pan, and consequently it offers but a negative proof as to the existence of the counting-board at that time.

23. The last use on coins of these numerals made of straight lines occurs in the sixth century, on the *Wu chu* =5 *chu* coins of the Liang dynasty (A.D. 502-557),[53] with

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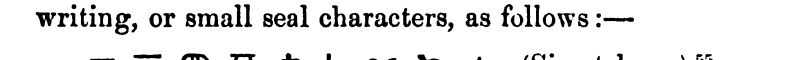
[52] An ancient Chinese work, the 數術記遺 *Shu-shuh-ki-y*, might have helped our investigations. It was written by 徐岳 Siu Yoh of the Han dynasty, and in a rather obscure style; it gives details of the Buddhist numeration, and particularizes fourteen professedly ancient systems of calculation. But it has disappeared after the Tang dynasty, and the work now existing under that title is supposed to be a spurious fabrication. *Vid.* A. Wylie, *Notes on Chinese Literature*, p. 92.

[53] *Cf. Ku tsuen hwei*, Li, k. vii.

the series nearly complete but with some slight alterations as follows:—

where the 9 only is missing. And excepting 𝍠 for 23, which is scarcely satisfactory, we have no means of verifying if the value of position was still in practical and daily use.

24. A long while before the disappearance of the ancient (tally) numerals from the coinage, we meet the ordinary characters used to write the numbers. They occur in isolated cases not in series, from the time of the Han dynasty downwards; and on the coins of the Sung dynasty (420-477 A.D.) they are nearly like their modern shape.[54] They are found first according to the *Siao tchuen* style of writing, or small seal characters, as follows:—

 (Siao tchuen)[55]

一 二 三 四 五 六 七 八 九 十 (modern form)

But these numerals have nothing to do with a knowledge or ignorance of the value of position; they are phonetic expressions used to write the numbers; the appellation of

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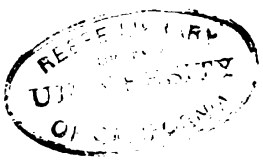
[54] *Cf. Ku tsuen hwei*, Li, k. vi. ff. 5, 6.

[55] Through their oldest shapes (*Ku-wen*) I have traced up the pedigree of these characters to their phonetic origin as intended compounds or as adopted words of the language. The sign for FOUR in its oldest shape is most likely an alteration of a character now written 𠂤 *tse* "regulation of affairs." For FIVE it was adapted from 乂 *ngai* "grass cut." For *Six* it was the primitive character for "mushroom," now fixed into 㚔 *lok* by the addition of the determinative 屮 "a sprout." For *Seven*, it was a phonetic compound of 十 and 乙=SHip+AT or *shat*. For *Eight* it was the word 八 *pat* "separate." For *Nine* it was an alteration of 久 *ku* "ancient." With the exception of the specially made compound character for "seven" all are nothing else than the adaptation of mere homophones.



classes, tens, hundreds, thousands, etc., have to be written after their corresponding number. For instance 1883 is written: ONE *thousand* EIGHT *hundred* EIGHT *tens* THREE. The order is from top to bottom in columns, or from right to left in horizontal lines, as the ordinary writing. They have no connection whatever with the system of the *swan-pan* nor with that of the mathematicians of the middle ages.[56]

25. Coins of a much later period (thirteenth century) exhibit for 1, 2 numerals called *Tasieti shumuh tze*[57] or

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[56] The *Note on the Chinese and Indo-Arabic numeral symbols* by J. R. Logan (App. C to ch. vi. of his *Ethnology of the Indo-Pacific Islands, Language, Part II.* Singapore, 1855, 8vo.), is now antiquated; the learned author in his endeavour to trace a Chinese ancestorship for the European numerals was not aware of the extensive ground covered by this problem, and the solution he proposes is entirely at variance with the facts.—Since this paper was in the hands of the printers, Mr. G. Kleinwachter, unaware of the unsuccessful attempt of Logan, has also ventured the derivation of the European numerals from the Chinese figures. Cf. his articles in the *China Review*, May–June 1883, *The Origin of the Arabic Numerals*, pp. 379–381; July–August, 1883, *More on the Origin of the Arabic numerals and the Introduction of the Sino-Arabic numerals in Europe*, pp. 25–30. Without entering into the many errors of detail in these papers, the transformations of shape proposed by the author are so violent as to allow the derivation by the same process of any character whatever from any other character; if the supposition of so many alterations was admitted without the slightest documentary evidence, there would be an end of scientific method in palæographic matters. But his system is open *ab initio* to a still more sweeping objection, viz. that there is no room for it. The historical and graphical derivation of the European numerals, through Kabul and Persia, from one Indian notation by alphabetical characters, is proved in the whole, and secondary points only remain to be settled. Sir E. Clive Bayley in the *J.R.A.S.* vol. xiv. and xv. *On the Genealogy of Modern Numerals*, parts i. and ii., is the last writer on the subject, and he has done a great deal towards the definitive solution.

[57] 大寫的數目字. *Cf.* Philo. Sinensis (E. Gützlaff), *Notices on Chinese Grammar* (Batavia, 1842, 8vo.), p. 66.

"numerals in capital writing," which consist of a selection of characters similar in sound, but of various meanings, used in official and important documents, to prevent their alteration,[58] or for the sake of ornament and the display of learning. The series runs as follows:—

壹 貳 參 肆 伍 陸 柒 捌 玖 拾.
參 肆 久

The first five of these numerals with the variants here indicated instead of the upper figures which are now current and are the complete forms, occur in books of the sixteenth century; but nothing is known exactly as to the time or when they were finally systematized.

26. The *K'ang-hi Tze-tien*[59] does not give any information on these figures, but under the entry of *Pah* 捌 "Eight," where it says that it is the fictitious character for *eight* in the official documents. The *Tching tze tung*[60] says that in the laws of Ts'in,[61] for the isolated characters of numerals, they took flourished and elegant characters and changed 一 "one" into 壹; 二 "two" into 貳. On this the editors of the *K'ang-hi Tze-tien* remark, "the inscriptions of Ts'in make 一, 二, 三 and from 4 use the ordinary characters, and as *Siu she*[62] is the first who did employ 捌 for 'eight,' it follows that the actual series from 1 to 10 is not made of the ancient characters of Ts'in."

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[58] As being less liable to alteration of any kind, they are used on drafts, pawn-tickets, etc. *Cf.* Herbert A. Giles, *A Glossary of Reference* (Hongkong, 1878, 8vo.), p. 179.

[59] *Cf.* Pu 64+7, f. 53.

[60] An important dictionary published at Nanking in 1634.

[61] The short-lived dynasty of the founder of the Chinese Empire, 255–206 B.C.

[62] 徐氏, who lived under the Posterior Tang dynasty, tenth century.

This is true as far as it goes, for if we look at the inscriptions of the Ts'in period, of which the fac-simile is reproduced in the epigraphical collection *Tsih-Ku-tsi Tchung-ting i-ki kwan-tsi*,[63] we find only a series represented by the following numbers: 一 = 1; 三 = 3; 丿乀 = 8; 卄 = 26; 卌 = 24, etc.

27. Now, if we look at more ancient texts, we are bound to recognize that the habit of writing complicated characters of the same sound instead of the ordinary numerals had begun earlier, but not as a regular series. The examples we find are much more ancient than the time of the Ts'in, and have been handed down by the native palæographers from the oldest MSS. recovered after the Burning of the Books (213 B.C.),[64] and from the ancient inscriptions. Besides the ordinary numerical characters, we find, as might be expected from analogy with the fate of the other characters or words of the writing, three kinds of substitutes: combined phonetics transcribing the spoken sound, homophones, and ornamented characters.[65] As they do not occur on coins, it will be sufficient, for our present purpose of showing the origin of this system, to transcribe the examples in (*Kiai-shu*) strokes of the modern style of writing. 1, 2, 3, are often written 弌, 弍, 弎; for 6 we find 𦒹; for seven 𢒄,

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[63] *Vid.* 積古齋鐘鼎彝器款識, k. ix.

[64] And reproduced in the excellent palæographical dictionary *Luh shụ t'ung* by Min tsi kih (1661), k. ix. ff. 15, 16. Of course the same forms are given in other dictionaries, as the *Luh shu fen luy*, by Fu Lwan Tsiang, and the *Tchuen Tze-Wei* by Tung Wei-fu (1691), in which the characters being classified according to the 214 *pu*, do not require reference of book and page.

[65] On this principle of archaic orthography and the symbolico-syllabic use of the characters in compound, *vid.* my paper *On the history of the Archaic Chinese writing and texts*, p. 4 (London, 1882), *The oldest book of the Chinese*, § 23, 2, 3 (London, 1883).

which is an homonym; for 2 = *nit*, we find the phonetic transcriptions 若九 = N*ok*-T*uk* and 二刂 = N*i*-T*ao*, or N-T, the latter being also an homonym with a distinct meaning; and also for 6 = *luk* we find the phonetic transcription 若干 = N*ok*-K*an* or NK (n=l) for L-K.[66] The symbolico-syllabic characters in compounds had the value for their initials.

They do not present in any way the slightest connection with the system of the swan-pan.

C.—Mathematical Evidence.

28. It is most important to remark that the value of position increasing from right to left (as on the *Swan-pan*), about which so many things have been said, was known at least six centuries ago by the Chinese. In his valuable paper on Chinese Arithmetic,[67] a great scholar, speaking of a native work on mathematics written at the close of the twelfth century, says: "It is not a little remarkable, that while it has been gravely asserted by most respectable authorities in Europe, that the Chinese are ignorant of the meaning of local value, we find here on the contrary, that they have pushed the principle to a degree of refinement unpractised in the West. It may be noticed, too, that instead of the old form of writing the equation, used in Europe, $x^3+15x^2+66x=360$ —the method invented by Hariot, of placing all the significant terms on one side,—is precisely that used by the

[66] *Cf.* Min tsi kih, *Luh shu t'ung*, kk. ix. f. 18; vii. f. 6v.; iv. f. 61v.; vii. f. 7; v. f. 29; ix. f. 6; ix. f. 15; ix. f. 29; vi. f. 31; x. f. 25v.

[67] *The science of the Chinese.* Arithmetic. By O — (A. Wylie). Extracted from the *North China Herald*, 1852, in *The Chinese and Japanese Repository*, 1864, vol. i. pp. 411–417, 448–457, 494–500; vol. ii. pp. 69–73.

Chinese some five centuries earlier;[68] and though this is in itself but a variation in algebraical language, yet it is said by De Morgan to have been the foundation of most important branches in the science. In (Tsin Kiu Shao's *Su shu kiu tchang*, A.D. 1247) Tsin's original work, positive and negative numbers are distinguished by the former being in red ink, and the latter in black; and this custom seems to have been in use long before his time; for we find Liu Huei[69] referring to it in the middle of the third century. It is said to represent the bamboo tally numerals, used in ancient times."[70]

29. In the said work of Tsin Kiu-Shao, the numeral expressions are all written horizontally, and it is from his time that the process seems to have been introduced. And as the same work contains obvious evidence of a direct or indirect North Indian influence,[71] it may be asked, if this influence is not to be recognized in the notation.[72]

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[68] Here is the equation :

丨	1 Cube of Monad.
丨𠆢	15 Square of Monad.
丅工元	66 Monad.
三丅Q太	360 Natural number.

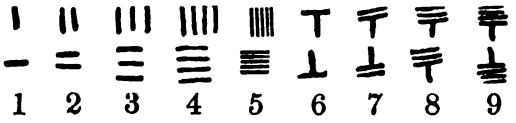
[69] About 263 A.D.

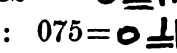
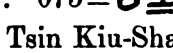
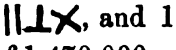
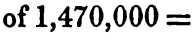
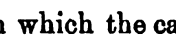
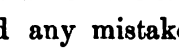
[70] I have quoted word for word, excepting the dates, etc., which I have borrowed from another and later work of the same author; cf. *Notes on Chinese Literature*, pp. 91, 93.

[71] It is in this work that is given for the first time a new formula for the resolution of indeterminate problems, called *Ta yen* 大衍, being analogous to the better known Hindoo process *Cuttaca* which Colebrooke translates "Pulverizer." *Cf.* Wylie, *O.C.* p. 93.

[72] It is in the eighth century that the Hindus were in possession of the value by position and the use of the zero. And it is not unlikely, that the advantage of the abridgment, which as a fact produces the value of position but which the Chinese have not carried to a regular system in practice, had been perceived

The *Yh ku yen tsuan*, by Li Yay, published in 1282,[73] contains in the notes[74] two sets of numbers, made of horizontal or vertical[75] lines as follows:



and the numbers are written as shown in the following instances: 83592=  ; 1082=  ; 20000 =; and the fractions: 075=  , etc., a system which is the same as that of Tsin Kiu-Shao, where we find 64,464 written thus:  , and 1,405,536 =  making a total of 1,470,000 =  , which proves the respective value of the numerals. All this shows the ease with which the calculators could vary their numerals to avoid any mistakes. The last three numerals could be written  =7, 8, 9, also, and interchange with the other forms.

An interesting feature of the preceding examples is the appearance at that period (1247) of the numeral ×′for 4,

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in Kashmir at the time when Chinese numerals were there known, and that the usual contraction of the *sip* + figure for ten into a point or a small circle, has not been without some influence on the improvement of the Indian notation and the systematisation of the zero. Indeed, the name itself is rather suggestive, and I leave to the specialists the care of carrying further the suggestion given in this note.

[73] *Cf.* Wylie, *O.C.*, p. 94.

[74] *Cf.* Ed. Biot, *Note sur la connaissance que les Chinois ont eue de la valeur de position des Chiffres* (*Journal Asiatique*, Décembre, 1839, IIIe série, vol. viii. pp. 497–502). The learned author shows that the Chinese had the knowledge of the value of position at the Mongol period; he had no earlier material at his disposal.

[75] In the *Seng li ta tsuen*, k. xxv. f. 3, are given the numerals with vertical strokes, and not the others. This work was published in 1415 under the Ming dynasty.

which is one of the Ma-tze or cursive numerals of the present time in China, and the main reason why some persons have sought for a Bactrian origin for this modern set of Chinese numerals, a question which we shall consider hereafter.[76]

30. The notation exemplified from the native works of mathematicians of the thirteenth century was totally different from the ancient numismatic numerals, not so much in the shapes of the figures, which excepting 〤〓 are all variations of the old forms, than in direction; it increases from right to left and has to be read from left to right as our numerals.[77] This is a change of primary

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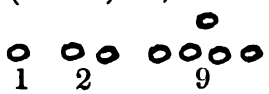
[76] "It may not be generally known that these signs are not Chinese, but Bactrian or Phœnician," *Notes and Queries on China and Japan*, vol. iv. p. 6 (Hong Kong, 1870, 8vo.). See also Herbert A. Giles, *A Glossary of Reference*, p. 179, who wrongly maintains this statement against, which Dr. Bushell had already protested in the same *N. and Q.* vol. iv. n. 103, p. 102. Cf. our remarks below, § 32.

[77] It may prove interesting to read the following account which I translate from the *Grammaire Coréenne* (p. 44) published by the French Missionaries at Yokohama in 1881, about the Corean "Counting Rods" (가 지 산 *Ka-tji san* 枝算). "The Coreans to make their calculations use small rods in non-fixed number, which are placed from right to left, isolated or in groups, in order to represent the units, tens, hundreds, etc. Disposed vertically they indicate each a unit of the order of the units or of the tens, etc.; disposed horizontally, they are worth five units of the same order. Ex.:

𝍠 𝍡 𝍢 𝍣 𝍤 =83,961 (8 3 9 6 1) 𝍥 𝍦 𝍧 𝍨 =2,248 (2 2 4 8)

Instead of rods, they use also small stones, but more often *sapéques* (or coins) and, according to the same rules. For instance:



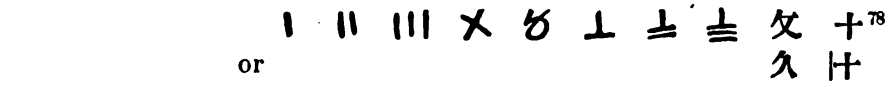
1 2 9

Another system of numeration is indicated by the popular saying: 쳔 헉 빅 립 *Tchyen-hsing Pĕik-rip*, etc., *i.e.* thousand hori-

importance, against the old tradition, and which cannot with any probability have been initiated in China.

31. The cursive numerals now in current use resemble the tally numerals of the middle ages not only in shape, which is nothing else than their cursive alterations with the addition of a new compound for *nine* and a cursive form of the regular numeral for five, but also in order; the numerals increase from right to left and have to be read as in our notation from left to right, in the same way as in the works of the mathematicians we have quoted. The main difference consists in this that the value of position is not implied as understood, and the names of the classes have to be written underneath. For instance 6544 has to written $^{6}_{\text{thousands}}\ ^{5}_{\text{hundreds}}\ ^{4}_{\text{tens}}\ ^{4}$. This is an ample evidence that the value of position in mathematical works is confined to the learned and does not in reality exist in the Chinese mind; the suppression of the written indications of classes is but a temporary dropping for the sake of brevity, a process of constant practice for everything in speech and writing among the sons of Han.

The series runs as follows:—



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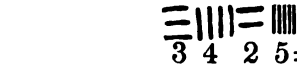
zontal, hundred upright; ten horizontal, units upright." Whence the formula:



3 4 2 5=3,425

But the first system, as widely known as this one, is nearer the abacus which the Corean traders use as the Chinese."

[78] We find them so shaped in the pagination of native works and also in the *Swan fah t'ung tsung* of 1593, k. 1 f. 3. E. C. Bridgman (*Chinese Chrestomathy*, Macao, 1841, 4to.) gives a still more cursive shape than those of our text; the only difference is in 廿 for 10 instead of 十.

32. It is quite clear from a close examination of these shapes, that we do not want to look outside China for their explanation and origin. It looks as if they were a partial revival of the most ancient figures, combined with a cursive alteration and combination of the regular ones. The forms for 1, 2, 3, do not require any remark. That for 4 is an abridged form of 𝌢, the old combination for this numeral; the upper horizontal stroke is ornamental, then remain four strokes which most likely have been simplified into two crossing each other, as in the process of simplification the four rays of such a cross may have been considered as sufficiently suggestive of the required number. That cross-shape for 4,[79] as we have seen, occurs since the 13th century, it does not seem that we have to suppose for it a Bactrian or a Phœnician origin for several reasons. First, the Phœnicians had long disappeared and had never known this shape. Second, the Bactrians, like the Phœnicians, had also ceased to exist long before the adoption by the Chinese mathematicians of this disputed numeral. Therefore we must consider it as a mere coincidence and worthless similarity. On the other hand, we find it with a totally new direction of notation, in which we might see an Indian influence, but India had not this shape and cannot have given it.[80]

The form of 5 is obviously the cursive of 五.

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[79] Tai Tung, the author of the *Luh shu ku*, who lived in the thirteenth century, describes the X, the old sign for five, and does not allude to the use of the same symbol for 4 in his time. See the passage in L. C. Hopkins, *The six Scripts*, a translation (of the introduction of the *Luh shu ku*, Amoy, 1881, 8vo.), p. 15, n.

[80] *Cf.* the comparative table I., given by Sir E. Clive Bayley, *On the Genealogy of Modern Numerals*, Part i. in *J.R.A.S.* vol. xiv. (1882).

For 6, 7, and 8 we have the familiar combinations of straight lines, met with on early coins and in the mathematical works of the middle ages.

The form for 9 is a composite made of the preceding shapes of 5 and 4 superposed, and as to 10 it is the ordinary cross which does not vary.[81]

33. Very little information is available on these cursive numerals. Their name *Su-tcheu Ma shumuh tze*, or "Ma numerals of Su-tcheu,"[82] indicates perhaps the place where they were invented. As Su-tcheu, the great and rich manufacturing town of Kiang-su province, received its name under the Ming dynasty,[83] and as we find these cursive numerals quoted in the *Swan fa tung tsung* of 1593, it seems that we have in this concurrence *primâ facie* evidence of the sixteenth century being the time of their invention.[84] On the other hand, they have various other names, but they are of no help in the solution of the question.

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[81] They are abbreviated forms used to facilitate the writing and expedite the drawing of accounts.

[82] 蘇州碼數目字 *cf.* Philosinensis, *O.C.* p. 67, who indicates also another name 花碼的數字 *Hwa-ma-ti Shu-tze*.

[83] It bore the same name under the Sui dynasty (sixth century), but was called otherwise under the subsequent dynasties. *Cf.* G. M. H. Playfair, *The Cities and Towns of China* (Hong Kong, 1879, 8vo.), n. 6666.

[84] In the name *Hwa-Ma-ti Shu-tze*, *Ma* means "weight," and it seems to me that *hwa* is here a word of disparagement showing that they are of a lower standard than the plain numerals; *hwa* should be taken here with its meaning of "indistinct vision." This view is justified by another name *Ngan-ma* 暗馬 instead of *hwa ma*, in Wylie's "Compendium of Mathematics" (in Chinese 數學啟蒙 *Shu hioh k'i mung*, Shanghai, 1853); *ngan* means "obscure, secret," and corresponds to *hwa*; though *ma* is there deprived of the ideographic determinative (石 "stone") which precises its sound in the sense of "weight," we have no doubt that *hwa-ma* and *ngan-ma* have the same meaning in the com-

D.—Historical Evidence.

34. The historical traditions concerning the counting-rods and the *swan-pan*, though very few, are not without interest.

We can dismiss without difficulty the statement according to which the *swan-pan* was invented by Li Sheu, the chief mathematician of the mythic emperor Nai Hwang-ti

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pound name. We do not see any sufficient reason to suppose with Bazin (*Grammaire Mandarine*, p. 31) that *ngan-ma* is the name of an unknown foreign country. In the most ancient work where we find them, they bear that name of *ngan-ma* (*cf. Swan fah t'ung tsung* (1593) k. i. f. 3v.) and also *ngan-tze ma-shu* 暗子馬數, which most likely, as the above-quoted names, means nothing else than abridged numerals for weights. We find also the Ma-tze, called 碼字, 馬字, 瑪字 and also 馬式. A non impossible supposition to explain the variations of *ma*, should be that this would be the name of a mathematician who made these abbreviations, or at least who improved them and gave them the regularity and convenience they present for quick calculation. And it is not unlikely that an European influence should have acted there. The *Swan fah ting tsung* was published in 1593, or eleven years after the arrival of Matteo Ricci, in Chinese 利瑪竇 Li Ma-teu, who devoted his first efforts to the subject of mathematics, and translated a treatise on European arithmetic as a preliminary step. He also translated Euclid. It will be remarked that the first character of his surname, Ma-teu, is also used for the Ma-tze numerals. The author of the article *Arithmetic* in the *Encyclopædia Britannica* (eighth edit.) has not overlooked a European influence when he writes the following interesting passage: "About the close of the seventeenth century the Jesuit missionaries Bouvet, Gerbillon, and others, then residing at the court at Peking, and able mathematicians, appear to have still further improved the numeral symbols of the Chinese traders, and reduced the whole system to a degree of simplicity and elegance of form scarcely inferior to that of our modern ciphers. With these abbreviated characters they printed at the Imperial Press, Vlacq's 'Table of Logarithms,' extending to ten places of decimals, in a beautiful volume, of which a copy was presented by Father Gaubil, on his return to Europe, about the year 1750, to the Royal Society of London."

(Nakhunta). This tradition, which we find still repeated in recent European books of sinology,[85] has been developed out of another one which has not any better foundation,[86] and according to which the said Emperor Hwang-ti commissioned Li Sheu to invent mathematics and to lay down the principles of calculation.[87] The mythological growth of the legend is here apparent, the translation of the name of *Li sheu* "chief mathematician" speaks for itself, and the "swan-pan" business has made its appearance through the single word 算 *swan* "to reckon," used in the text of the record,[88] according to the process we have mentioned above (§ 2).

35. The invention of the "Nine sections of arithmetic"

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[85] *Vid.* for instance P. Perny, *Grammaire Chinoise*, vol. i. p. 108.

[86] Tch'ing Ta-wei in his work *Swan fah t'ung tsung*, published in A.D. 1593, states very clearly that the ancients did not know the Swan-pan. *Vid. kiuen* xii. f. 9v.

[87] In the great historical compilation, called *T'ung kien kang muh* under the direction of the celebrated Tchu Hi (A.D. 1130–1200), and which is considered as the standard History of China (*cf.* Mayers' *Chinese Reader's Manual*, i. n. 79), it is stated, under the reign of Hwang-ti, that he caused his minister, Li-sheu, to form the *Kiu-tchang* "Nine sections of Arithmetic." These nine sections, which have formed the nucleus of arithmetical science in China, contain several things which deserve attention. Divided into 20 phrases by the great sinologist A. Wylie, they exhibit allusions to the quadrature of the circle and to plane mensuration; one gives the ratio of the hypothenuse to the short sides of a right-angled triangle; others state that a quadrangle bounding the three angles contains double the area of the said triangle, and that the whole is equal to the sum of the several parts; one is the well-known 47th proposition of the first book of Euclid; the application of trigonometry to the measurement of distant objects, and the fundamental principle upon which the area of the circle is calculated, were also known to the author, as well as some ancient instrument for representing the appearance of the heavens and earth.

[88] *Vid. Liu Fung sze she*, in *Kih tche king yuen* 格致鏡原, k. 49, f. 7. A Cyclopædia compiled in 1735.

is attributed to Tcheu Kung,[89] and he is reputed to have also invented the beginnings of the Swan-pan.[90] This statement, which is to be considered as an ingenious device of a recent writer to combine the reputed authorship of the ancient sage with the more sober notions of later times, has no other support, than the part played by Tcheu kung in the exposition of the said "nine sections," and we hear no more of the Swan-pan.

It is of the counting-rods that we hear in history. The oldest references refer to the fourth century B.C., and it seems by the rather contemptuous manner with which they are mentioned, that they were considered as a new invention or improvement for the facility of calculation by the unlearned. This is important because it agrees with the indications derived from palæography and numismatics, which tend to show that the counting-rods do not seem to be older than the fourth century B.C. With this new information from a different quarter, the question may be considered as pretty well settled.

36. Hiao-tze, the ruler of Ts'in from 361 to 337 B.C., who had proclaimed[91] offers of high reward to men of ability from other States, considered[92] that good mathematicians ought not to use counting stalks (籌策). It will be remarked that the written expressions 算, 筭, do not seem to have existed at the time.

Tchwang tze (born about 330 B.C.), the author of such childish and useless speculations,[93] is reputed not to have

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[89] On this great man, see above § 10.

[90] 制算盤之始. *Vid. Liu Fung sze she*, ibid.

[91] *Cf.* Mayers, *Chinese R.M.*, i. n. 845.

[92] *Cf. Tai Ping yü lan*, k. 750, f. 3v.

[93] The work which goes by his name, though many parts are not his (*cf.* Wylie, *Notes on Chinese Literature*, p. 174), the *Nan*

used the counting-rods.[91] This second tradition about the counting-rods does not come from the same work, nor the same class of information as the preceding. We have not to connect the second with the first, and we have only to consider it as a negative indication, which might prove that the practical knowledge of the implements had not yet reached the region of Liang, the native country of the philosopher.

37. The next tradition in chronological order that we meet with contains some material information on these famous rods. Tchao T'o (B.C. 240–137),[95] formerly officer of the first Emperor She Hwang-ti, and who ruled over Kwang-tung and Kwang-si as an independent sovereign, had several sorts of counting-rods made to suit his fancy, when he went to the South. This would be about 215 B.C. The Emperor Ngan (397–419 A.D.) of the Eastern Tsin dynasty kept them preciously in his Museum of Antiquities.[96] They were one cubit long, some were white, made of bone, others were black, made of horn. Under the reign of Wu-Ti (140–86 B.C.) of the Han dynasty, Sang Hung was renowned about 118 B.C. for his ability in using the counting-rods for his calendaric calculations.[97]

38. In the annals of the Western Tsin dynasty (A.D. 265–317), it is recorded that Wang Jung, a minister of Hwei-Ti who ruled A.D. 290–307, when he had taken in

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hwa tchen king, has been rewritten lately in English by Mr. F. H. Balfour, *The Divine Classic of Nan hwa*, Shanghai, 1881, 8vo.

[94] *Vid. Yuen kien Lei han*, k. 331, f. 12v.

[95] On Tchao T'o Prince of Yüeh, *vid.* a short biographical notice in Mayers' *Chinese R.M.*, i. n. 50.

[96] *Cf. Tai Ping yü lan*, k. 750, f. 3v. The expression here used is: 筭策; not yet the *swan* of Swan-pan.

[97] *Cf. Tai Ping yü lan*, k. 750, f. 1. The expression used is 筭.

hand his ivory tallies, he spent his nights in calculating, as if he could not stop.[98] This is quoted as a proof of his great wealth, as he used his tallies to calculate his income.[99] The expression "reckon with ivory tallies" 牙籌計, which still remains in literature as an allusion to wealth,[100] is extracted from the above statement.

During the reign of the Emperor Tch'eng (326–343 A.D.) of the following dynasty (the Eastern Tsin), the counting-rods were made of wood, of ivory or of iron. Under the Wei dynasty the Emperor Siuen Wu (500–516 A.D.) regulated the currency, and made counting-rods cast in iron for the use of the people.[101]

39. It will be deemed unnecessary to accumulate such proofs of the use of the counting-rods during the following centuries; it is plain enough that the swan-pan was not known, and we shall jump at once to the last period where we find the counting-rods in use; it is most likely when the Swan-pan made its appearance. In the *Meng K'i pih t'an*, a work compiled about the middle of the eleventh century,[102] we hear of an able mathematician,[103] who could move his bamboo tallies *as if they were "flying men"* (飛人 =puppets?), and this so quickly that the eye could not

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[98] *Cf. Kih tche king yuen*, k. 49, f. 7. *Cf.* also *Tai Ping yü lan*, k. 750, f. 2.

[99] *Vid.* a short biography of this man, one of the Seven Worthies, in Mayers' *Chinese R. M.*, i. n. 799.

[100] Callery, *Dictionnaire Encyclopédique de la langue Chinoise*, (an adaptation of the *Pei wen yun fu*) vol. i. (only published Paris, 1844, 8vo), p. 53–54, has been mistaken on the origin of this expression.

[101] *Cf.* 淸異錄 in *Kih tche king yuen*, ibid. Here the expression used is 算子.

[102] A. Wylie, *Notes on Chinese Literature*, p. 131.

[103] Wei Poh, a man of Hwai-nan.

follow the moving nor see anything before the account or result was obtained.[104]

Now this description is exactly what we should expect of a man knowing the Swan-pan, perhaps only by hearsay, and who wants to show that the tallies had been practised with the same peculiarities as those offered by the counting-board, where the beads are really moved as flying men, and the result only can be seen.[105] This assumption of ours is confirmed, to a certain extent, by this fact, that at the beginning of the Kin dynasty (twelfth century),[106] mention is made, with praise, of the ability to move the bamboo tallies and dispose the written strokes in order that the eyes might see the calculation. As if in opposition to the Swan-pan which has not this advantage.

These conflicting descriptions show most distinctly the appearance of the counting-board, and the usual struggle of Chinese conservatism, to uphold their ancient systems and to find in them the qualities which are conspicuous by their display or absence in the new.

40. Afterwards we hear no more of the counting-rods or bamboo tallies, as they were ousted by the more convenient swan-pan, but we find nowhere a record of its introduction, for which the Chinese are most likely indebted to foreigners. It is under the Mongol period that we find the notation in strokes, horizontally to be read from left to right as in our notation, and as in the swan-pan practice. The two processes seem to be dependent one on the other or at least narrowly connected, and their parallelism is highly suggestive.

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[104] *Yuen kien lei han*, k. 331, f. 11.
[105] *Yuen kien lei han*, k. 331, f. 11v.
[106] The *Kin* dynasty ruled over the North of China from 1115 to 1234 A.D.

41. An acute student and one of the most voluminous writers on astronomical and mathematical matters, Mei Wuh-ngan, at the end of the seventeenth century, directed his inquiries to ancient calculating instruments;[107] in a special work in which are embodied the results of his researches, which I have not seen, he shows that the use of the abacus in China is comparatively recent, probably not earlier than the twelfth century.

The *Swan fah t'ung tsung*, published in 1593, enables me to give a confirmation of the statement of Mei Wuh-ngan; in a bibliographical list of works on mathematics which I find in the last book or *kiuen* of this treatise are quoted[108] the titles of two works published during the period Shun-hi (*i.e.* A.D. 1174–1190), the *Pan tchu tsih*[109] and the *Tseu pan tsih*,[110] which are the earliest describing the counting *pan* or board. It will be remarked that the compound expression *swan-pan* was not yet made, and that the implement is still described as late as the sixteenth century by the name of *Pan shih*[111] "board to measure." The two works just quoted are called "Collected notes on the board—beads," and "Collected notes on moving the board;" these titles show how necessary it was to describe the new implement.

Conclusion.

42. The various inquiries we have instituted in Linguistics and Palæography, ancient and modern numis-

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[107] In his work 古算器考 "Inquiry regarding ancient calculating instruments." *Vid.* A. Wylie, *Notes on Chinese Literature*, p. 91.

[108] *Vid.* kiuen xii. f. 25v.

[109] 盤珠集.

[110] 走盤集.

[111] 盤式. *Cf. Swan fah t'ung tsung*, k. ii. f. 1.—Cf. *páṭhi*, the common Sanskrit term for the abacus.

matics, mathematical works and historical traditions, have all tended to the same result, a convergence of negative evidence against the supposed antiquity of the Swan-pan in China.

Its name itself is quite modern, and the ancient words meaning "to calculate or reckon" indicate the use of counting rods and tallies from the fourth century B.C. and nothing of any board of any kind.

The coin legends of the fourth and third centuries B.C. show a curious and special series of numerals, obviously connected with the counting-rods by their shapes made of straight strokes, and exhibiting a knowledge of place-value which seems to be supported by an example of two centuries earlier. These numerals, as well as the regular ones, were disposed like the writing from right to left, or vertically, and not at all in the *swan-pan* order. The regular numerals which occur on coins since the first century A.D. no more exhibit series over ten. But the knowledge of the value of position and a limited use of the zero, both gained by abridgment, though imperfect as they were, have no more been lost. The numbers on these coins were written with the ornamented numerals which we have seen are nothing else than the written words, and they have nothing to do with any system connected with the *Swan-pan*.

In another line of research we have seen that a great scholar, Ts'in Kiu Shao of the thirteenth century, almost the only mathematician of the Sung dynasty, did use numerals, like the ancient numismatic ones, made of straight strokes with place-value, but in reversed order and increasing from right to left and read as the swan-pan and our notation; it is worth remarking that the learned author does not introduce these numerals as newly in-

vented or improved, and that his silence on the subject implies an extensive and practical use of them by all his readers in his time. The cursive characters which later on are introduced to his readers by the author of the *Swan fah t'ung tsung* (1593) are abridged from the preceding, but their practical use does not necessarily imply place-value, and they do not seem to be of foreign origin.[112]

The historical traditions, excepting those of the mythical period, bear only on the counting-rods, which were used with great ability from the fourth century B.C. to the twelfth century A.D., where a curious conflict of testimony in favour of the ancient tallies shows without doubt that the counting-board being introduced at that time, the conservative Chinese endeavoured to uphold the qualities of the ancient instruments, and to find in them not only the same but also some more advantages than in the newly introduced implement. In accordance with this result, the oldest works describing the counting-board appeared at the end of the twelfth century, and an eminent Chinese mathematician, who investigated the matter two hundred years ago, and could dispose of other material than we do, has arrived at the same conclusion, viz. that the *Swan-pan* did not exist in China previously to the twelfth century.

43. Now the question arises to know from where the Chinese have obtained the abacus, as there is no doubt that it is a foreign introduction. Not direct from India; it is principally about or before the time[113] of the celebrated Buddhist astronomer Ih Heng 一行 (717 A.D.) that the

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[112] On a probable European influence on this simplification, see above n. 84.

[113] *Vid.* A. Wylie, in *Chinese and Japanese Repository*, p. 416. The other traces of Indian influence on Chinese mathematics are posterior to the introduction of the Swan-pan. *Vid.* above § 28.

Chinese received what little they seem to have obtained from India.[114] We have seen that the *Swan-pan* was not known till four centuries afterwards, and if we consider what China was at the time, not only surrounded in the north, but half swallowed by two powerful states, the Tangut or Ho-si and the Liao and Kin Tartars, in which learning and improvements were far from being despised, and that one had relations with central and western Asia, the probability of its western origin begins to appear. The Tangutans were great traders and carried extensive relations for that purpose; their civilization had a good deal of Indian in it; it is not unlikely that through them the use of this useful instrument was carried to the knowledge of some Chinese. On the other hand, there is a general remark to make on this late appearance of the Swan-pan. It occurs after the Tang dynasty, after so many foreign elements and notions had found their way to China. The Arabs and Persians at Canton in the eighth century, the Nestorians in the north-west, the relations with Central Asia not interrupted since the same dynasty, present so many channels by which the Chinese have obtained many notions and elements of progress.

The *Swan-pan* was undoubtedly used in China at the time of Ser Marco Polo, but no mention is made of it in his marvellous book. However, the other absence of reference

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[114] When M. Reinaud (*Mémoire sur l'Inde*, p. 301; Th. H. Martin, *Recherches Nouvelles concernant les Origines de notre numération écrite*, p. 605, in *Revue Archéologique*, 1857, Paris, 8vo.) said that the Chinese borrowed the system of the value by position and the use of the zero, from the Indians after the fifth century, he was not aware of the peculiarities offered by the numeration on ancient coins which we have seen above §§ 17–21. The reverse may have been the case to a certain extent. See above n. 72.

by the great Venetian traveller to the Chinese art of printing, which was very flourishing in his time, does not permit us to draw any inference from it. We might suggest that he had some dubious reasons to do so; as it is a curious and rather suspicious coincidence that a connection of some kind might be traced up to Venice, for the European art of printing and the knowledge of a peculiarity of the Chinese abacus in the practice of the Bank[115] of the bankers.

Having shown in the preceding pages that the Chinese abacus is not of Chinese origin, and that, on the contrary, it is of comparatively modern introduction (twelfth century) into the Middle Kingdom, two results opposed to the opinion hitherto expressed in Europe, we should pass beyond the scope of the present paper and our own capacity, should we go into the rather complicated historical problem presented by the European Abacus and

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[115] The system of calculating on the *Swan-pan* is much like that which the authors of the fifteenth century called *super lineas et per projectiles*, used in the middle age by the *Argentarii* (Mr. Trentlein has explained this calculation in his work *Das Rechnen im 16ten Jahrhundert*, 1st fasc. of *Abhandlungen zur Geschichte der Mathematik*); their calculations were made on a small table, a *bank*, whence "faire la banque" and later on the qualification of *bankers* given to the *Argentarii*. On the table were traced horizontal lines representing the different orders of decimal units; pebbles or *projectiles* were placed on the lines to indicate the units of the line, but the pebbles placed over were worth 5. This peculiarity has been considered lately (*cf.* L. Rodet, *Le Souan-pan des Chinois et la Banque des Argentiers*, pp. 165, 166, 168, in *Bulletin de la Société Mathématique de France*, t. viii. (1880, Paris, 8vo.) as of prime importance, inasmuch as it exists in the Swan-pan calculation, without any attempt at a connection, which, however, does not seem unlikely, if the Venetian traders in Central Asia have brought back that notion to their brethren. Are not the Venetians those who have renewed the ancient trade and began again the business of the Argentarii?

the Indian *Páṭhi*, with its further complication of the undefined (improved combination of the earlier dust-writing and pebbles- (or cowries-)heaping boards) Asiatic antecedent of the latter and of the Chinese *Swan-pan*, and the questions of derivation, improvement, or parallel descent implied by their obvious connection.

TERRIEN DE LACOUPERIE.

LONDON, *May*, 1883.

For easier reference, I add a summary of the whole paper.

PROLEGOMENA.—§§ 1. Curious difficulty of researches in Chinese literature.—2. Commentaries and texts.—3. Ready-made solutions in Europe. Another difficulty.—4. The Swan-pan known in Europe.—5. In Russia, France, England; in Japan.—6. No special record of the Swan-pan.—7. We shall follow the negative method.

A. LINGUISTIC AND PALÆOGRAPHICAL EVIDENCES.—§§ 8. No special pictorial character for the "counting-board."—9. *Pan* and *swan* in the rituals.—10.—The three Rituals, spurious compilations of the Han period.—11. *Swan*=counting-rods.—12. Another character *Swan*.—13. The *Sheu* or tallies.

B. NUMISMATIC NUMERALS EVIDENCE.—§§ 14. Evidence of ancient Chinese coins; first time adduced.—15. Barter at early period.—16. Coinage begins in the sixth century B.C.—17. Characteristics of the legends.—18. Table of the numerals of the fourth and third centuries B.C.—19. Remarks on the single figures.—20. The double figures exhibit the *value by position*.—21. An example of the value by position in the sixth century B.C.—22. Order of these numerals contrary to the *Swan-pan*.—23. These numerals still used in the sixth century A.D.—24. Numerals in small seal character contrary to the *Swan-pan*.—25. Ornamented numerals.—26. Their origin according to the K'ang-hi Tze-tien.—27. According to palæography. No connection with the Swan-pan.

C. Mathematical evidence.—28. Value by position known in the thirteenth century.—29. Instance of numbers and numerals. X for 4.—30. Different in order from the ancients.—31. The cursive numerals.—32. Their Chinese origin.—33. Their various names.

D. Historical evidence.—§§ 34. Mystic traditions worthless.—35. References to the counting-rods.—36. Hiao-tze (fourth century), Tchwang tze (fourth century).—37. Tch'ao T'o (third century), Sang Hung (second century B.C.).—38. Wang Jung (third century A.D.) and others of the fourth and sixth centuries.—39. Conflicting evidence of the twelfth century.—40. Corresponds with the new direction of numerals.—41. Other proofs in favour of the twelfth century.

Conclusions.—§§ 42. Résumé of the various evidence.—43. The Swan-pan known in China from Central Asia in the twelfth century.